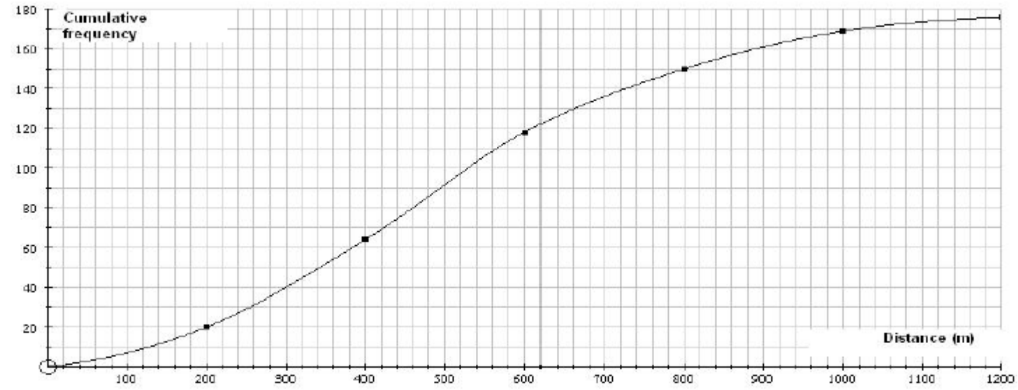
Practice Exam 3 (Business)

1 The cumulative frequency graph below illustrates the distance that 176 students live from their university.



Use the graph to estimate to the nearest 10 metres:

(i) the median distance from school; **[1]**

(ii) the lower quartile, the upper quartile and the inter-quartile range. **[3]**

The graph used the following cumulative frequency grouped data:

Distance (metres)	200	400	600	800	1000	1200
Cumulative frequency	20	64	118	150	169	176

(iii) Copy and complete the grouped frequency table below for this data **[3]**

Distance (d metres)	Frequency (f)
$0 < d \le 200$	20
$200 < d \le 400$	44
....	....

(iv) Estimate the mean distance using your table. **[4]**

(v) Estimate the standard deviation. **[4]**

2 The function f is defined by $f(x) = 1 + \sqrt{x}$ $x \ge 0$,
and the function g is defined by $g(x) = x^2$ $x \in R$.

(i) Find the domain of the inverse function f^{-1}. **[2]**

(ii) Find an expression for $f^{-1}(x)$. **[3]**

(iii) Sketch the graphs of $f(x)$ and $f^{-1}(x)$, using the same axes. **[4]**

(iv) Find and simplify an expression for $fg(x)$ when $x \ge 0$. **[2]**

(v) Explain clearly why the value of $fg(-2)$ is 3. **[1]**

(vi) Sketch the graph of $y = fg(x)$, for both positive and negative values of x, and give the equation of this graph in simplified form. **[3]**

3 (a) Differentiate **[2]**

$$x^3 \cos 3x \qquad \text{with respect to } x.$$

(b) Find the stationary value (turning point) of **[5]**

$$y = \tan x - 8\sin x \qquad \text{for} \qquad 0 < x < \frac{\pi}{2}.$$

(c) Find the equation of the tangent to the curve **[4]**

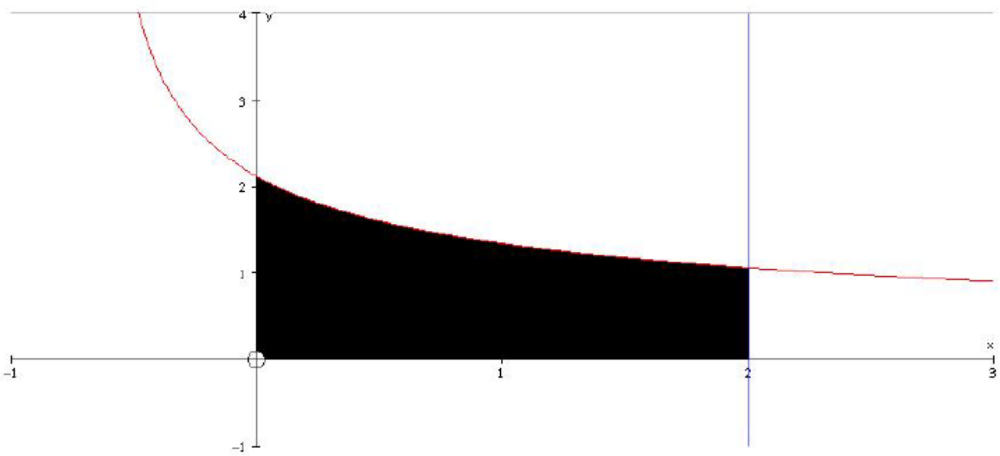
$$y = (x^2+1)^4 \qquad \text{at the point } (1,16).$$

