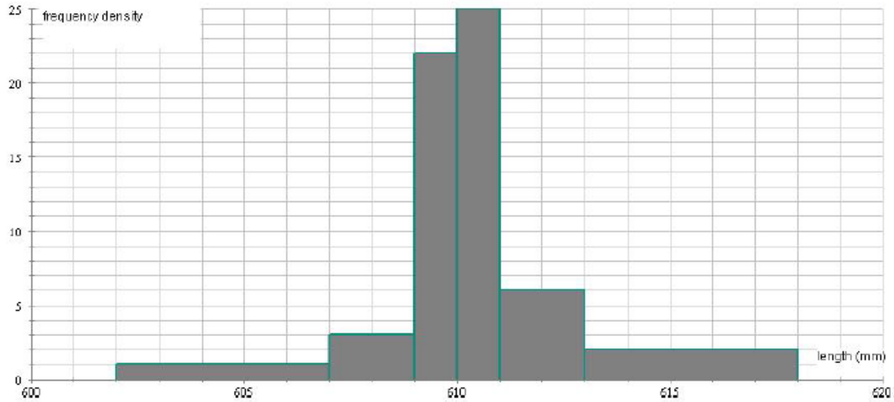
Practice Exam 2 (Business)

1 (a) A man makes garden fence posts which should be 610 mm long.
The lengths of 80 posts are measured.
Their lengths are illustrated in the histogram below.



(i) Copy and complete the grouped frequency table below, stating the number of posts in each of the classes in the histogram. **[3]**

Length	Frequency
602 to 607	
….	

(ii) Estimate the mean length of the posts. Give your answer correct to 1 decimal place. **[3]**

(iii) Estimate the standard deviation of the lengths of the posts. **[4]**

(b) Two hundred and fifty policemen have the following heights.

Height (cm)	No. of men
$160 \le \text{height} < 165$	18
$165 \le \text{height} < 170$	37
$170 \le \text{height} < 175$	60
$175 \le \text{height} < 180$	65
$180 \le \text{height} < 185$	48
$185 \le \text{height} < 190$	22

Find the cumulative frequencies and plot the cumulative curve. **[3]**
Use the curve to estimate:
(i) the median height; **[1]**
(ii) the lower quartile height. **[1]**

2 (a) The function f is defined by

$$f(x) = x^2 + 3.$$

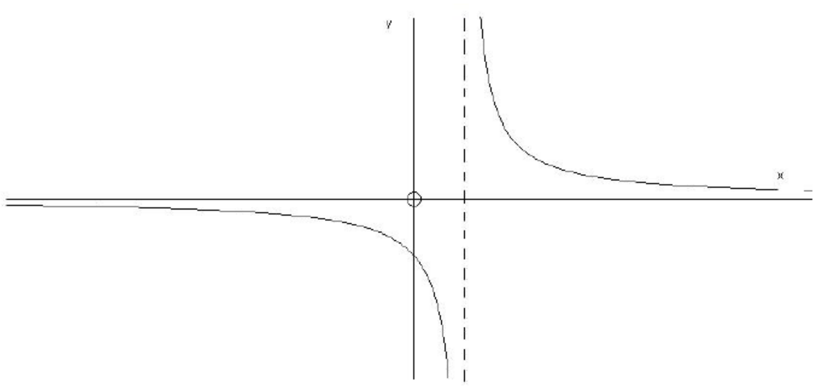
(i) Write down the range of $f(x)$. **[1]**

(ii) Explain why $f(x)$ has no inverse. **[2]**
Suggest a suitable domain so that $f^{-1}(x)$ does exist.

(b) The function g is defined by

$$g(x) = \frac{5}{x-2}, \qquad x \neq 2.$$

The diagram shows a sketch of $y = g(x)$.



