Practice Exam 5 (Business)

Question 1

The grouped frequency table below shows the number of laptops sold per day in a computer store during a period of 200 working days.

Number of laptops sold	Frequency
$46-60$	20
$61-65$	30
$66-70$	55
$71-75$	50
$76-80$	35
$81-95$	10

(i) Draw a histogram to illustrate this set of data. **[6]**

(ii) Calculate the mean number of laptops sold per day. **[4]**

(iii) Find the standard deviation of the data. **[5]**

Question 2

The functions f and g are defined as follows:

$$f(x) = \sin(\pi x) \qquad 0 < x < 2$$
$$g(x) = 1 + x^2 \qquad x \geq 0$$

(i) Write down the range of $f(x)$.

Solve the equation $f(x) = \frac{1}{2}$.

Explain why the function f does not have an inverse. **[4]**

(ii) The inverse of g is g^{-1}. Find $g^{-1}(x)$.

Write down the domain of $g^{-1}(x)$. **[3]**

(iii) Show that $g(|f(x)|) = \dfrac{3-\cos(2\pi x)}{2}$. **[5]**

(iv) Sketch the graph of $g(|f(x)|)$. **[3]**

Question 3

(a) Differentiate $e^x \ln x$ with respect to x. **[2]**

(b) Find the stationary value (turning point) of $y = e^{-x}\cos x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$,

and decide if this is a maximum or a minimum. **[7]**

(c) Find the equation of the normal to the curve $y = (2x-3)^3$

at the point $(2,1)$. **[6]**

Question 4

(a) Use the substitution $u = \ln x$ to evaluate $\int_{e}^{e^2} \frac{1}{x(\ln x)^2} dx$. **[6]**

(b) Let $f(x) = x(3-x)$.

(i) Sketch the curve $y = f(x)$ where $y \geq 0$. **[2]**

(ii) Find the area under this curve in this region. **[3]**

(iii) The area is rotated completely about the x-axis.

Find the exact value of the volume of the solid formed. **[4]**

Question 5

(a) (i) Express $\frac{x+5}{(x^2-1)(x+2)}$ as a sum of partial fractions. **[4]**

(ii) Hence evaluate

$$\int_{2}^{4} \frac{x+5}{(x^2-1)(x+2)} dx.$$ **[6]**

(b) Use integration by parts to find the exact value of $\int_{0}^{\pi/2} x^2 \sin x dx$. **[5]**

Question 6

(a) The probability of events A and B are $P(A)$ and $P(B)$ respectively.

If $P(A|B) = \frac{4}{5}$, $P(B) = \frac{1}{2}$ and A and B are independent find

(i) $P(A \cap B)$ **[2]**

(ii) $P(A' \cap B')$ **[2]**

(iii) Explain why A and B are not mutually exclusive. **[2]**

(iv) Illustrate your results on a Venn diagram,

labelling each of the four regions with their probabilities. **[2]**

(b) Simon wants to join a rugby team.

The probability he is chosen for the Bees team is 0.3,

and for the Wasps team is 0.1.

Otherwise he must play for the Hornets team.

The probability that the Bees team wins is 0.7,

that the Wasps team wins is 0.6,

but the probability that the Hornets team **loses** is 0.9.

(i) Draw a probability tree to illustrate the above situation. **[2]**

(ii) What is the probability that Simon's team wins? **[3]**

(iii) Given that Simon's team loses,

what is the probability that he is in the Wasps team? **[2]**

Question 7

(a) The number of chocolate bars sold to an individual customer in a sweet shop is modelled by the probability distribution

$$P(X=r) \quad = a \qquad r=1$$
$$= \frac{1}{5r} \qquad r=2,3,4,5,6$$
$$= 0 \qquad r \geq 7$$

(i) Copy and complete the table below showing the probability distribution and determine the value of a. **[4]**

r						
$P(X=r)$						

(ii) Calculate $E(X)$ and $\mathrm{Var}(X)$. **[4]**

(iii) Calculate $E(7X–2)$ and $\mathrm{Var}(7X–2)$.