(d) Given that $x = \cos y$, prove that $\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{(1-x^2)}}$. **[4]**

4 (a) Use the substitution $u^2 = 1 + x$ to show that **[7]**

$$\int_0^1 \frac{x}{\sqrt{(1+x)}}\,dx = \frac{2}{3}(2-\sqrt{2}).$$

(b)



The diagram above shows the curve

$$y = \frac{3}{\sqrt{(3x+2)}}$$

The shaded region is enclosed by the curve, the two co-ordinate axes and the line $x = 2$.

(i) Show that the exact area of the shaded region is $2\sqrt{2}$. **[4]**

(ii) The shaded region is rotated completely about the x-axis. Find the exact value of the volume of the solid formed. **[4]**

5 (a) By finding values of A, B and C so that **[5]**

$$\frac{x^2}{x^2-1} = A + \frac{B}{x-1} + \frac{C}{x+1}$$

find $\displaystyle\int \frac{x^2}{x^2-1}\,dx$. **[4]**

(b) Use integration by parts to evaluate the integral **[6]**

$$\int_0^{\frac{\pi}{6}} x\cos 3x\,dx.$$

6 (a) The probability of events A and B are $P(A)$ and $P(B)$ respectively.

If $P(A)=\frac{7}{10}$, $P(A\cap B)=\frac{2}{5}$, $P(A\cup B)=q$, then find, in terms of q,

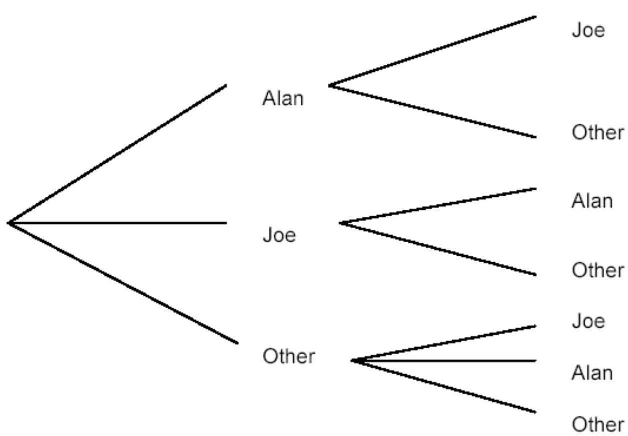
(i) $P(B)$, **[2]**

(ii) $P(A|B)$. **[2]**

(iii) Given that A and B are independent events, **[3]**
Find the value of q.

(iv) Illustrate your results on a Venn Diagram. **[2]**

(b) A table tennis team consists of 8 students, two of whom are Alan and Joe. Two of the team are chosen at random to play the teachers.



By considering the above tree diagram, or otherwise,

(i) find the probability that both Alan and Joe are chosen, **[2]**

(ii) find the probability that at least one of Alan and Joe are chosen, **[2]**

(iii) find the probability that both Alan and Joe are chosen, given that at least one of Alan and Joe is chosen. **[2]**

7 (a) The discrete random variable X has probability function given by

$$P(X=x)=\begin{cases}(\frac{1}{2})^x & x=1,2,3,4,5\\ k & x=6\\ 0 & otherwise\end{cases}$$

where k is a constant.

(i) Copy and complete the table showing the probability distribution and determine the value of k. **[4]**

X	1	2	3	4	5	6
$P(X=x)$						

(ii) Find the mode of x. **[1]**

(iii) Find the mean of x. **[3]**

(iv) Given E(Y) = 8 and Var(Y) = 5, find E(3Y – 1) and Var(3Y – 1).