(i) Sketch the curve $y = g^{-1}(x)$. Write the equations of the asymptotes. **[5]**

(ii) Calculate the exact x coordinates of the points for which $g(x) = x$. **[3]**

(iii) Find the composite function $gf(x)$, where $f(x)$ is the function in (a) above, and determine its range. **[4]**

3 (a) Differentiate $e^{-x}\sin 2x$ with respect to x. **[2]**

(b) Given that $y = \tan x + 2\cos x$, find the exact value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$. **[4]**

(c) Find the gradient of the curve **[5]**

$$y = \frac{1}{(4x+3)^2}$$

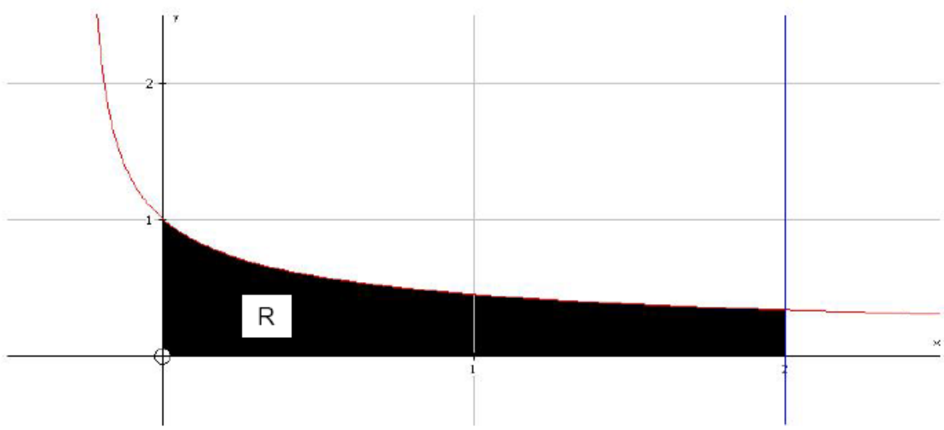
at the point where $x = \frac{1}{4}$.

(d) Given that $x = \tan \frac{1}{2} y$, prove that $\frac{dy}{dx} = \frac{2}{1+x^2}$. **[4]**

4 (a) Use the substitution $u = x - 1$ to evaluate **[7]**

$$\int_2^5 \frac{x}{\sqrt{(x-1)}}\,dx .$$

(b)



The diagram shows the curve

$$y = \frac{1}{\sqrt{(4x+1)}} .$$

The shaded region, R, is enclosed by the curve, the axes and the line $x = 2$.

(i) Show that the exact value of the area of the region R is 1. **[4]**

(ii) The region R is rotated completely about the x axis. Find the exact value of the volume of the solid formed. **[4]**

5 (a) Let $I = \int \frac{1}{x(1+\sqrt{x})^2} dx$

(i) Show that the substitution $u = \sqrt{x}$ transforms I to $\int \frac{2}{u(1+u)^2} du$. **[3]**

(ii) Express $\frac{2}{u(1+u)^2}$ in the form $\frac{A}{u} + \frac{B}{(1+u)} + \frac{C}{(1+u)^2}$ **[4]**

(iii) Use your result from (ii) to find I. **[3]**

(b) Find $\int_0^1 x e^{-2x} dx$, giving your answer in terms of e. **[5]**

6 (a) John and Anne are a brother and sister in a school. The probability that a brother plays basketball is 7/10 and the probability that a sister plays basketball is 2/5. The probability that a sister plays basketball given that her brother plays is 1/2.
A brother and sister are chosen at random.

(i) Show that the probability that both of them play basketball, if John plays, is 7/20. **[2]**

(ii) Draw a Venn diagram to represent these data. **[2]**

Find the probability that:

(iii) only one of them plays basketball; **[2]**

(iv) neither of them plays basketball. **[2]**

(b) Tom classifies the weather on a day as either fine or wet.

From past records he suggests that
* if a day is fine then the probability that the next day is fine is 0.8,
* if a day is wet then the probability that the next day is wet is 0.5

In a particular week it is wet on Monday.

