# Practice Exam 6 (Business)

**Question 1**

The grouped frequency table below shows the number of cars sold per week in a car showroom during a period of $200$ working days.

| Number of cars sold | Frequency |
|---|---|
| $26-40$ | $20$ |
| $41-45$ | $30$ |
| $46-50$ | $55$ |
| $51-55$ | $50$ |
| $56-60$ | $35$ |
| $61-75$ | $10$ |

(i) Draw a histogram to illustrate this set of data. **[6]**

(ii) Calculate the mean number of cars sold per day. **[4]**

(iii) Construct a cumulative frequency table for the data. **[2]**

(iv) Find the lower quartile of the data. **[2]**

(v) State the modal class of the number of cars sold each week. **[1]**

**Question 2**

The functions $f$ and $g$ are defined as follows:

$$f(x) = x^2 - 1$$
$$g(x) = \cos \pi x$$

(i) Show that $f$ has no inverse.

State why the domain $x \geq 0$ does permit an inverse of $f$.

In this case find an expression for the inverse of $f(x)$. **[4]**

(ii) Which of the following statements is true:

the function $g(x)$ is even,

the function $g(x)$ is odd,

the function $g(x)$ is neither even nor odd?

Give reasons for your answer. **[3]**

(iii) One of the functions $f$ and $g$ is periodic.

Explain which one is, and state its period. **[3]**

(iv) Show that $f(g(x)) = \frac{1}{2}(\cos 2\pi x - 1)$

Sketch its graph for the domain $1 \leq x \leq 4$. **[5]**

**Question 3**

(a) Differentiate $x \tan 3x$ with respect to $x$. **[2]**

(b) Find the stationary value (turning point) of $y = \frac{1}{x}\ln x$ for $x > 0$,

and decide if this is a maximum or a minimum. **[8]**

(c) Find the equation of the tangent to the curve $y = \frac{1}{(3x-4)^3}$

at the point $(1,-1)$. **[5]**

**Question 4**

(a) Use the substitution $u = 1 + x^3$ to evaluate $\int_0^2 x^2\sqrt{1+x^3}\,dx$. **[6]**

(b) Let $f(x) = \sin x$ for $0 \le x \le \pi$.

(i) Sketch the curve $y = f(x)$. **[2]**

(ii) Find the area under this curve in this region. **[2]**

(iii) The area is rotated completely about the $x$-axis.

Find the exact value of the volume of the solid formed. **[5]**

**Question 5**

(a) (i) Express $\frac{x-7}{(x-2)(x^2+1)}$ in the form $\frac{A}{x-2} + \frac{B}{x^2+1} + \frac{Cx}{x^2+1}$. **[4]**

(ii) Use your result from (i) to show that

$$\int_0^1 \frac{x-7}{(x-2)(x^2+1)}\,dx = \frac{3}{2}\ln 2 + \frac{3\pi}{4}.$$ **[6]**

(b) Use integration by parts to find the exact value of $\int_1^2 x^2 \ln x\,dx$. **[5]**

**Question 6**

(a) The probability of events $A$ and $B$ are $P(A)$ and $P(B)$ respectively.

If $P(A) = 0.6$ $P(B|A) = 0.7$ and $A$ and $B$ are independent find

(i) $P(A \cap B)$ **[2]**

(ii) $P(A' \cap B')$ **[2]**

(iii) Explain why $A$ and $B$ are not mutually exclusive. **[2]**

(iv) Illustrate your results on a Venn diagram,

labelling each of the four regions with their probabilities. **[2]**

(b) Chris wants to join an ice hockey team.

The probability he is chosen for the Polar Bears team is $0.4$,

and for the Seals team is $0.1$.

Otherwise he must play for the Walruses team.

The probability that the Polar Bears team wins is $0.7$,

that the Seals team wins is $0.6$,

but the probability that the Walruses team **loses** is $0.9$.

