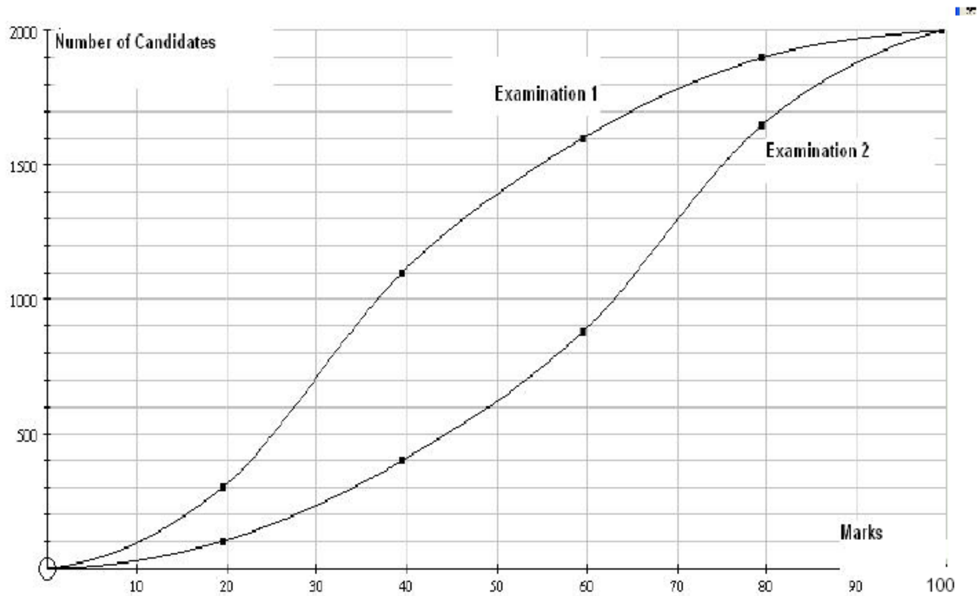
Practice Exam 4 (Business)

1 The diagram below shows the cumulative frequency graphs for the marks scored by the candidates in two examinations. The 2000 candidates each took 2 examinations.



(i) Use the diagram to estimate the median mark for examination 1 and examination 2, and the inter-quartile range for examination 1. **[5]**

(ii) State, with a reason, which of the two examinations you think was the easier one. **[2]**

(iii) Copy and complete the grouped frequency table shown below for examination one and estimate the mean and standard deviation of the marks for examination 1.

Marks	Cumulative frequency	Frequency
0 - 19	300	
20 - 39		
…		

[8]

2 The functions f and g are defined by

$$f(x) = \frac{4}{3+x}, \quad x > 0$$

$$g(x) = 9 - x^2, \quad x \in \mathbb{R}.$$

a) Find $fg(x)$. **[3]**

b) (i) Solve the equation $g(x) = 5$ **[2]**
(ii) Explain why the function g does not have an inverse. **[1]**

c) Sketch the graph of $y = g(x)$. **[2]**
Using the same axes, sketch the graph $y = |g(x)|$. **[2]**

d) (i) Find the range of $f(x)$, where $f(x)$ is the function above. **[2]**

(ii) The inverse of f is f^{-1}. Find $f^{-1}(x)$. **[3]**

3 (a) Differentiate xe^{2x} with respect to x. **[2]**

(b) Given $y=(\ln x)^2$ $\quad x>0$

(i) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. **[3]**

(ii) Find the coordinates of the point P at which the *gradient* takes its maximum value. **[4]**

(c) (i) Given that $y=\tan^{-1}x$, express x in terms of y.

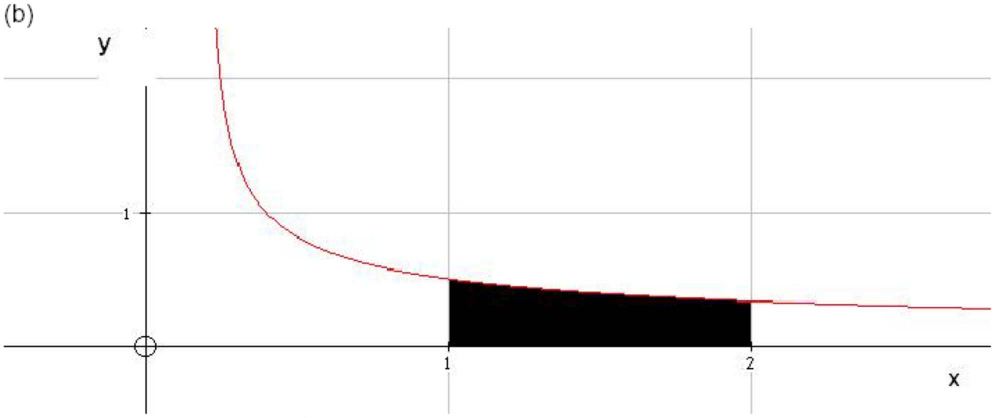
Find $\dfrac{dx}{dy}$ in terms of y.