(i) Draw a probability tree diagram for fine or wet days on Tuesday and Wednesday. **[2]**

(ii) Find the probability that Tuesday and Wednesday both have the same weather. **[2]**

(iii) Find the probability that the weather is wet on Wednesday
Given that it is wet on Wednesday, find the conditional probability that it was fine on Tuesday. **[3]**

7 (a) The number of passengers in taxis coming into a city centre is modelled by the probability distribution

$$P(X = r) = \frac{k}{r} \qquad \textit{for} \quad r = 1, 2, 3, 4.$$

(i) Copy and complete the table showing the probability distribution and determine the value of k. **[4]**

r				
$P(X=r)$				

(ii) Calculate $E(X)$ and $Var(X)$. **[4]**

(iii) Calculate $E(3X+1)$ and $Var(3X+1)$.

(b) The table below gives the distance from London to a number of places in Britain and the journey time.

From London to:	Distance (miles) x	Time (mins) y
Birmingham	101	118
Brighton	49	53
Manchester	164	199
Newcastle	248	284
Plymouth	193	238
Hull	156	210
Caenarfon	209	274
Carlisle	260	305
Edinburgh	331	428

where
$\sum x = 1711,\ \sum y = 2109,\ \sum x^2 = 383429,\ \sum y^2 = 589019,\ \sum xy = 474511$

(i) Calculate the correlation coefficient between the two sets of data **[5]**

(ii) What does your answer to (i) tell you about the suitability of drawing the line of best fit to predict y from x? (You are not asked to calculate the line). **[2]**

8 (a) A police team examines the tyres of a large number of lorries.
They find that 17% of lorries have defective tyres.
Six lorries are stopped at random by the police team.

(i) Write down the appropriate distribution to model the number of lorries with defective tyres. **[1]**

(ii) Find the probability that none of the lorries has defective tyres. **[2]**

(iii) Find the probability that exactly two lorries have defective tyres. **[2]**

(iv) Four sets of six lorries are stopped. Find the probability that at most one of these four sets contain exactly two lorries with defective tyres. **[3]**

(b) The police team examines the tyres of a large number of vans (small lorries).
They find that 20% of the vans have defective tyres.
Following a campaign to reduce the number of vehicles with defective tyres, 20 vans are stopped.
Just one of the vans has defective tyres.
You are to carry out a hypothesis test to investigate whether the campaign has been successful.

(i) State your hypotheses clearly. **[2]**

(ii) Carry out the test at the 5% significance level and state your conclusion. **[5]**

9 Browning Pencil Company Sales:

Year	Period	Sales (000s)
2002	Jan-Apr	41
	May-Aug	21
	Sep-Dec	48
2003	Jan-Apr	42
	May-Aug	25
	Sep-Dec	50
2004	Jan-Apr	47
	May-Aug	28
	Sep-Dec	53
2005	Jan-Apr	51
	May-Aug	32
	Sep-Dec	58

(a) Plot the data on a chart.
(b) Why do you think sales are lowest during the May-Aug periods?
(c) Calculate suitable moving averages and add these to the chart.
(d) Draw a suitable trendline for these moving averages on your chart.
(e) Calculate the mean seasonal deviation for Jan-Apr.
(f) Predict sales for Jan-Apr 2006.

10 (a) Given that the inverse of $A = \begin{pmatrix} 1 & 2 & -3 \\ -2 & 3 & 1 \\ -1 & -1 & -3 \end{pmatrix}$ is $A^{-1} = \dfrac{1}{p}\begin{pmatrix} -8 & 9 & 11 \\ -7 & q & 5 \\ 5 & -1 & 7 \end{pmatrix}$,

find p and q.