(b) The table below shows the number n of hot drinks sold each day between 3 pm and 4 pm during a week, together with the maximum temperature $T°$ C recorded each day:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
n	66	55	73	52	32	47	60
T	22	24	20	26	30	28	25

where $\sum T = 175, \sum n = 385, \sum T^2 = 4445, \sum n^2 = 22247, \sum Tn = 9360$

(i) Calculate the equation of the regression line of n on T. **[5]**

(ii) Use your equation to estimate the number of hot drinks you would expect to sell if the maximum temperature were $27°$ C. **[2]**

Question 8

In a factory which produces light bulbs, 10% are defective.

A sample of 15 bulbs is tested each day.

(i) Find the probability that the sample contains 2 defectives. **[3]**

(ii) Find the probability that the sample contains 4 or more defectives. **[2]**

(iii) Over five days what is the probability that 4 or more defectives are found on exactly 3 occasions? **[4]**

(iv) A new process is introduced to reduce the number of defectives. A random sample of 15 bulbs is tested and none is defective. Carry out a suitable hypothesis test at the 5% significance level, stating your hypothesis and conclusions carefully. **[5]**

(v) Would you recommend any improvement to the test? **[1]**

9

Sales of petrol at the Star petrol station:

Week	Day	Litres
1	Monday	28
	Tuesday	16
	Wednesday	24
	Thursday	44
	Friday	65
	Saturday	82
	Sunday	30
2	Monday	33
	Tuesday	21
	Wednesday	29
	Thursday	49
	Friday	70
	Saturday	87
	Sunday	35
3	Monday	35
	Tuesday	23
	Wednesday	31
	Thursday	51
	Friday	72
	Saturday	89
	Sunday	37

(a) Plot the data on a chart.

(b) Why do you think more petrol is sold on Saturdays?

(c) Calculate moving averages and put these on your chart.

(d) Use the moving averages to draw a suitable trendline and extrapolate it to Wednesday of week 4.

(e) Calculate the average daily deviation for Wednesday.

(f) Predict the petrol sales for Wednesday of week 4.

10 (a) Find the determinant and inverse of the following matrix:

$$A = \begin{pmatrix} 7 & 1 & -1 \\ 4 & 0 & 3 \\ 1 & -1 & 4 \end{pmatrix}$$

(b) Hence, solve for *x*, *y* and *z* in the following set of simultaneous equations.

$$7x + y - z = 4$$

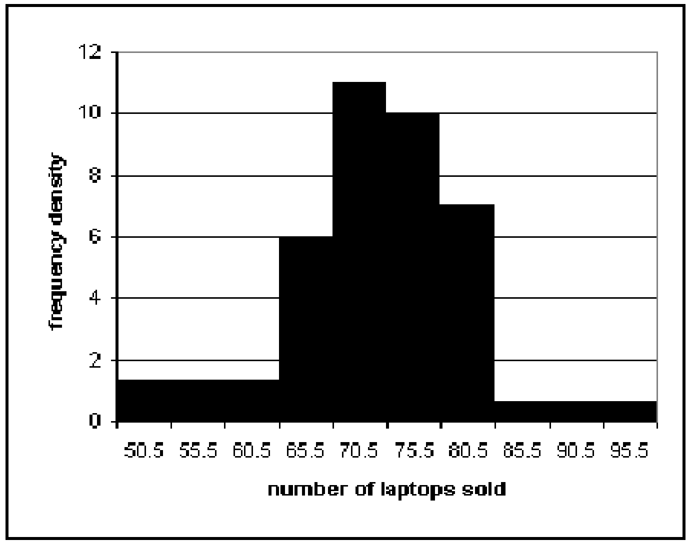
$$4x + 3z = 6$$

$$x - y + 4z = 2$$

Practice Exam 5 Answers

1(i)

from	to	w	f	f/w
45.5	50.5	5	6.67	1.333
50.5	55.5	5	6.67	1.333
55.5	60.5	5	6.67	1.333
60.5	65.5	5	30	6
65.5	70.5	5	55	11
70.5	75.5	5	50	10
75.5	80.5	5	35	7
80.5	85.5	5	3.33	0.667
85.5	90.5	5	3.33	0.667
90.5	95.5	5	3.33	0.667