Show that $\dfrac{dy}{dx}=\dfrac{1}{1+x^2}$. **[4]**

(ii) Evaluate $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$ **[2]**

4

(b)



The diagram above shows the curve

$$y=\frac{1}{\sqrt{(5x-1)}}.$$

The shaded region is enclosed by the curve, the x-axis, the line $x=1$ and the line $x=2$.

(i) Show that the exact value of the area of the shaded region is $\tfrac{2}{5}$. **[5]**

(ii) The shaded region is rotated completely about the x-axis. Find the exact value of the volume of the solid formed. **[4]**

5 (a) (i) Express $\dfrac{2}{x(x+1)(x+2)}$ as the sum of partial fractions **[4]**

(ii) Hence show that $\displaystyle\int_2^4 \frac{2}{x(x+1)(x+2)}\,dx = 2\ln 3 - 2\ln 5$ **[4]**

(b) (i) Using the identity $\tan x \equiv \dfrac{\sin x}{\cos x}$, $\quad -\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$,

show that $\dfrac{d}{dx}(\tan x)=\sec^2 x$. **[3]**

(ii) Use integration by parts to find

$$\int x\sec^2 x\,dx$$ **[4]**

6 (a) The events A and B are such that $P(A \mid B) = 0.5$, $p(B \mid A) = 0.25$, and $P(A \cap B) = 0.2$.

(i) Calculate the value of $P(B)$. **[2]**

(ii) Give a reason why A and B are not independent. **[1]**

(iii) Calculate the value of $P(A \cap B')$. **[3]**

(iv) Draw a Venn diagram to illustrate your results. **[2]**

(b) Every year two basketball teams, the Bulls and the Tigers, meet in a cup tournament. From past results it seems that when the Bulls team win, the probability of them winning the next year is 0.7 and in the years when the Tigers team win, the probability of them winning the next year is 0.5. The Bulls won in 2005.

(i) Draw a probability tree for the two years up to 2007. **[2]**

(ii) Find the probability that the Tigers team will win in 2007. **[2]**

(iii) Supposing that the Tigers team win in 2007, what is the probability that it will be their first win for 2 years? **[3]**

7 (a) The discrete random variable X has probability function given by

$$P(X = x) = \begin{cases} kx^3 & x = 1, 2, 3, 4, \\ 0 & otherwise \end{cases}$$ where k is a constant.

(i) Copy and complete the table below showing the probability distribution and determine the value of k. **[3]**

x	1	2	3	4
$P(X=x)$				

(ii) Find the mean and variance of x. **[4]**
(iii) Find $E(5X - 3)$. **[2]**

(b) The table shows the number of pieces in a model, n, and the price paid, £p.

Name	Model 1	Model 3	Model 4	Model 5	Model 6
n	11	21	28	37	75
p	11	26	34	41	88

$$\sum n = 172, \ \sum p = 200, \ \sum n^2 = 8340, \ \sum p^2 = 11378, \ \sum np = 9736.$$

(i) Calculate the equation of the regression line of p on n. **[5]**

(ii) Use your equation to estimate the price of a model with 15 pieces. Answer correct to the nearest pound. **[1]**

8 In a national survey, it was found that 30% of young people failed a standard fitness test. A random sample of 20 people is taken.