(i) Draw a probability tree to illustrate the above situation. **[2]**

(ii) What is the probability that Chris's team wins? **[3]**

(iii) Given that Chris's team loses,

what is the probability that he is in the Seals team? **[2]**

**Question 7**

(a) The number of people sitting at tables in a restaurant is modelled by the probability distribution

$$P(X = r) \quad = a \qquad r = 1$$
$$= \frac{1}{2^{r-1}} \qquad r = 2,3,4,5,6$$
$$= 0 \qquad r \geq 7$$

(i) Copy and complete the table below showing the probability distribution and determine the value of $a$. **[4]**

| $r$ | | | | | | |
|---|---|---|---|---|---|---|
| $P(X = r)$ | | | | | | |

(ii) Calculate $E(X)$ and $\text{Var}(X)$. **[4]**

(b) The table below shows the number $n$ of doughnuts sold each day between 3 pm and 4 pm during a week, together with the maximum temperature $T°$ C recorded each day:

| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| $n$ | 45 | 48 | 31 | 63 | 37 | 74 | 66 |
| $T$ | 22 | 24 | 20 | 26 | 30 | 28 | 25 |

where $\sum T = 175, \sum n = 364, \sum T^2 = 4445, \sum n^2 = 20460, \sum Tn = 9232$

(i) Calculate the correlation coefficient between the two sets of data. **[5]**

(ii) By considering the correlation coefficient in (i), comment on the relationship between the number of doughnuts sold and the maximum temperature recorded. **[2]**

**Question 8**

A football team expects to lose $30\%$ of its matches.

They play $15$ matches each season.

(i) Find the probability that they lose exactly $4$ matches in the season. **[3]**

(ii) Find the probability that they lose $6$ or more matches in the season. **[3]**

(iii) Over $4$ seasons what is the probability that they lose $6$ or more matches in exactly $2$ seasons? **[3]**

(iv) In an effort to improve their performance they appoint a new manager. In the following season they lose just $2$ matches. Using a hypothesis test at the $5\%$ level and stating your hypotheses clearly, decide if this provides adequate evidence that the new manager has been a success. What advice would you give to the chairman? **[6]**

**Question 9**

A gloves company has recorded their sales, in 100's, during the period 2004-2007:

| Year | Period | Sales |
|---|---|---|
| 2004 | Jan-Apr | 12 |
| 2004 | May-Aug | 4 |
| 2004 | Sep-Dec | 15 |
| 2005 | Jan-Apr | 13 |
| 2005 | May-Aug | 5 |
| 2005 | Sep-Dec | 17 |
| 2006 | Jan-Apr | 16 |
| 2006 | May-Aug | 7 |
| 2006 | Sep-Dec | 20 |
| 2007 | Jan-Apr | 18 |
| 2007 | May-Aug | 8 |
| 2007 | Sep-Dec | 23 |

a. Plot the data.
b. Calculate moving averages and add these to your chart.
c. Draw a suitable trendline for these moving averages.
d. Calculate the mean seasonal deviation for the Jan-Apr periods.
e. Forecast the sales for Jan-Apr 2008.

**Question 10**

(a) Given that the inverse of $A = \begin{pmatrix} 2 & 1 & 1 \\ -1 & -1 & 1 \\ 3 & 2 & -2 \end{pmatrix}$ is $A^{-1} = \frac{1}{p}\begin{pmatrix} 0 & 4 & 2 \\ 1 & -7 & q \\ 1 & -1 & -1 \end{pmatrix}$,

find $p$ and $q$.

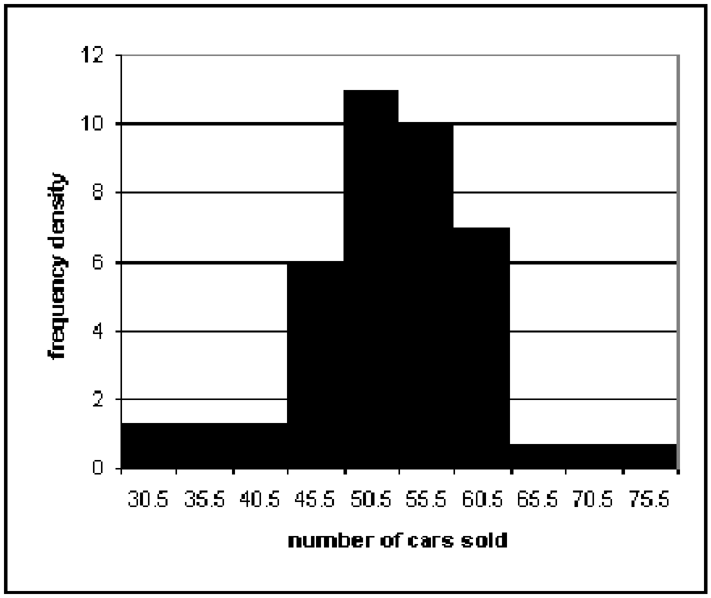
(b) Hence, solve the following simultaneous equations:

$$2x + y + z = 9$$
$$-x - y + z = 5$$
$$3x + 2y - 2z = -8$$

# Practice Exam 6 Answers

1

| from | to | w | f | f/w |
|---|---|---|---|---|
| 25.5 | 30.5 | 5 | 6.667 | 1.333 |
| 30.5 | 35.5 | 5 | 6.667 | 1.333 |
| 35.5 | 40.5 | 5 | 6.667 | 1.333 |
| 40.5 | 45.5 | 5 | 30 | 6 |
| 45.5 | 50.5 | 5 | 55 | 11 |
| 50.5 | 55.5 | 5 | 50 | 10 |
| 55.5 | 60.5 | 5 | 35 | 7 |
| 60.5 | 65.5 | 5 | 3.333 | 0.667 |
| 65.5 | 70.5 | 5 | 3.333 | 0.667 |
| 70.5 | 75.5 | 5 | 3.333 | 0.667 |




Labels on horizontal axis. **[1]**
Horizontal scale (accept 40/40.5/41 on boundary). **[1]**
Labels on vertical axis. **[1]**
Vertical scale. **[1]**
Bars right height and width. **[2]**

(ii)

| | | x | f | xf |
|---|---|---|---|---|
| 25.5 | 40.5 | 33 | 20 | 660 |
| 40.5 | 45.5 | 43 | 30 | 1290 |
| 45.5 | 50.5 | 48 | 55 | 2640 |
| 50.5 | 55.5 | 53 | 50 | 2650 |
| 55.5 | 60.5 | 58 | 35 | 2030 |
| 60.5 | 75.5 | 68 | 10 | 680 |
| | | | 200 | 9950 |

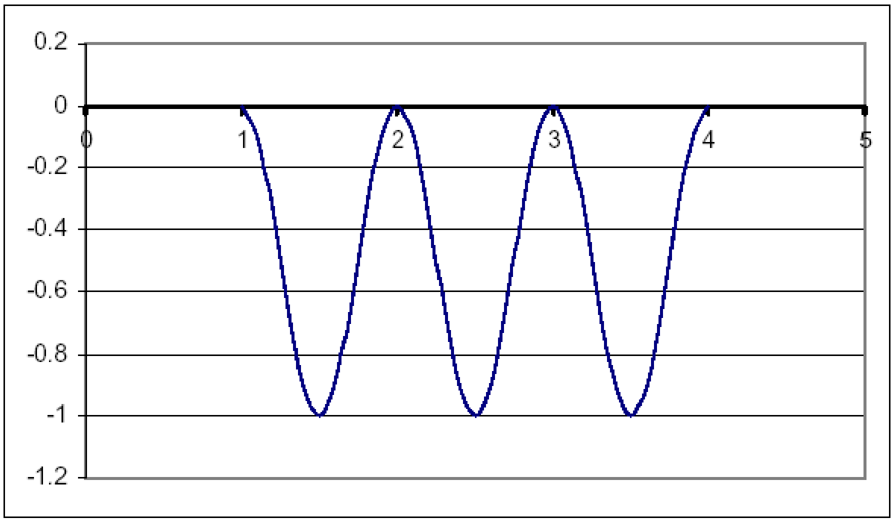
$x$ column **[1]**
$xf$ column **[2]**
mean =

$$\frac{\sum xf}{\sum f} = \frac{9950}{200} = 49.75\,.$$ **[1]**

| lcb | ucb | f | cf |
|---|---|---|---|
| | | | 0 |
| 25.5 | 40.5 | 20 | 20 |
| 40.5 | 45.5 | 30 | 50 |
| 45.5 | 50.5 | 55 | 105 |
| 50.5 | 55.5 | 50 | 155 |
| 55.5 | 60.5 | 35 | 190 |
| 60.5 | 75.5 | 10 | 200 |