Labels on horizontal axis. **[1]**
Horizontal scale (accept 60/60.5/61 on boundary). **[1]**
Labels on vertical axis. **[1]**
Vertical scale. **[1]**
Bars right height and width. **[2]**

(ii)

		x	f	xf	xxf
45.5	60.5	53	20	1060	56180
60.5	65.5	63	30	1890	119070
65.5	70.5	68	55	3740	254320
70.5	75.5	73	50	3650	266450
75.5	80.5	78	35	2730	212940
80.5	95.5	88	10	880	77440
			200	13950	986400

x column **[1]**
xf column **[2]**

$$\text{mean} = \frac{\sum xf}{\sum f} = \frac{13950}{200} = 69.75\,.$$ **[1]**

xxf column **[2]**

$$\text{sd} = \sqrt{\frac{\sum x^2 f}{\sum f} - \left(\frac{\sum xf}{\sum f}\right)^2} = \sqrt{\frac{986400}{200} - 69.75^2} = 8.18$$ **[M2A1]**

2(i) The range is $-1 \le y \le 1$. **[1]**

$x = \frac{1}{6}$ or $\frac{5}{6}$. (Accept 0.167 or 0.833.) **[2]**

The function is not one-one, as we have just established. **[1]**

(ii) $g^{-1}(x) = \sqrt{x-1}$ **[2]**

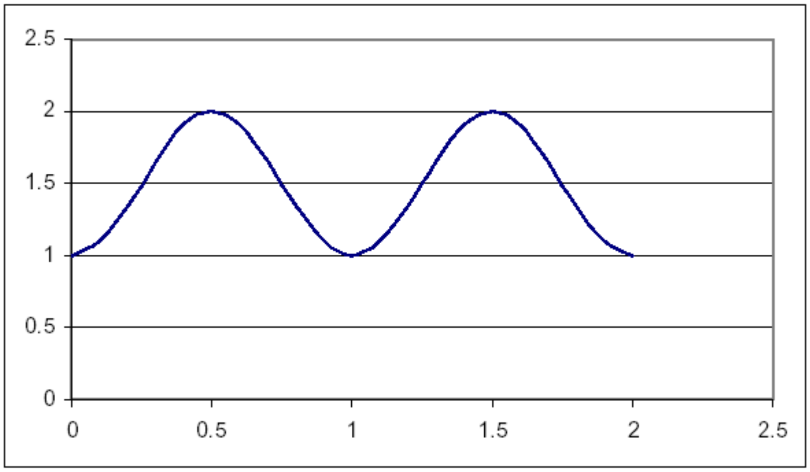
Its domain is $1 \le x < \infty$ or simply $1 \le x$. **[1]**

(iii) $g(|f(x)|) = 1 + (f(x))^2 = 1 + \sin^2(\pi x)$ **[2]**

$= 1 + \frac{1}{2}(1 - \cos(2\pi x))$ **[2]**

$= \frac{3 - \cos(2\pi x)}{2}$ **[1]**

(iv)



Shape **[1]**

position on x-axis **[1]**

position on y-axis **[1]**

3(a) $\frac{dy}{dx} = e^x \ln x + e^x \frac{1}{x}$ **[2]**

(b) $\frac{dy}{dx} = -e^{-x}\cos x - e^{-x}\sin x$ **[1]**

$= 0$ at a stationary point. **[1]**

Then $\cos x = -\sin x$, $\tan x = -1$, $x = -\frac{\pi}{4} (\approx -0.785)$. **[1]**

Here $y = e^{\pi/4}\frac{1}{\sqrt{2}} (\approx 1.55)$. **[1]**

$\frac{d^2y}{dx^2} = e^{-x}\cos x + e^{-x}\sin x + e^{-x}\sin x - e^{-x}\cos x = 2e^{-x}\sin x$.