(i) Explain why, for such random samples, the mean number of those who failed the fitness test is 6. **[1]**

(ii) Find the probability of the mean number occurring. **[2]**

(iii) Find the probability that at least 4 people fail the test. **[2]**

(iv) Samples of 20 people are taken from 5 areas. What is the probability that in 3 of these samples exactly 6 people fail the test? **[4]**

(v) A school organises a fitness programme for its students for two months.
The school wishes to investigate whether the fitness of the students has improved.
A random sample of 20 students contains 2 who failed the test.
Carry out a suitable hypothesis test at the 5% significance level, stating your hypothesis and conclusions carefully. **[6]**

9 Steve runs 5000 m every day. His times for 12 days are as follows:

Day	Time (min.)
1	23.2
2	25.1
3	24.3
4	23.0
5	24.3
6	23.6
7	23.0
8	24.2
9	23.4
10	22.7
11	24.0
12	23.8

a. Calculate suitable moving averages.
b. Plot the daily times along with the moving averages on a graph.
c. Draw a suitable trendline and extrapolate it to day 16.
d. Calculate the average daily variation for days 4, 7 and 10.
e. Forecast the time for day 16.

10 (a) Express the following simultaneous equations as a matrix equation:

$$-2x + 3y - z = 0$$

$$x - y + 2z = 5$$

$$x + y = -7$$

(b) Hence, solve the simultaneous equations.

Practice Exam 4 Answers

1 (i) The median mark for examination 1 : 38 (+/- 1) **[1]**
median examination 2: 63 (+/- 1) **1**
Examination 1: UQ; 56 (+/- 1) **1**
LQ ; 25 (+/- 1) **1**
the interquartile range for examination 1 = 31 or follow through **1**

(ii) Examination 2 appears to be the easier one **[1]**
as the median mark is higher **1**
Or as the marks in all the groups is higher

(iii) Complete the grouped frequency table for examination one and estimate the mean and standard deviation of the marks for examination 1.

Length	Cumulative frequency	Frequency
0 - 19	300	300
20 - 39	1100	800
40 - 59	1600	500
60 - 79	1900	300
80-100	2000	100

[2]

Mid-points: 9.5,29.5,49.5,69.5,89.5 (Discrete data) **1**
Mean = (300x 9.5+ 800 x 29.5 + 500 x 49.5 + 300 x 69.5 + 100 x 89.5)/2000 = 81000/2000 **1**
= 40.5 (only 1 mark lost if mid points 10,30 … used) **1**
Var = 4198500/2000 – 40.5^2 = 359 **2**
s.d. = 18.9 **1`**

2 $f(x)=\dfrac{4}{3+x},\quad x>0$ $\qquad g(x)=9-x^2,\quad x\in\mathbb{R}.$

a) $fg(x)$ = f(9-x^2) **[1]**

$= \dfrac{4}{3+9-x^2}=\dfrac{4}{12-x^2}$, **[1]**

($9-x^2$) >0, |x| < 3 **1**

b) (i) $g(x)=5$

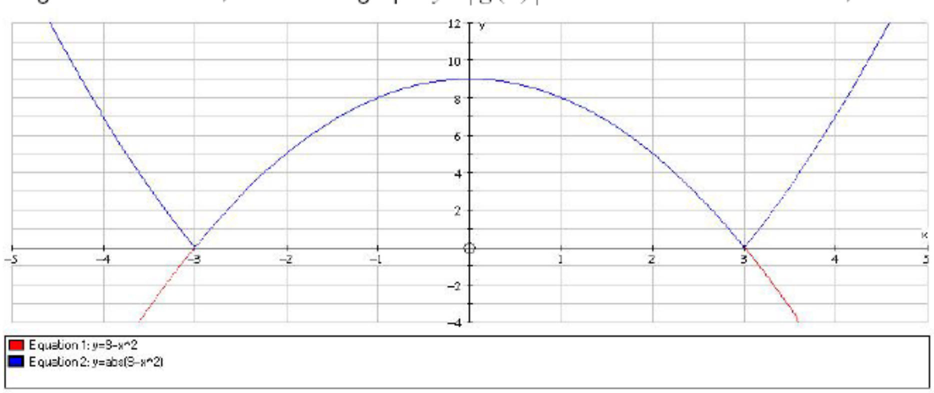
$9-x^2=5$ **[1]**

x^2 =4

x= ± 2 **[1]**

(ii) function g does not have an inverse as g is a many-one function (or equivalent). **[1]**

c) Sketch the graph of $y=g(x)$. Shape- 1, Max point 1 **[2]**
Using the same axes, sketch the graph $y=|g(x)|$. Centre section 1, sides 1 **[2]**