(b) The table shows unemployment figures and wage increases for a 10 year period.

Percentage unemployed (x)	3.6	4.2	4.3	3.7	3.6	4.1	4.6	3.7	3.5	3.6
Annual percentage increase in wages (y)	5.0	3.2	2.7	2.1	4.1	2.7	2.9	4.6	3.5	4.4

(i) Given: $\bar{x}=3.89$, $\bar{y}=3.52$, $\sum x^2=152.61$, $\sum y^2=132.22$, $\sum xy=135.1$, calculate the product-moment correlation coefficient, correct to 2 decimal places. **[5]**

(ii) By considering the correlation coefficient obtained in (i), comment upon the relationship between wage increases and unemployment. **[2]**

8 Of the spare aircraft parts produced by a factory, 35% are defective. 20 parts are made each day.

(a) Write down the appropriate distribution to model the number of parts that are defective each day. **[1]**

(b) Find the probability that exactly 4 are defective. **[2]**

(c) Find the probability that more than 3 are defective. **[2]**

(d) For a working week of five days, find the probability that on one of these days, exactly 4 defective machines are made. **[4]**

(e) The company aims to improve the standard of production. The following day, 20 parts are made and the number of parts which are defective is 2. Test at the 5% level whether there is significant evidence that the number of defective parts has been reduced. **[6]**

9 Sales of Pac-man (computer game):

Year	Period	Sales (000s)
1999	1	30
	2	35
	3	35
	4	40
	5	50
	6	60
2000	1	30
	2	40
	3	38
	4	35
	5	52
	6	60
2001	1	35
	2	33
	3	37
	4	49
	5	50
	6	65

(a) Calculate 6-period centered moving averages.
(b) Why do you think sales are highest during period 6 (Nov-Dec)?
(c) Construct a graph that includes period sales and the moving average.
(d) Draw the trendline and extrapolate it to the third period of 2002.
(e) Find the mean seasonal variation for the third period, and use this figure along with the trendline to forecast sales for the third period of 2002.

10 The following simultaneous equations

$$4x + 3y - z = 1$$
$$-2x - 2y + z = 1$$
$$x + 5y + z = 4$$

can be expressed in the form

$$(M)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

Find M and M^{-1}. Hence, solve the simultaneous equations.

Practice Exam 3 Answers

1

(a) Use the graph to estimate to the nearest 10 metres:

(i) the median distance from school: accept answers 480-500 **[1]**

(ii) the lower quartile, accept answers 300-320 **[1]**

the upper quartile : accept answers 650-670 **1**

the interquartile range. Follow through from the above **1**

(iii) **[3]**

Distance (d metres)	Frequency (f)
$0 < d \le 200$	20
$200 < d \le 400$	44
$400 < d \le 600$	54
$600 < d \le 800$	32
$800 < d \le 1000$	19
$1000 < d \le 1200$	7
	Sum = 176

(iv) Estimate the mean distance = $\sum f.(mid-po\text{int})/\sum f$ **M1**

= (20 x 100 + 44 x 300 + 54 x 500 + 32 x 700 + 19 x 900 + 7 x 1100)/176 **2**

= 507.95 = 508 (3 s.f.) **1**

(v) Estimate the standard deviation = $\sqrt{\sum fd^2/176 - mean^2}$ **M1**

$\sum fd^2 =$ 57200000 **1**

s.d.^{2} = 32500 – 507.95^2 = 66986.8 **1**

s.d. = 258.8 = 259 (3 s.f.) **1**

2 The function f is defined by $f(x) = 1 + x^{\frac{1}{2}}$ $x \ge 0$,

and the function g is defined by $g(x) = x^2$ $x \in R$.