When $x = -\pi/4$, $\frac{d^2y}{dx^2} = 2e^{\pi/4}\sin(-\pi/4) = -\sqrt{2}e^{\pi/4} < 0$. **[2]**

So the stationary value is a maximum. **[1]**

(c) $\frac{dy}{dx} = 3(2x-3)^2 2 = 6(2x-3)^2$ **[1]**

$= 6$ at the point (2,1). **[1]**

The gradient of the normal is therefore $-\frac{1}{6}$. **[1]**

The equation of the normal is $y = -\frac{1}{6}x + c$. **[1]**

$1 = -\frac{1}{6} \times 2 + c$, $c = \frac{4}{3}$. **[1]**

$y = -\frac{1}{6}x + \frac{4}{3}$.

$x + 6y = 8$. **[1]**

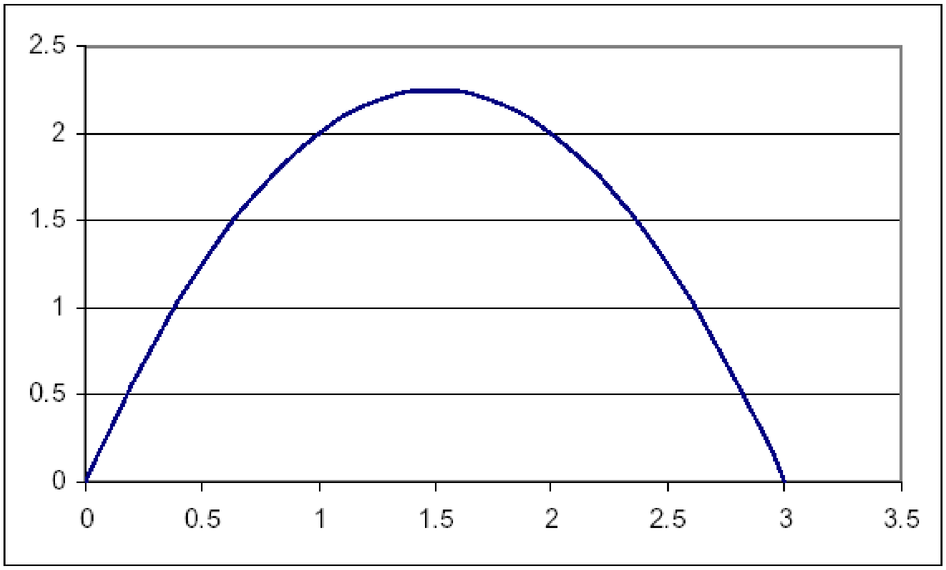
4(a) $u = \ln x,\ x = e^u,\ du = \dfrac{dx}{x}$. **[2]**

$$\int_e^{e^2} \frac{1}{x(\ln x)^2}\,dx = \int_1^2 \frac{1}{u^2}\,du$$ **[2]**

$$= \left[-\frac{1}{u}\right]_1^2$$ **[1]**

$$= -\frac{1}{2} + 1 = \frac{1}{2}.$$ **[1]**

(b)(i)



Shape **[1]**
limits **[1]**

(ii) $$\int_0^3 (3x - x^2)\,dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$ **[1]**

$$= \left[\frac{27}{2} - \frac{27}{3}\right]$$ **[1]**

$$= \frac{9}{2}$$ **[1]**

(iii) $$\pi\int_0^3 (3x - x^2)^2\,dx = \pi\int_0^3 (9x^2 - 6x^3 + x^4)\,dx$$ **[1]**

$$= \pi\left[\frac{9x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right]_0^3$$ **[1]**

$$= \pi\left[81 - \frac{243}{2} + \frac{243}{5}\right]$$ **[1]**

$$= \frac{81\pi}{10}(\approx 25.4).$$ **[1]**

5(a)(i) $\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+2}=\frac{A(x+1)(x+2)+B(x-1)(x+2)+C(x-1)(x+1)}{(x^2-1)(x+2)}$. **[1]**