(b) Hence, solve the following simultaneous equations:

$$x + 2y - 3z = 12$$
$$-2x + 3y + z = -7$$
$$-x - y - 3z = 1$$

Practice Exam 2 Answers

1 a)

Lengths	Frequency
602 to 607	5
607 to 609	6
609 to 610	22
610 – 611	25
611-613	12
613-618	10

[2] freq

[1] total 80

F	x (mid-point)	fx	fx^2
5	604.5	3022.5	1827101
6	608	3648	2217984
22	609.5	13409	8172786
25	610.5	15262.5	9317756
12	612	7344	4494528
10	615.5	6155	3788403
Totals			
80		48841	29818558

[1] x

[1] fx

[1] mean

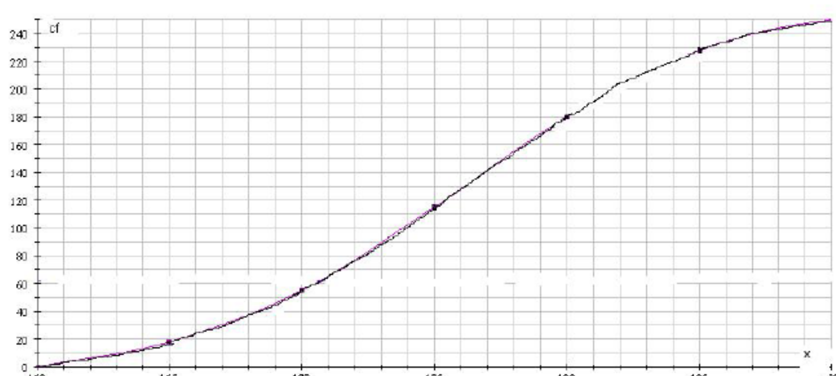
[1] fx^2

Mean = 48841/80 = 610.5 (1d.p.)

Var = 29818558/80 - 610.5^2 = 6.456 **[1] var**

Standard Deviation = 2.54 (3 s.f.) **[1] sd**

(b) Plot (160,0), (165,18), (170,55), (175,115), (180.180), (185,228), (190,250) **[1] uci**



[1] axes
[1] plot

[1] median

Median (cf = 125): 175.8 cm (+/- 0.4)
Lower quartile (c,f, = 62.5): 170.6 cm (+/- 0.4)

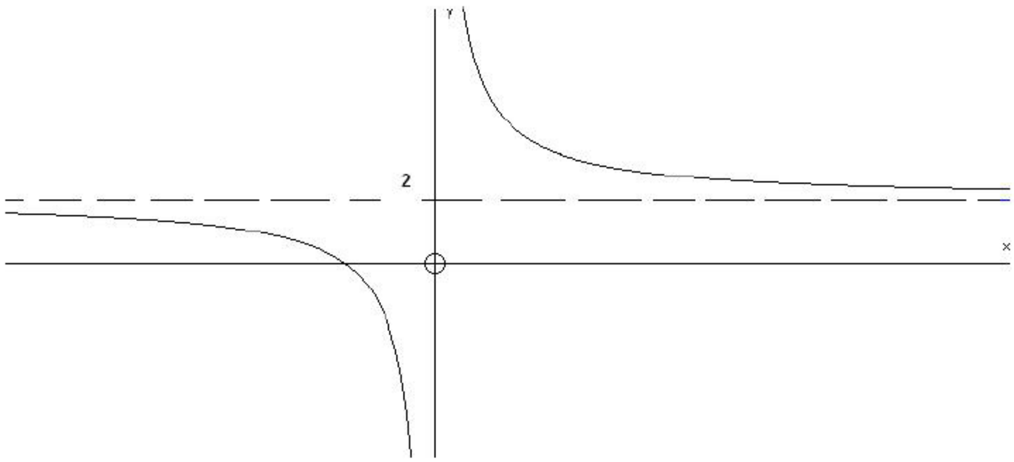
[1] LQ

2 (i) $f(x) \geq 3$ **[1]**

(ii) $f(x)$ has no inverse as it is a many-one function
or as $f(x)$ is not a one-one function…or equivalent **[1]**
Suggest a suitable domain so that $f^{-1}(x)$ does exist: $x \geq 0$ **[1]**

(b)

Sketch the curve $y = g^{-1}(x)$.
Two branches drawn **[1]**
Both branches drawn to the asymptotes **[2]**
Write the equations of the asymptotes: y=2, x=0 **[2]**



ii) Calculate the exact x coordinates of the points for which $g(x) = x$.

$$x = \frac{5}{x-2}$$

$x^2 -2x -5 = 0$ **[1]**
x = (2 +/- (4+20)^0.5)/2 **[1]**
x = 1 +/- √6 **[1]**

iii) Find the composite function $gf(x)$ and determine its range.