(i) Domain of f^{-1} is the range of $f(x)$:

$f^{-1}(x) \ge 1$ **[2]**

(ii) y = 1+ √x

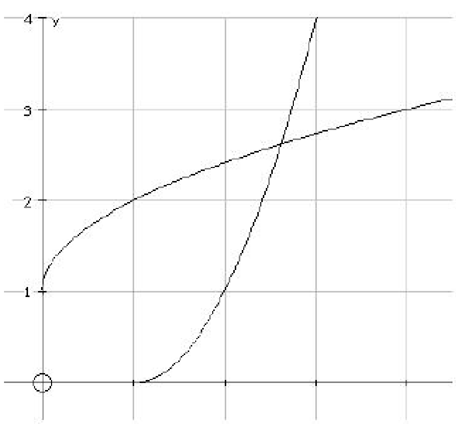
√x = y – 1 **1**

x = (y – 1)^2

$f^{-1}(x) = (x-1)^2$ (or x^2 -2x +1) **1**

1

(iii)Sketch the graphs of $f(x)$ and $f^{-1}(x)$, using the same axes.



s
2

Shape
2

Find and simplify an expression for $fg(x)$ when $x \geq 0$.

$f(x^2)$ **1**

$= 1 + x \qquad x \geq 0$ **1**

Explain clearly why the value of $fg(-2)$ is 3.

$f(4)$ **1**

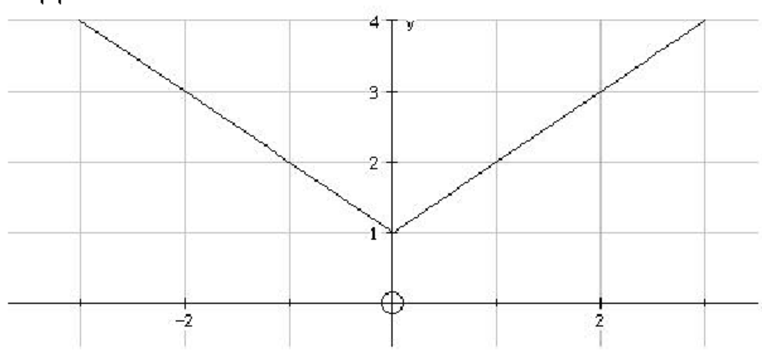
$= 1 + \sqrt{4}$ **1**

$= 3$

Sketch the graph of $y = fg(x)$, for both positive and negative values of x, and give the equation of this graph in simplified form.

$f(x^2) \qquad x \in R$

$= 1 + |x| \qquad x \in R$ **1**



2 (1 for each branch)

3a

$$\frac{d}{dx}(x^3 \cos 3x) = 3x^2 \cos 3x - 3x^3 \sin 3x$$ 2

b

$$y = \tan x - 8\sin x \qquad 0 < x < \frac{\pi}{2}$$ 1

$$\frac{dy}{dx} = \sec^2 x - 8\cos x$$

$$\frac{1}{\cos^2 x} - 8\cos x = 0$$ 1

$$\frac{1 - 8\cos^3 x}{\cos^2 x} = 0$$

$$8\cos^3 x = 1 \qquad \cos x \neq 0$$ 1

$$\cos x = \tfrac{1}{2}$$

$$x = \tfrac{\pi}{3} \qquad y = \sqrt{3} - 8\left(\frac{\sqrt{3}}{2}\right) = -3\sqrt{3}$$ 1 1

c

$y = (x^2+1)^4$ at the point $(1,16)$.

$$\frac{dy}{dx} = 4(x^2+1)^3 2x = 8x(x^2+1)^3$$ 2

When $x = 1, \frac{dy}{dx} = 64$ 1

Equation of the tangent at (1,16): y-16 = 64(x-1)
y = 64x -48 1

d

Given that $x = \cos y$ prove that $\frac{dy}{dx} = -\frac{1}{\sqrt{(1-x^2)}}$

$$\frac{dx}{dy} = -\sin y$$ 1

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$ 1

$$\frac{dy}{dx} = -\frac{1}{\sqrt{(1-\cos^2 y)}}$$ 1

$$\frac{dy}{dx} = -\frac{1}{\sqrt{(1-x^2)}}$$ 1

4

$u^2 = 1 + x$

$\frac{dx}{du} = 2u$ 1

When x=0, u=1 and at x=1, u=$\sqrt{2}$ 1

$\int_0^1 \frac{x}{\sqrt{(1+x)}}dx = \int_1^{\sqrt{2}} \frac{(u^2-1)}{u}2u\,du$ 1