(iii) See above **[2]**
(iv) lower quartile = 45.5 (accept 45 or 46) **[2]**
(v) modal class is 46 – 50 **[1]**

| | | |
|---|---|---|
| 2(i) | $f(1)=0,\ f(-1)=0$, so the function is not one-one. | **[1]** |
| | If $x_1 \geq 0$ and $x_2 \geq 0$ and $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ | |
| | (or other suitable explanation). | **[1]** |
| | $f^{-1}(x)=\sqrt{x+1} \qquad x \geq -1$ | **[2]** |
| (ii) | Since $\cos(\pi x)=\cos(-\pi x)$, | **[2]** |
| | the function $g(x)$ is even. | **[1]** |
| (iii) | Since $g(x+2)=\cos(\pi(x+2))=\cos(\pi x+2\pi)=\cos(\pi x)=g(x)$ | **[1]** |
| | the function $g(x)$ is periodic with period 2. | **[2]** |
| (iv) | $f(g(x))=(\cos(\pi x))^2-1=\cos^2(\pi x)-1=-\sin^2(\pi x)=\frac{1}{2}(\cos 2\pi x-1)$ | **[2]** |




| | | |
|---|---|---|
| | shape | **[1]** |
| | position on $x$-axis | **[1]** |
| | position on $y$-axis | **[1]** |
| 3(a) | $\frac{dy}{dx}=\tan 3x + x\sec^2 3x \times 3 = \tan 3x + 3x\sec^2 3x$ | **[2]** |
| (b) | $\frac{dy}{dx}=-\frac{1}{x^2}\ln x+\frac{1}{x}\frac{1}{x}=\frac{1}{x^2}(1-\ln x)$ | **[2]** |
| | $=0$ at a stationary point. | **[1]** |
| | Then $\ln x=1,\ x=\mathrm{e}(\approx 2.72)$. | **[1]** |
| | Here $y=\frac{1}{\mathrm{e}}\ln(\mathrm{e})=\frac{1}{\mathrm{e}}(\approx 0.368)$. | **[1]** |
| | $\frac{d^2y}{dx^2}=-\frac{2}{x^3}(1-\ln x)+\frac{1}{x^2}\left(0-\frac{1}{x}\right)=-\frac{1}{x^3}(2-2\ln x+1)=-\frac{3-2\ln x}{x^3}$. | |
| | When $x=\mathrm{e}$, $\frac{d^2y}{dx^2}=-\frac{3-2\ln(\mathrm{e})}{\mathrm{e}^3}=-\frac{1}{\mathrm{e}^3}<0$ | **[2]** |
| | So the stationary value is a maximum. | **[1]** |
| (c) | $\frac{dy}{dx}=\frac{-3}{(3x-4)^4}3=\frac{-9}{(3x-4)^4}$ | **[1]** |
| | $=-9$ at the point $(1,-1)$. | **[1]** |
| | The equation of the tangent is $y=-9x+c$. | **[1]** |
| | $-1=-9\times 1+c,\ c=8$. | **[1]** |
| | $y=-9x+8$ | **[1]** |

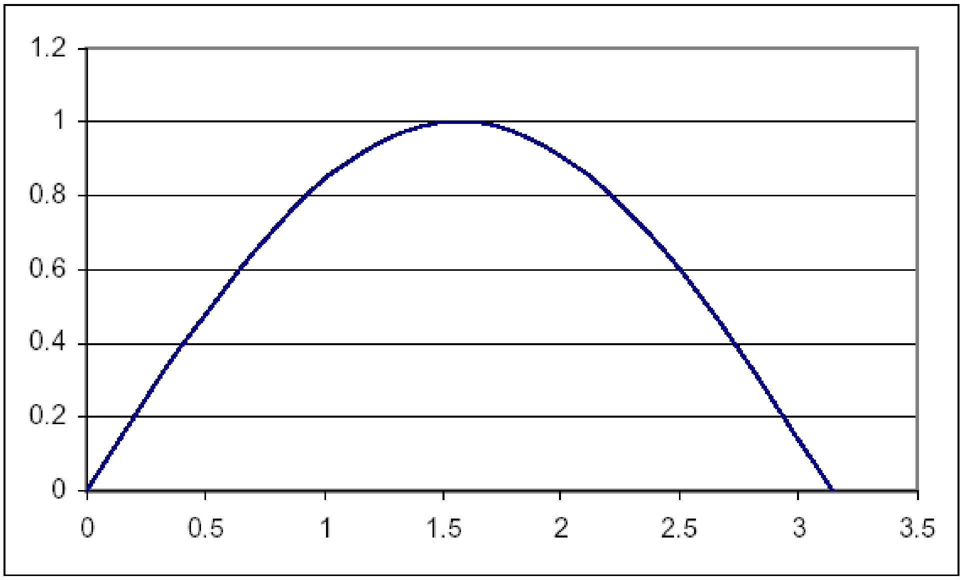
4(a) $u = 1 + x^3$, $\mathrm{d}u = 3x^2\mathrm{d}x$. **[2]**