$gf(x)$ = g(x^2 + 3) **[1]**

$$= \frac{5}{x^2+3-2}$$

$$= \frac{5}{x^2+1}$$ **[1]**

Max value when x=0, $gf(x)$ = 5 **[1]**
Min value when x $\to \infty$, $gf(x) \to 0$ **[1]**
Range: $0 < gf(x) \leq 5$

3 (a) Differentiate $e^{-x}\sin 2x$ with respect to x.

$$\frac{d}{dx}(e^{-x}\sin 2x) = -e^{-x}\sin 2x + e^{-x}2\cos 2x$$ **2**

(b) $y = \tan x + 2\cos x$,

$$\frac{dy}{dx} = \sec^2 x - 2\sin x$$ **2**

$$At\ x = \frac{\pi}{4},\ \frac{dy}{dx} = 2 - \sqrt{2}$$

2

(c) $y = \dfrac{1}{(4x+3)^2}$ = $(4x + 3)^{-2}$ **1**

$$\frac{dy}{dx} = -2(4x+3)^{-3}4$$ or substitute u = (4x+3) **2**

$$\frac{dy}{dx} = -8(4x+3)^{-3}$$

when $x = 1/4$, $\dfrac{dy}{dx} = -8(4)^{-3}$ **1**

$$\frac{dy}{dx} = -\frac{1}{8}$$ (or 0.125)

1

(d) $x = \tan\frac{1}{2}y$

$$\frac{dx}{dy} = \frac{1}{2}\sec^2\left(\frac{y}{2}\right)$$ **1**

$$\frac{dy}{dx} = \frac{2}{\sec^2(\frac{1}{2}y)}$$ **1**

$$= \frac{2}{1+\tan^2(\frac{1}{2}y)}$$ **1**

$$= \frac{2}{1+x^2}$$ **1**

4 (a) $u = x - 1$

$\frac{du}{dx} = 1$ **1**

At x=2, u=1 and at x=5, u=4 **1**

$\int_2^5 \frac{x}{\sqrt{(x-1)}}dx = \int_1^4 \frac{u+1}{u^{\frac{1}{2}}}\left(\frac{dx}{du}\right)du$ **1**

$= \int_1^4 (u^{\frac{1}{2}} + u^{-\frac{1}{2}})du$ **1**

$= \left[\frac{2u^{\frac{3}{2}}}{3} + \frac{2u^{\frac{1}{2}}}{1}\right]_1^4$ **1**

$= 6\frac{2}{3}$ **2**

(b) (i) Area = $\int_0^2 \frac{1}{\sqrt{(4x+1)}}dx$ **1**

$= \int_0^2 (4x+1)^{-1/2}dx$ **1**

$= \left[\frac{2}{4}(4x+1)^{1/2}\right]_0^2$ **1**

$= \frac{3}{2} - \frac{1}{2}$

= 1 **1**

(ii) Volume = $\int_0^2 \pi y^2 dx$ **1**

$= \pi\int_0^2 \frac{1}{4x+1}dx$ **1**

$= \left[\frac{\pi}{4}(\ln|4x+1|)\right]_0^2$ **1**

$= \frac{\pi}{4}\ln 9$ **1**

5 (a) $I = \int \frac{1}{x(1+\sqrt{x})^2} dx$

1

(i) $u = \sqrt{x}$, $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, $\frac{dx}{du} = 2\sqrt{x}$ or $x = u^2$, $\frac{dx}{du} = 2u$

M1 A1

$I = \int \frac{1}{x(1+\sqrt{x})^2} \frac{dx}{du} du = \int \frac{1}{u^2(1+u)^2} 2u \, du$