This equals $\frac{x+5}{(x^2-1)(x+2)}$

$x=1\Rightarrow 6A=6\Rightarrow A=1$. **[1]**

$x=-1\Rightarrow -2B=4\Rightarrow B=-2$. **[1]**

$x=-2\Rightarrow 3C=3\Rightarrow C=1$. **[1]**

Given expression equals $\frac{1}{x-1}-\frac{2}{x+1}+\frac{1}{x+2}$.

(ii) Given integral equals

$\int_2^4\left(\frac{1}{x-1}-\frac{2}{x+1}+\frac{1}{x+2}\right)dx=\left[\ln(x-1)-2\ln(x+1)+\ln(x+2)\right]_2^4$ **[2]**

$=\left[\ln 3-2\ln 5+\ln 6-\ln 1+2\ln 3-\ln 4\right]$

$=\left[(1+1+2)\ln 3-2\ln 5+(1-2)\ln 2\right]$

$=4\ln 3-2\ln 5-\ln 2=\ln\left(\frac{81}{50}\right)(\approx 0.482)$. **[M2A2]**

(b) $\int_0^{\pi/2}x^2\sin x\,dx=\left[-x^2\cos x\right]_0^{\pi/2}+\int_0^{\pi/2}2x\cos x\,dx$ **[1]**

$=\left[-x^2\cos x+2x\sin x\right]_0^{\pi/2}-\int_0^{\pi/2}2\sin x\,dx$ **[1]**

$=\left[-x^2\cos x+2x\sin x+2\cos x\right]_0^{\pi/2}$ **[1]**

$=0+\pi+0+0-0-2$ **[1]**

$=\pi-2$. **[1]**

6(a)(i) $P(A) = \frac{4}{5}$, **[1]**

$P(A \cap B) = P(A)P(B) = \frac{2}{5}$ **[1]**

(ii) $P(A') = \frac{1}{5}$, $P(B') = \frac{1}{2}$, **[1]**

$P(A' \cap B') = \frac{1}{10}$ **[1]**

(iii) If A and B are mutually exclusive, $P(A \cap B) = 0 \neq \frac{2}{5}$. **[2]**

(iv)

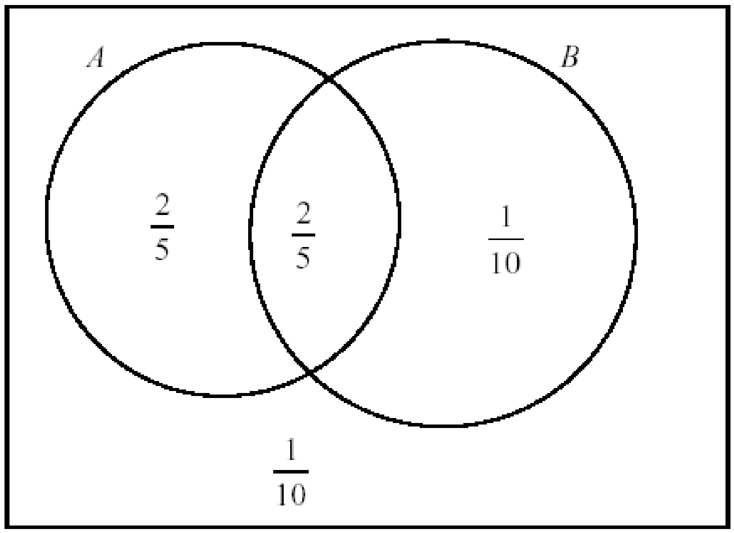
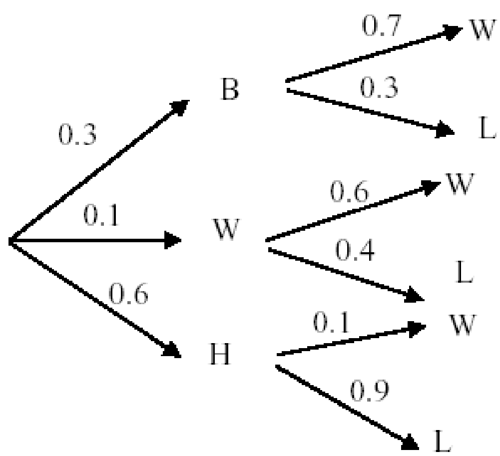