$= \int_1^{\sqrt{2}} (2u^2-2)du$ 1

$= \left[\frac{2u^3}{3} - 2u\right]_1^{\sqrt{2}}$ 1

$= \left(\frac{4\sqrt{2}}{3} - 2\sqrt{2}\right) - \left(\frac{1}{3} - 1\right)$ 1

$= \frac{4}{3} - \frac{2\sqrt{2}}{3} = \frac{2}{3}(2-\sqrt{2})$ 1

(b) (i) Area = $\int_0^2 \frac{3}{\sqrt{(3x+2)}}dx$ 1

$= \int_0^2 3(3x+2)^{-1/2}dx$ 1

$= \left[\frac{3}{3.(1/2)}(3x+2)^{1/2}\right]_0^2$ 1

$= 2\sqrt{8} - 2\sqrt{2}$ 1

$= 4\sqrt{2} - 2\sqrt{2} = 2\sqrt{2}$

(ii) Volume = $\int_0^2 \pi y^2 dx$ 1

$= \pi\int_0^2 \frac{9}{3x+2}dx$ 1

$= \left[\frac{9\pi}{3}(\ln|3x+2|)\right]_0^2$ 1

$= 3\pi(\ln 8 - \ln 2) = 3\pi\ln 4$ 1

5 $$\frac{x^2}{x^2-1} = A+\frac{B}{x-1}+\frac{C}{x+1}$$

$$x^2 = A(x^2-1)+B(x+1)+C(x-1)$$ 1

Let x=1 $\Rightarrow$ B=1/2 1

Let x=0 $\Rightarrow$ A + C = ½

1

Let x = 2 $\Rightarrow$ 3A + C = 5/2
Subtract; 2A = 2, A = 1
Substitute; C = -1/2 1

$$\frac{x^2}{x^2-1} = 1+\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$$

$$\int\frac{x^2}{x^2-1}dx = \int 1+\frac{1}{2(x-1)}-\frac{1}{2(x+1)}dx$$

$$x+\frac{1}{2}\ln|x-1|-\frac{1}{2}\ln|x+1| + c$$ 3

$$= x+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$$ 1

(b) $\int x\cos 3x\,dx$

1

Let u = x and dv/dx = cos 3x

$$v = \frac{\sin 3x}{3}$$ 1

$$\int x\cos 3x\,dx = \left[x\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \frac{\sin 3x}{3}dx$$ 2

$$= \left[\frac{x}{3}\sin 3x+\frac{\cos 3x}{9}\right]_0^{\frac{\pi}{6}}$$ 2

$$= \frac{\pi}{18}+0-(0+1/9)$$

$$=\frac{\pi}{18}-1/9$$ 1

6

(a) $P(A)=\frac{7}{10},\ P(A\cap B)=\frac{2}{5},\ P(A\cup B)=q$

(i) $P(B)$ = q – P(A) + $P(A\cap B)$ M1

= q – 7/10+2/5 = q – 3/10 = q – 0.3 A1

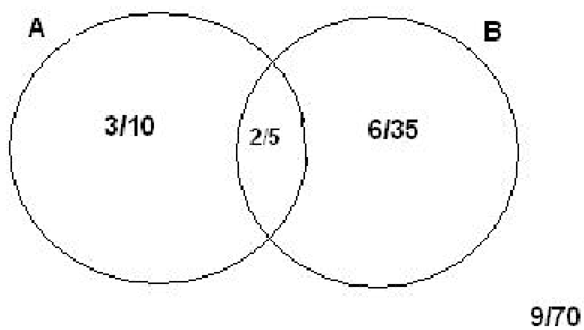
(ii) $P(A|B)$ = $P(A\cap B)$ / $P(B)$ M1