(i) 0 < f(x) < 4/3 **2**

(ii) $f^{-1}(x)$.: $y=\dfrac{4}{3+x}$

$x=\dfrac{4}{y}-3$ **[1]**

$f^{-1}(x)=\dfrac{4}{x}-3$, 0<x<4/3 **1,1**

3

(a) $\frac{d}{dx}xe^{2x} = e^{2x} + 2xe^{2x}$ with respect to x. **2**

(b) $y = (\ln x)^2$ $x>0$

(i) $\frac{dy}{dx} = 2\ln(x).\frac{1}{x}$ **1**

$\frac{d^2y}{dx^2} = \frac{2}{x^2} - \frac{2\ln(x)}{x^2} = \frac{2(1-\ln x)}{x^2}$ **2**

(ii) The *gradient* takes its maximum value when $\frac{d^2y}{dx^2} = 0$. **1**

$\frac{2(1-\ln x)}{x^2} = 0$ **1**

$1 = \ln x$

$x = e$ **1**

y = 1 **1**

(c) (i) $y = \tan^{-1}x$, x = tan y

$\frac{dx}{dy}$ = $\sec^2 y$ **1**

$\sec^2 y = 1 + \tan^2 y$ **1**

$= 1 + x^2$

Show that **1**

$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$ **1**

(ii) $\int_0^1 \frac{1}{1+x^2}dx = \left[\tan^{-1}x\right]_0^1$ **1**

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$ **1**

4

(b) (i)

$\int_1^2 \frac{1}{\sqrt{(5x-1)}}dx = \int_1^2 (5x-1)^{-1/2}dx$ **1**

$= \left[\frac{2(5x-1)^{1/2}}{5}\right]_1^2$ **2**

$= 6/5 - 4/5$ **1**

$= 2/5$ **1**

(ii) Volume = $\int_1^2 \pi y^2 dx$ **1**

$= \pi\int_1^2 \frac{1}{5x-1}dx$ **1**

$= \left[\frac{\pi}{5}(\ln|5x-1|)\right]_1^2$ **1**

$= \frac{\pi}{5}\ln(9/4)$ **1**

5

(a) (i) $$\frac{2}{x(x+1)(x+2)} \equiv \frac{A}{x}+\frac{B}{(x+1)}+\frac{C}{(x+2)}$$

$2 \equiv A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$ **1**

$x=0 \Rightarrow \quad 2=2A, \quad A=1$ **1**

$x=-1 \Rightarrow \quad 2 = -B, \quad B=-2$ **1**

$x = -2 \Rightarrow \quad 2 = 2C, \quad C=1$ **1**

$$\frac{2}{x(x+1)(x+2)} \equiv \frac{1}{x}-\frac{2}{(x+1)}+\frac{1}{(x+2)}$$

(b)

(ii) Hence show that

$$\int_2^4 \frac{2}{x(x+1)(x+2)}dx = 2\ln 3 - 2\ln 5 .$$

$$\int_2^4 \frac{2}{x(x+1)(x+2)}dx = \int_2^4 \frac{1}{x}-\frac{2}{(x+1)}+\frac{1}{(x+2)}dx$$ **1**

$= \left[\ln|x| - 2\ln|x+1| + \ln|x+2|\right]_2^4$ **1**

$= \ln 4 - 2\ln 5 + \ln 6 \quad -(\ln 2 - 2\ln 3 + \ln 4)$ **1**

$= \ln 4 - 2\ln 5 + \ln 2 + \ln 3 - \ln 2 + 2\ln 3 - \ln 4$

$= 3 \ln 3 - 2 \ln 5$ q.e.d. **1**

(a) (i) $$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x}$$ **1**

$$= \frac{1}{\cos^2 x}$$ **1**

$= \sec^2 x$ **1**

(ii) $\int x\sec^2 x\,dx$

Let u = x and dv/dx = $\sec^2$ x, **1**

v=tan x **1**

I = x tanx - $\int \tan x\,dx$ **1**

I = x tan x + ln cos x + C **1**

6 (a) $P(A \mid B) = 0.5$, $p(B \mid A) = 0.25$, $P(A \cap B) = 0.2$.