Diagram **[1]**
All 4 probabilities shown **[1]**

(b)(i)



[2]

(ii) P(S wins) = P(S in B and W) + P(S in W and W) + P (S in H and W) **[1]**
$= 0.3 \times 0.7 + 0.1 \times 0.6 + 0.6 \times 0.1$ **[1]**
= 0.21 + 0.06 + 0.06 = 0.33 **[1]**

(iii) P(S in W | S loses) = P(S in W and S loses)/P(S loses) **[1]**
= 0.04 / 0.67 = 0.0597 **[1]**

7(a)(i)

r	1	2	3	4	5	6
P(X=r)	a	1/10	1/15	1/20	1/25	1/30

[2]

$$\sum P(X=r) = a + \frac{1}{10} + \frac{1}{15} + \frac{1}{20} + \frac{1}{25} + \frac{1}{30} = a + \frac{29}{100} = 1$$

so $a = \frac{71}{100} (= 0.71)$. **[M1A1]**

(ii)

x	p	xp	x^2p
1	71/100	71/100	71/100
2	1/10	1/5	2/5
3	1/15	1/5	3/5
4	1/20	1/5	4/5
5	1/25	1/5	1/1
6	1/30	1/5	6/5
sum	1	171/100	471/100

$E(X) = \sum xp = \frac{171}{100} = 1.71.$ **[M1A1]**

$Var(X) = \sum x^2 p - (E(x))^2 = \frac{17859}{10000} (\approx 1.79)$ **[M1A1]**

(iii) $E(7X-2) = 7E(X) - 2 = 7(1.71) - 2 = 9.97$
$\text{Var}(7X-2) = 7^2\text{Var}(X) = 49(1.7859) = 87.51$

(b)(i) The means are, respectively,
25, 55, 635, 3178, 1337 **[1]**
$S_T = 3.162$ **[1]**
$S_{Tn} = -37.86$ **[1]**
The regression line is $n = -3.786T + 149.6$. (Accept $n = -3.79T + 150$.) **[2]**

(ii) Putting $T = 27$ in the above equation gives $n = 47.43$
You would expect to sell 47 or 48 drinks. **[M1A1]**

8(i) Looking in the binomial distribution tables in the formula booklet, the probability that the bulbs are defective is 10%, so the probability that less than or equal to 2 are defective is 0.8159, **[1]**
that less than or equal to 1 is defective is 0.5490, **[1]**
and the difference is 0.267. **[1]**

(ii) The probability that less than or equal to 3 are defective is 0.9444, **[M1A1]**
so the probability that 4 or more are defective is 0.0556. **[1]**

(iii) We need to work out ${}^5C_3 p^2 q^2$ **[1]**

$= \frac{5\times 4}{1\times 2} 0.9444^2 \times 0.0556^3$ **[1]**

$= 0.00153$. **[1]**

(iv) The null hypothesis is that the new bulbs are no better than the old, so we assume a 10% failure rate. If the probability of getting less than or equal to the observed failure rate is less than 5% then we reject the null hypothesis and accept that the new bulbs are better.
Looking down the column headed $p = 0.10$ with $n = 15$ we see that it is impossible to get a probability less than 5% even when x is 0. Thus we must accept the null hypothesis and say that the test is inconclusive.