$= \frac{2}{5(q-\frac{3}{10})} = \frac{4}{(10q-3)}$ A1

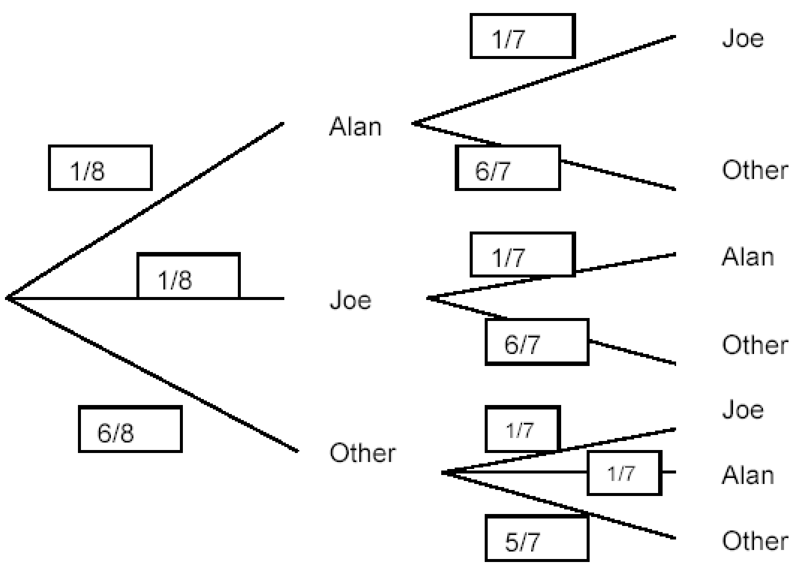
(iii) P(A) = $P(A|B)$ M1

$\frac{7}{10}=\frac{4}{10q-3}$ A1

q = 61/70



(b) First Choice Second choice



M1

(i) P(both Alan and Joe are chosen) = 1/8 x 1/7 + 1/8 x 1/7 = 1/28 A1

M1

(ii) P(at least one of Alan and Joe are chosen) = 1 – 6/8 x 5/7 = 26/56 = 13/28 A1

M1

(iii) P(both Alan and Joe are chosen | at least one of Alan and Joe is chosen) = 1/28 / 13/28 = 1/13 A1

7 (a) 9(i)

x	1	2	3	4	5	6
$P(X=x)$	1/2	¼	1/8	1/16	1/32	k

[2]

$\sum P(X=x)=1$ M1

$\frac{31+32k}{32}=1$ A1

k = 1/32

(ii) The mode of x =1 1

(iii) The mean of x = $\sum xP(X=x)$ M1

= ½ + ½ + 3/8 + ¼ + 5/32 + 6/32

= $1\frac{31}{32}$ A1

(iv.) E(3Y – 1) = 3E(Y) – 1 = 23, Var(3Y – 1) = 3^2Var(Y) = 45

(b) (i) $\bar{x}=3.89$, $\bar{y}=3.52$, $\sum x^2=152.61$, $\sum y^2=132.22$, $\sum xy=135.1$

$s_{xy}=\frac{135.1}{10}-(3.89)(3.52)=-0.1828$ 1

$s_x=\sqrt{\frac{152.61}{10}-3.89^2}=0.3590$ 1

$s_y=\sqrt{\frac{132.22}{10}-3.52^2}=0.9119$ 1

$r=\frac{-0.1828}{0.3590*0.9119}=-0.56$ 1 A1

(ii) Consider the correlation coefficient and comment on the relationship between wage increases and unemployment.

Weak correlation/ some negative correlation 1

Not a strong linear relationship / weak (some) indication that as the percentage of unemployed (x) increases the wage increase (y) falls. / I would need a scatter diagram to comment. 1

8 (a) X = the number of parts that are defective each day

X ∩ B(20,0.35) 1

P(X=4) = P(X<=4) – P(x<=3) = 0.1182 – 0.0444 = 0.074 (tables) M1

or $C_4^{20}0.35^4 0.65^6$ = 0.074 A1

(b) P(X>3) = 1-P(X<=3) = 1-0.0444 = 0.956 M1 A1

(c) For a working week of five days, find the probability that on one of these days, exactly 4 defective machines are made.