$$\int_0^2 x^2\sqrt{1+x^3}\,\mathrm{d}x = \int_1^9 \frac{1}{3}\sqrt{u}\,\mathrm{d}u$$ **[2]**

$$= \left[\frac{2}{9}u^{3/2}\right]_1^9$$ **[1]**

$$= \frac{2}{9}[27-1] = \frac{52}{9}(\approx 5.78)$$ **[1]**

(b)(i)




shape **[1]**

limits **[1]**

(ii) $$\int_0^\pi \sin x\,\mathrm{d}x = [-\cos x]_0^\pi$$ **[1]**

$$= [1+1] = 2$$ **[1]**

(iii) $$\pi\int_0^\pi (\sin x)^2\,\mathrm{d}x = \pi\int_0^\pi \left(\frac{1}{2}(1-\cos 2x)\right)\mathrm{d}x$$ **[2]**

$$= \pi\left[\frac{1}{2}x - \frac{1}{4}\sin 2x\right]_0^\pi$$ **[1]**

$$= \pi\left[\frac{\pi}{2} - 0 - 0 + 0\right]$$ **[1]**

$$= \frac{\pi^2}{2}(\approx 4.93).$$ **[1]**

5(a)(i) $\frac{A}{x-2}+\frac{B}{x^2+1}+\frac{Cx}{x^2+1}=\frac{A(x^2+1)+B(x-2)+Cx(x-2)}{(x-2)(x^2+1)}$. **[1]**

This equals $\frac{x-7}{(x-2)(x^2+1)}$

$x=2 \Rightarrow 5A=-5 \Rightarrow A=-1$. **[1]**

$x=0 \Rightarrow A-2B=-7 \Rightarrow B=3$. **[1]**

Equating coefficients of $x^2 \Rightarrow A+C=0 \Rightarrow C=1$. **[1]**

Given expression equals $\frac{-1}{x-2}+\frac{3}{x^2+1}+\frac{x}{x^2+1}$.

(ii) Given integral equals

$$\int_0^1\left(\frac{-1}{x-2}+\frac{3}{x^2+1}+\frac{x}{x^2+1}\right)dx=\left[-\ln|x-2|+3\tan^{-1}x+\frac{1}{2}\ln(x^2+1)\right]_0^1$$ **[3]**

$$=\left[-\ln(1)+3\tan^{-1}1+\frac{1}{2}\ln(2)+\ln(2)-3\tan^{-1}0-\frac{1}{2}\ln(1)\right]$$

$$=-0+\frac{3\pi}{4}+\frac{1}{2}\ln 2+\ln 2-0-0=\frac{3\pi}{4}+\frac{3}{2}\ln 2$$ **[M2A1]**

(b) $$\int_1^2 x^2\ln x\,dx=\left[\frac{1}{3}x^3\ln x\right]_1^2-\int_1^2\frac{1}{3}x^3\frac{1}{x}dx=\left[\frac{1}{3}x^3\ln x\right]_1^2-\frac{1}{3}\int_1^2 x^2dx$$ **[2]**

$$=\left[\frac{1}{3}x^3\ln x-\frac{1}{9}x^3\right]_1^2$$ **[1]**

$$=\frac{8}{3}\ln 2-\frac{8}{9}-0+\frac{1}{9}$$ **[1]**

$$=\frac{8}{3}\ln 2-\frac{7}{9}.$$ **[1]**

6(a)(i) $P(B) = 0.7$, **[1]**

$P(A \cap B) = P(A)P(B) = 0.42$ **[1]**

(ii) $P(A') = 0.4$, $P(B') = 0.3$, **[1]**

$P(A' \cap B') = 0.12$ **[1]**

(iii) If $A$ and $B$ are mutually exclusive, $P(A \cap B) = 0 \neq 0.42$. **[2]**

(iv)