$= \int \frac{2}{u(1+u)^2} du$

(ii) $\frac{2}{u(1+u)^2} = \frac{A}{u} + \frac{B}{(1+u)} + \frac{C}{(1+u)^2}$

$2 = A(1+u)^2 + Bu(1+u) + Cu$ **1**

$u=0 \Rightarrow 2 = A$

$u = -1 \Rightarrow 2 = -C, \quad C = -2$

$u = 1 \Rightarrow 2 = 8 + 2B - 2, \quad B = -2$ **3**

$\frac{2}{u(1+u)^2} = \frac{2}{u} - \frac{2}{(1+u)} - \frac{2}{(1+u)^2}$

(iii) $I = 2\int \left(\frac{1}{u} - \frac{1}{(1+u)} - \frac{1}{(1+u)^2} \right) du$

$= 2(\ln u - \ln|1+u| + 1/(1+u))$ **2**

$= 2\ln\left(\frac{\sqrt{x}}{1+\sqrt{x}} \right) + \frac{2}{1+\sqrt{x}} + C$ **1**

(b) Find $\int_0^1 xe^{-2x} dx$, giving your answer in terms of e.

Let u = x and dv/dx = e^{-2x} **1**

$v = \frac{e^{-2x}}{-2}$ **1**

$\int_0^1 xe^{-2x} dx = \left[x\frac{e^{-2x}}{-2} \right]_0^1 - \int_0^1 \frac{e^{-2x}}{-2} dx$ **1**

$= \left[\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4} \right]_0^1$ **1**

$= -\frac{e^{-2}}{2} - \frac{e^{-2}}{4} - (0 - \frac{1}{4})$

$= \frac{1}{4} - \frac{3e^{-2}}{4}$ **1**

6 (a) $P(B) = \frac{7}{10}, \quad P(G) = \frac{4}{10}, \quad P(G \mid B) = \frac{1}{2}$

(i) $P(B \cap G) = P(B).P(G \mid B) \quad = \frac{7}{10}.\frac{1}{2} = \frac{7}{20}$ **M1** **A1**

(ii)

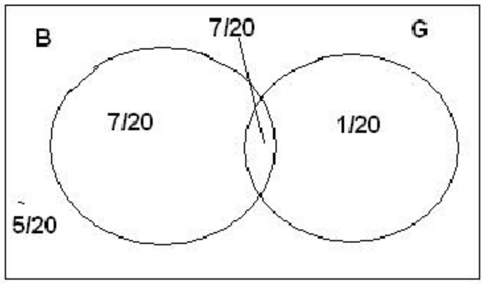
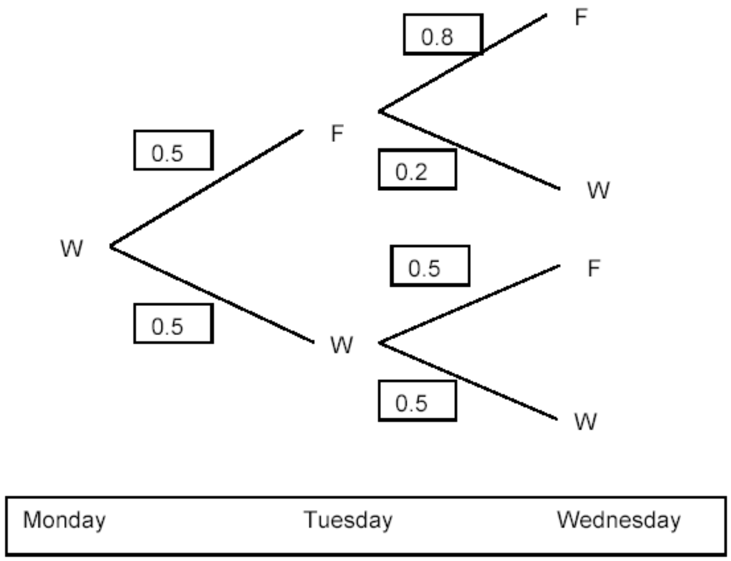