$H_0 : P(\text{failure}) = 0.10$ **[1]**

$H_1 : P(\text{failure}) < 0.10$ **[1]**

X is the number of bulbs which fail.

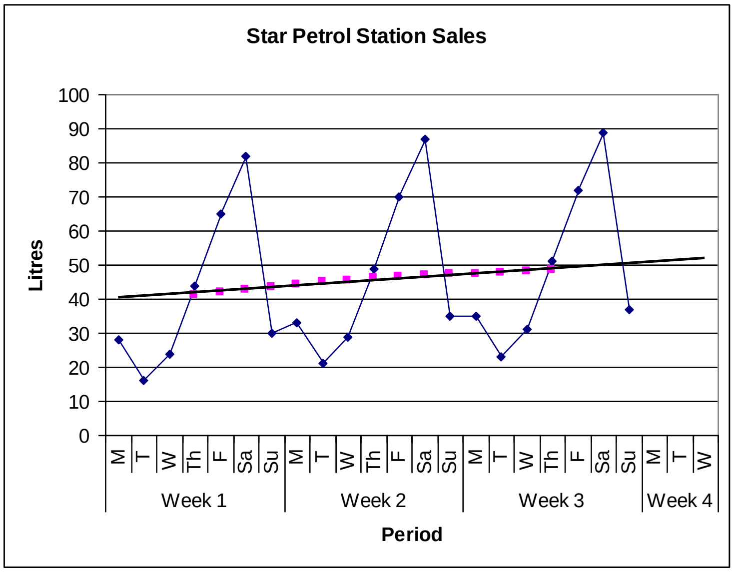
$X \sim B(15, 0.10)$ **[1]**

$P(X = 0) = 0.2059 > 5\%$ **[1]**

So we must accept H_0 at the 5% level and say that the test is inconclusive. **[1]**

(v) To get a useful result we would need a much larger sample. **[1]**

9



More petrol is sold on Saturdays because people like to go out and drive on Saturdays.

Week	Day	Litres	Moving Av.	Deviation
1	Monday	28		
	Tuesday	16		
	Wednesday	24		
	Thursday	44	42.29	
	Friday	65	42.00	
	Saturday	82	42.71	
	Sunday	30	43.43	
2	Monday	33	44.14	
	Tuesday	21	44.86	
	Wednesday	29	45.57	–16.57
	Thursday	49	46.29	
	Friday	70	46.57	
	Saturday	87	46.86	
	Sunday	35	47.14	
3	Monday	35	47.43	
	Tuesday	23	47.71	
	Wednesday	31	48.00	–17.00
	Thursday	51	48.29	
	Friday	72		
	Saturday	89		
	Sunday	37		

Average daily variation for Wednesday = $\dfrac{-16.57-17.00}{2}=-16.79$

Value from trendline for Wednesday of week 4 = 52.0

Prediction for petrol sales for Wednesday of week 4 = 52.0 – 16.79 = 35.2 litres

10 (a) det$A$ = 7(0+3) – 1(16-3) – 1(–4–0) = 12

$$A^{-1}=\frac{1}{12}\begin{pmatrix}3 & -13 & -4\\ -3 & 29 & 8\\ 3 & -25 & -4\end{pmatrix}^T=\frac{1}{12}\begin{pmatrix}3 & -3 & 3\\ -13 & 29 & -25\\ -4 & 8 & -4\end{pmatrix}$$

(b) $$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{12}\begin{pmatrix}3 & -3 & 3\\ -13 & 29 & -25\\ -4 & 8 & -4\end{pmatrix}\begin{pmatrix}4\\ 6\\ 2\end{pmatrix}=\frac{1}{12}\begin{pmatrix}0\\ 72\\ 24\end{pmatrix}=\begin{pmatrix}0\\ 6\\ 2\end{pmatrix}$$

$x=0, y=6, z=2$