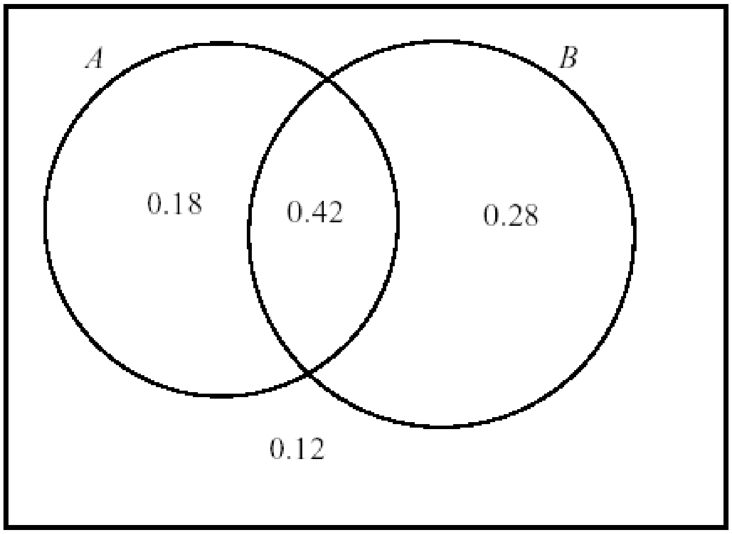
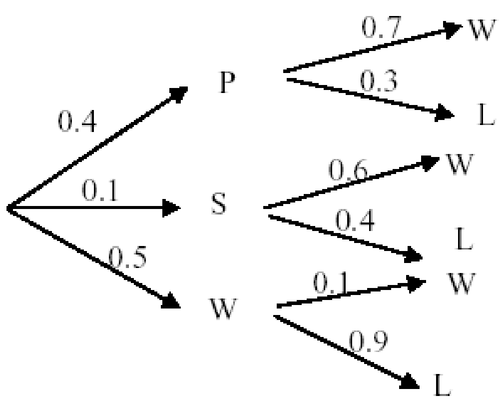



Diagram **[1]**

All 4 probabilities shown **[1]**

(b)(i)




**[2]**

(ii) $P$(C wins) = $P$(C in P and W) + $P$(C in S and W) + $P$ (C in W and W) **[1]**

$= 0.4 \times 0.7 + 0.1 \times 0.6 + 0.5 \times 0.1$ **[1]**

= 0.28 + 0.06 + 0.05 = 0.39 **[1]**

(iii) $P$(C in S | C loses) = $P$(C in S and C loses)/$P$(C loses) **[1]**

= 0.04 / 0.61 = 0.0656 **[1]**

7(a)(i)

| r | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P(X=r) | a | 1/2 | 1/4 | 1/8 | 1/16 | 1/32 |

**[2]**

$$\sum P(X=r) = a + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = a + \frac{31}{32} = 1$$

so $a = \frac{1}{32} (\approx 0.0313)$. **[M1A1]**

(ii)

| x | p | xp | $x^2p$ |
|---|---|---|---|
| 1 | 1/32 | 1/32 | 1/32 |
| 2 | 1/2 | 1 | 2 |
| 3 | 1/4 | 3/4 | 9/4 |
| 4 | 1/8 | 1/2 | 2 |
| 5 | 1/16 | 5/16 | 25/16 |
| 6 | 1/32 | 3/16 | 9/8 |
| sum | 1 | 73/32 | 201/32 |

$E(X) = \sum xp = \frac{73}{32} = 2.28.$ **[M1A1]**

$Var(X) = \sum x^2 p - (E(x))^2 = \frac{1103}{1024} = 1.08$ **[M1A1]**

(b)(i) The means are, respectively,
25, 52, 635, 2923, 1319 **[1]**

$S_T = 3.162$ **[1]**

$S_n = 14.79$ **[1]**

$S_{Tn} = 18.86$ **[1]**

$r = 0.403$ **[1]**

(ii) The correlation here is poor. There is no evidence that temperature is a significant factor in the sale of doughnuts. **[2]**

8(i) Looking in the binomial distribution tables in the formula booklet, the probability that the team lose is 30%,
so the probability that they lose less than or equal to 4 is 0.5155, **[1]**
that they lose less than or equal to 3 is 0.2969, **[1]**
and the difference is 0.2186. (Accept 0.219.) **[1]**

(ii) The probability that they lose less than or equal to 5 is 0.7216, **[2]**
so the probability that they lose 6 or more is 0.2784. (Accept 0.278.) **[1]**

(iii) We need to work out ${}^4C_2 p^2 q^2$ **[1]**

$= \frac{4 \times 3}{1 \times 2} 0.2784^2 \times 0.7216^2$ **[1]**

$= 0.242$. **[1]**

(iv) $H_0 : P(\text{failure}) = 0.30$ **[1]**

$H_1 : P(\text{failure}) < 0.30$ **[1]**

$X$ is the number of matches lost.