(i) $P(B) = \dfrac{P(A \cap B)}{P(A \mid B)}$ **M1**

= 0.2/0.5 = 0.4 **A1**

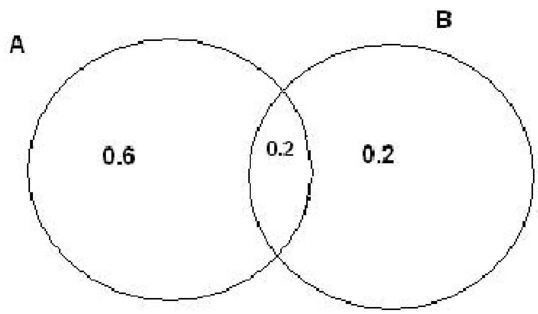
(ii) Give a reason why A and B are not independent. **1**

$P(B) \neq p(B \mid A)$

(iii) $P(A \cap B')$ = P(A) - $P(A \cap B)$ **M1**

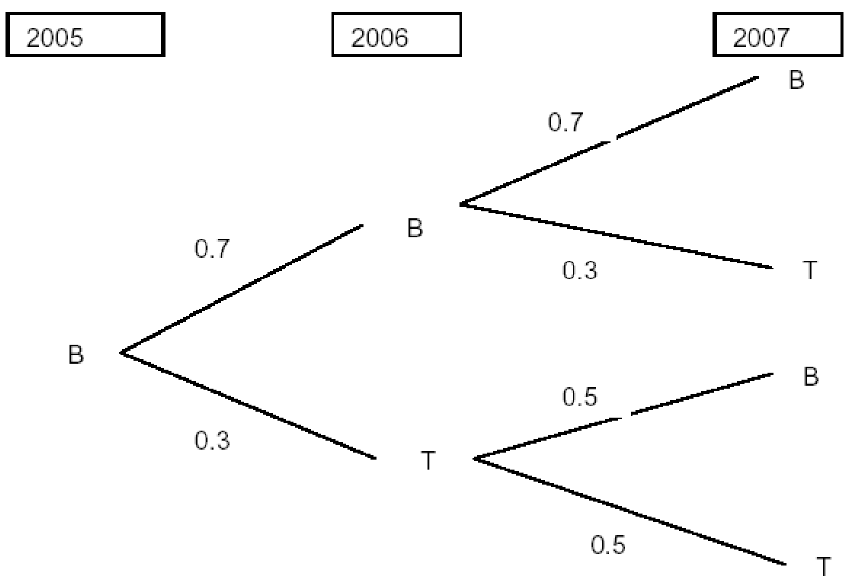
P(A) = $\dfrac{P(A \cap B)}{P(B \mid A)}$ = 0.2 / 0.25 = 0.8 **1**

$P(A \cap B')$ = 0.8 – 0.2 = 0.6 **1**



[2]

(iv)



(b) **[3]**

(b) P(Tigers win in 2007) = 0.7 x 0.3 + 0.3 x 0.5 **[2]**

= 0.36

[2]

(c) P(First/Tigers win 2007) = 0.21 / 0.36 = 7/12

7 (a) (i)

$$P(X = x) = \begin{cases} kx^3, & x = 1, 2, 3, 4, \\ 0 & otherwise, \end{cases}$$ where k is a constant.

x	1	2	3	4	
$P(X=x)$	k	8k	27k	64k	**1**

$\sum P(X = x) = 1$ **M1**

100k = 1 **A1**

k = 0.01

(ii) E(X) = 1(k) + 2(8k) +3(27k) + 4(64k) **M1**

= 354k = 3.54 **A1**

Var(X) = $E(X^2) - (E(X))^2$ = 13-12.542 = 0.468 **M1A**

(iii) $E(5X - 3)$ = 5E(X) – 3 = 14.7 **M1A**

(b) $\overline{n}$ = 172/5 = 34.4 **0.5**

$\overline{p}$ = 200/5 = 40 **0.5**

$s_{np} = \frac{9736}{5} - 34.4 \times 40 = 571$ **1**

$s_n{}^2 = \frac{8340}{5} - 34.4^2 = 484.64$ **1**

Regression line of p on n: **1**

$p - 40 = \frac{571}{484.64}(n - 34.4)$ **1**

p = 1.18 n - 0.530

n = 15, p = £17 (nearest £) **1**

8 (i) Number in the test, n=20 the probability of failing , p=0.3

mean = np = 20*0.3 = 6 **1**

(ii) Find the probability of the mean number occurring.