Diagram **1** **area probs 1**

(iii) P(only one of them plays basketball) = 7/20+1/20 = 8/20 = 4/10 = 0.4 **M1** **A1**

(iv) P(Neither of them plays basketball) = 1-(7/20 + 7/20 + 1/20)
= 5/20 = 0.25 **M1** **A1**

Tom classifies the weather on a day as either fine or wet.
From past records he suggests that
* if a day is fine then the probability that the next day is fine is 0.8,
* if a day is wet then the probability that the next day is wet is 0.5 .
In a particular week it is wet on Monday.



2

(ii) P(Tuesday and Wednesday both have the same weather) **M1**
= 0.5^2 + 0.5 x 0.8 = 0.65 **A1**

(iii) P(the weather is wet on Wednesday) = 0.5 x 0.2 + 0.5 x 0.5 = 0.35 **1**
P(FTues|W Wed) = P(FT $\cap$ W W)/P(W W) **M1**
= 0.5 x 0.2 /0.35 = 0.286 **A1**

7 (a) The number of passengers in taxis coming into a city centre is modelled by the probability distribution

$$P(X=r)=\frac{k}{r} \qquad for \;\; r=1,2,3,4.$$

(i) Copy and complete the table showing the probability distribution and determine the value of k.

r	1	2	3	4
$P(X=r)$	k/1	k/2	k/3	k/4

[2]

$\sum P(X=r) = 1$ **M1**

25k/12 = 1

k = 12/25 = 0.48 **A1**

(ii) $E(X) = \sum xp$ = k+k+k+k = 4k **M1**

= 48/25 = 1.92 **A1**

$Var(X) = \sum x^2 p - \overline{x}^2$ **M1**

= k+2k+3k+4k – 16k^2 = 4.8 – 3.6864 = 1.1136 = 1.11 (3s.f.) **A1**

(iii) $E(3X+1) = 3E(X)+1 = 3(1.92) + 1 = 6.76$

$Var(3X+1) = 3^2 Var(X) = 9(1.1136) = 10.02$

(b) n=9

$\sum x = 1711, \; \sum y = 2109, \; \sum x^2 = 383429, \; \sum y^2 = 589019, \; \sum xy = 474511$

(ii) $\overline{x}$ = 1711/9 = 190.11 **1**

$\overline{y}$ = 2109/9 = 234.33

s_x = 80.380 $\qquad s_y$ = 102.638 $\qquad s_{xy}$ = 8174.164 **3**

$$r=\frac{8174.164}{80.38 \text{ x } 102.638}=0.99$$ **1**

(b) **2**

High correlation ⇒ accurate predictions from the line of best fit.

8 (i) X= the number of lorries with defective tyres

$X \cap B(6, 0.17)$ **1**

(ii) P(X=0) = 0.83^6 = 0.327 **M1** **A1**

(iii) P(X=2) = $^6C_2$ 0.17^2 0.83^4 = 0.206 **M1** **A1**

(iv) Y=no with defective tyres per set of six lorries: $Y \cap B(4, 0.207)$

P(Y<=1) = P(Y=0) + P(Y=1) **M1**

= 0.793^4 + 4(0.207)(0.793)^3 = 0.81 **M1** **A1**

(b) (i) H_0 : p = 0.2 **2**

H_1 : p <0.2

(ii)