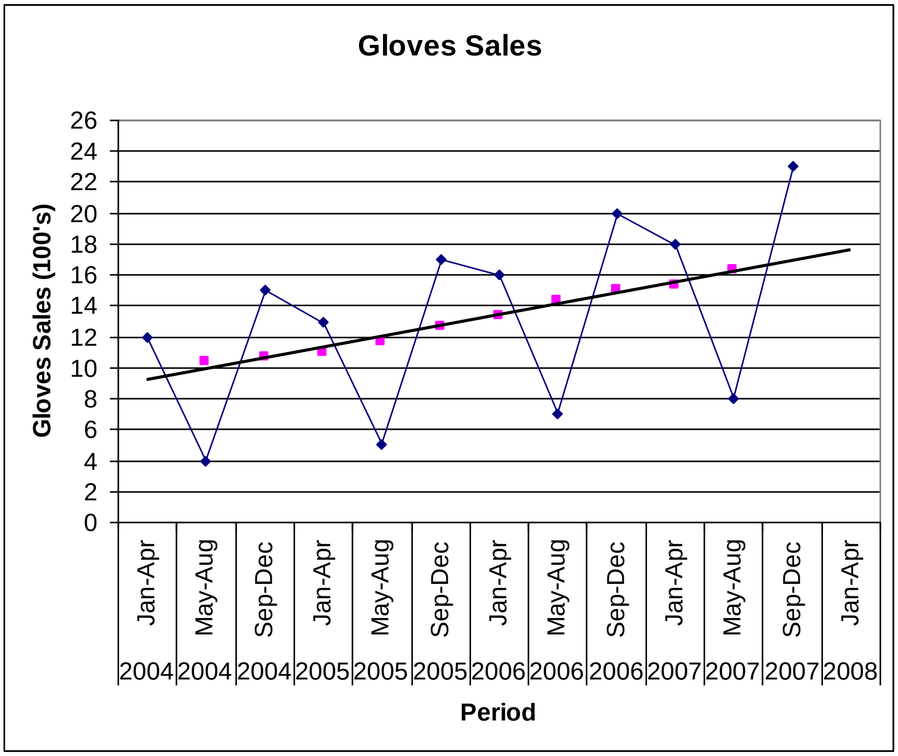
$X \sim B(15, 0.30)$ **[1]**

$P(X \le 1) = 0.0353$

$P(X \le 2) = 0.1268 > 5\%$ **[1]**

Looking down the column headed $p = 0.30$ with $n = 15$ we see that to get a probability less than 5% then $x$ must be less than or equal to 1. Therefore having lost 2 matches we do not have adequate evidence that the new manager is a success. We should recommend to the chairman that the manager is given more time to prove himself. **[2]**

**9**




| Year | Period | Sales | Moving Av. | Deviation |
|---|---|---|---|---|
| 2004 | Jan-Apr | 12 | | |
| 2004 | May-Aug | 4 | 10.33 | |
| 2004 | Sep-Dec | 15 | 10.67 | |
| 2005 | Jan-Apr | 13 | 11 | 2 |
| 2005 | May-Aug | 5 | 11.67 | |
| 2005 | Sep-Dec | 17 | 12.67 | |
| 2006 | Jan-Apr | 16 | 13.33 | 2.67 |
| 2006 | May-Aug | 7 | 14.33 | |
| 2006 | Sep-Dec | 20 | 15 | |
| 2007 | Jan-Apr | 18 | 15.33 | 2.67 |
| 2007 | May-Aug | 8 | 16.33 | |
| 2007 | Sep-Dec | 23 | | |

Average seasonal deviation for Jan-Apr periods = $\frac{2+2.67+2.67}{3}$ = 2.4

Value from trendline for Jan-Apr 2008 = 17.6
Forecast for Jan-Apr 2008 = 17.6 + 2.4 = 20.0 = 2000 sales

**10**

(a) $p = \det A = 2(2-2) - 1(2-3) + 1(-2+3) = 2$
$q = -[(2)(1) - (-1)(1)] = -3$

(b) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 4 & 2 \\ 1 & -7 & -3 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 9 \\ 5 \\ -8 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 4 \\ -2 \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

$x = 2, y = -1, z = 6$