Y=no of days with 4 defective parts

Y ∩ B(5,0.074)

P(Y=1) = $C_1^5 0.074^1\ 0.926^4$ M2 M1

= 0.2720 1

(d) H_o : p = 0.35

H_1 : p<0.35 1

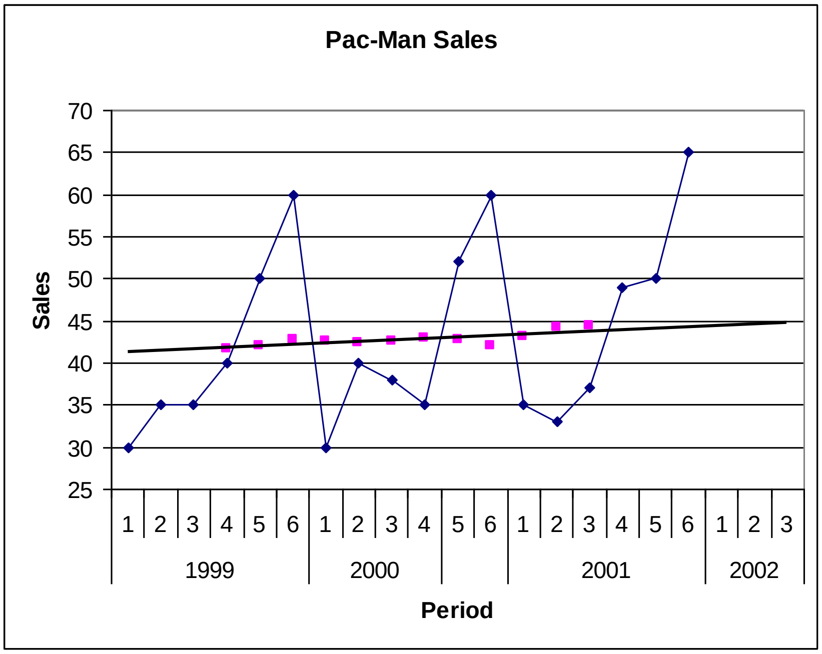
n=20, x=2 1

P(X<=2) = 0.01 <5% 1+1

Reject H_0 (or accept H_1) 1

The sample shows significant evidence that the standard has been improved/ that the number of defective parts has been reduced. 1

9



Sales are highest during period 6 because people send Pac-man as a Christmas gift.

Year	Period	Sales (000s)	Moving Av.	Cntrd M. Av.	Variation
1999	1	30			
	2	35			
	3	35			
			41.67		
	4	40		41.67	
			41.67		
	5	50		42.08	
			42.5		
	6	60		42.75	
			43		
2000	1	30		42.58	
			42.17		
	2	40		42.33	
			42.5		
	3	38		42.5	–4.5
			42.5		
	4	35		42.92	
			43.33		
	5	52		42.75	
			42.17		
	6	60		42.08	
			42		
2001	1	35		43.17	
			44.33		
	2	33		44.17	
			44		
	3	37		44.42	–7.42
			44.83		
	4	49			
	5	50			
	6	65			

Mean seasonal variation for period 3 = $\dfrac{-4.5-7.42}{2} = -5.96$

Value from trendline for 2002 period 3 = 44.8

Forecast for 2002 period 3 = 44.8 – 5.96 = 38.84 ≈ 39 sales

10

$$M=\begin{pmatrix}4 & 3 & -1\\ -2 & -2 & 1\\ 1 & 5 & 1\end{pmatrix}$$

$$\det M=4(-7)-3(-3)-1(-8)=-11$$

$$M^{-1}=\frac{1}{-11}\begin{pmatrix}-7 & 3 & -8\\ -8 & 5 & -17\\ 1 & -2 & -2\end{pmatrix}^{T}=\frac{1}{-11}\begin{pmatrix}-7 & -8 & 1\\ 3 & 5 & -2\\ -8 & -17 & -2\end{pmatrix}$$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{-11}\begin{pmatrix}-7 & -8 & 1\\ 3 & 5 & -2\\ -8 & -17 & -2\end{pmatrix}\begin{pmatrix}1\\ 1\\ 4\end{pmatrix}=\frac{1}{-11}\begin{pmatrix}-11\\ 0\\ -33\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 3\end{pmatrix}$$

$$x=1, y=0, z=3$$