X=the number who fail

$X \cap B(20, 0.3)$ **M1**

$P(X=6) = P(X \le 6) - P(X \le 5)$ = 0.6080 -0.4164 = 0.192 (tables) **A1**

or $C_6^{20} 0.3^6\, 0.7^{14}$ = 0.192

(iii) Find the probability that at least 4 people fail the test.

$P(X \ge 4) = 1 - P(X \le 3)$ **M1**

= 1 – 0.1071 = 0.893 **A1**

(iv) Samples of 20 people are taken from 5 areas. What is the probability that in 3 of these samples exactly 6 people fail the test?

Y = number where 6 fail the test **M2**

$Y \cap B(5, 0.192)$ **2**

P(Y = 3) = $C_3^5 0.1916^3\, 0.8084^2$ **1**

= 0.046

(v) H_0: p = 0.3 **1**

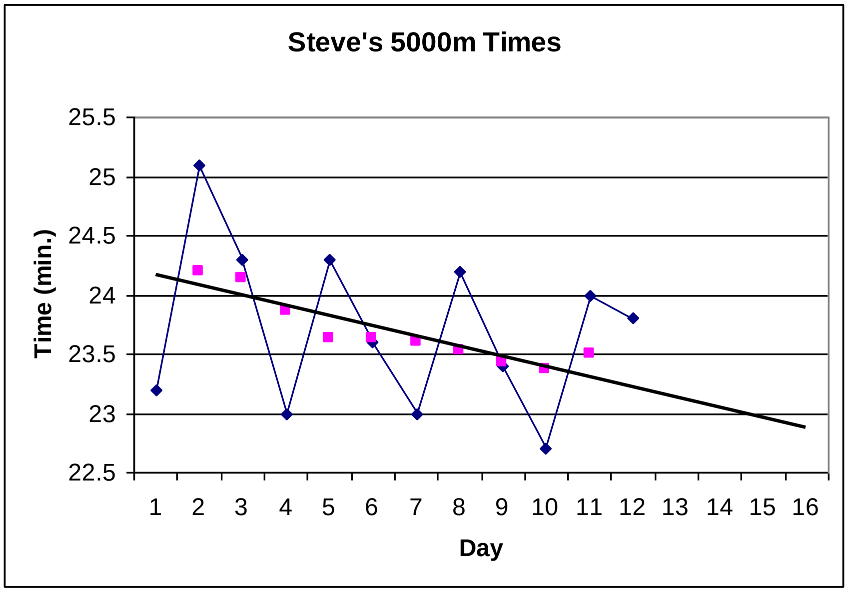
H_1 : p < 0.3

$P(X \le 2)$ 0.0355 <5% **1, 1**

Reject H_0 **1**

The fitness of the students has significantly improved. **1**

9



Day	Time	Moving Av.	Daily Var.
1	23.2		
2	25.1	24.2	
3	24.3	24.13	
4	23.0	23.87	–0.87
5	24.3	23.63	
6	23.6	23.63	
7	23.0	23.60	–0.6
8	24.2	23.53	
9	23.4	23.43	
10	22.7	23.37	–0.67
11	24.0	23.5	
12	23.8		

Average daily variation for days 1, 4, 7 and 10 = $\dfrac{-0.87-0.6-0.67}{3} = -0.71$

Value from trendline for day 16 = 22.9
Forecast for day 16 = 22.9 – 0.71 ≈ 22.2 min.

10

(a) $$\begin{pmatrix} -2 & 3 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -7 \end{pmatrix}$$

(b) $$\det\begin{pmatrix} -2 & 3 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix} = -2(0-2)-3(0-2)-1(1+1) = 8$$

$$\begin{pmatrix} -2 & 3 & -1 \\ 1 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}^{-1} = \frac{1}{8}\begin{pmatrix} -2 & 2 & 2 \\ -1 & 1 & 5 \\ 5 & 3 & -1 \end{pmatrix}^{T} = \frac{1}{8}\begin{pmatrix} -2 & -1 & 5 \\ 2 & 1 & 3 \\ 2 & 5 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -2 & -1 & 5 \\ 2 & 1 & 3 \\ 2 & 5 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 5 \\ -7 \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -40 \\ -16 \\ 32 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \\ 4 \end{pmatrix}$$

$x = -5, y = -2, z = 4$