X = the number of vans with defective tyres

$X \cap B(20, 0.2)$ **1**

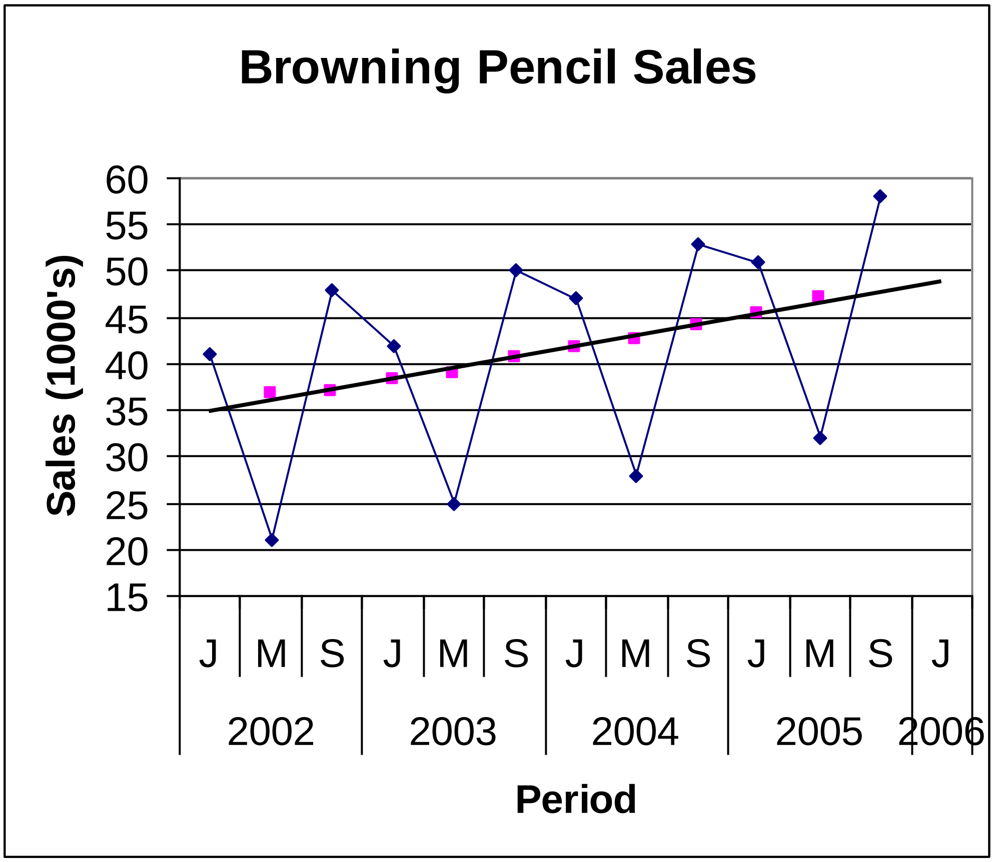
P(X<=1) = 0.069 **1**

which is >5% **1**

Do not reject H_0 (or accept H_0) **1**

No significant evidence of an improvement or campaign appears unsuccessful. **1**

9



Sales are lowest during the May-August periods because students have summer vacation.

Year	Period	Sales (000s	Moving Av.	Deviation
2002	Jan-Apr	41		
	May-Aug	21	36.67	
	Sep-Dec	48	37.00	
2003	Jan-Apr	42	38.33	3.67
	May-Aug	25	39.00	
	Sep-Dec	50	40.67	
2004	Jan-Apr	47	41.67	5.33
	May-Aug	28	42.67	
	Sep-Dec	53	44.00	
2005	Jan-Apr	51	45.33	5.67
	May-Aug	32	47.00	
	Sep-Dec	58		

Mean seasonal deviation for Jan-Apr = $\frac{3.67+5.33+5.67}{3} = 4.89$

Value from trendline for Jan-Apr 2006 = 48.5
Prediction for Jan-Apr 2006 = 48.5 + 4.89 = 53.4 (000) = 53400 pencils

10 (a) $p = \det A = 1(-9+1) - 2(6+1) - 3(2+3) = -37$
$q = (1)(-3) - (-1)(-3) = -6$

(b) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-37}\begin{pmatrix} -8 & 9 & 11 \\ -7 & -6 & 5 \\ 5 & -1 & 7 \end{pmatrix}\begin{pmatrix} 12 \\ -7 \\ 1 \end{pmatrix} = \frac{1}{-37}\begin{pmatrix} -148 \\ -37 \\ 74 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$$

$x = 4, y = 1, z = -2$