Practice Exam 1 (Business)

Section A. Answer ALL questions in this section. They are not equally weighted.

Question A1

Differentiate $x^2 \ln x^2$ with respect to x. **[3]**

Question A2

All the members of a small sports club like either cricket or tennis. Of these, 20 like both, 27 like cricket and 25 like tennis. With the help of a Venn diagram, find how many there are in the club.

[4]

Question A3

The random variable Y has mean 5 and variance 12. Find the mean and variance of $3Y+4$. **[4]**

Question A4

For the probability distribution

r	1	2	3	4
$P(X=r)$	0.2	0.4	0.3	0.1

calculate $E(X)$ and $\mathrm{Var}(X)$. **[4]**

Question A5

The table below shows the number n of ice creams sold each day between 3 pm and 4 pm during a week, together with the maximum temperature T° C recorded each day:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
n	44	57	35	63	81	90	85
T	22	24	20	26	30	28	25

where $\sum T = 175, \sum n = 455, \sum T^2 = 4445, \sum n^2 = 32265, \sum Tn = 11749$

Calculate the correlation coefficient between the two sets of data. **[5]**

Question A6

Given that the inverse of $A = \begin{pmatrix} 1 & -2 & 4 \\ 2 & -3 & 11 \\ -3 & 6 & -11 \end{pmatrix}$ is $A^{-1} = \begin{pmatrix} -33 & 2 & a \\ b & 1 & -3 \\ 3 & 0 & 1 \end{pmatrix}$,

find a and b.

Hence solve for x, y, z the following set of simultaneous equations:

$$\begin{aligned} x - 2y + 4z &= 1 \\ 2x - 3y + 11z &= -5 \\ -3x + 6y - 11z &= 3 \end{aligned}$$

[5]

Question A7

Find the equation of the tangent to the curve $y = \frac{1}{(2x-3)^5}$

at the point $(2,1)$. **[5]**

Question A8

Use integration by parts to find $\int_0^1 x^2 \, e^x \, dx$,

giving your answer in terms of e. **[5]**

Question A9

Evaluate $\int_1^2 \frac{\ln x}{x} dx$ **[5]**

Section B. Answer FOUR questions from this section. The questions are equally weighted

Question B1

The grouped frequency table below shows the number of mobile phones sold per day in a city centre store during a period of 200 working days.

Number of phones sold	Frequency
$66-80$	20
$81-85$	30
$86-90$	55
$91-95$	50
$96-100$	35
$101-115$	10

(i) Calculate the mean number of mobile phones sold per day. **[4]**

(ii) Draw the cumulative frequency curve for the data. **[5]**

(iii) Using the cumulative frequency curve, explain how to obtain the median of the data and find its value. **[3]**

(iv) Find the interquartile range of the data. **[3]**

Question B2

(a) The probability of events A and B are $P(A)$ and $P(B)$ respectively.

If $P(A|B) = 0.7$, $P(B) = 0.4$ and A and B are independent find

(i) $P(A \cap B)$ **[2]**

(ii) $P(A' \cap B')$ **[2]**

(iii) Explain why A and B are not mutually exclusive. **[2]**

(iv) Illustrate your results on a Venn diagram,
labelling each of the four regions with their probabilities. **[2]**

(b) Judy wants to join a netball team.

The probability she is chosen for the Ladybirds team is 0.2,

and for the Butterflies team is 0.5.

Otherwise she must play for the Moths team.

The probability that the Ladybirds team wins is 0.7,

that the Butterflies team wins is 0.6,

but the probability that the Moths team **loses** is 0.9.

(i) Draw a probability tree to illustrate the above situation. **[2]**

(ii) What is the probability that Judy's team wins? **[3]**

(iii) Given that Judy's team loses,
what is the probability that she is in the Butterflies team? **[2]**

Question B3

A heating oil merchant has recorded their sales during the period 2004 – 2007. The following table shows their sales in millions of litres:

Year	Period	Sales
2004	Jan – Apr	35
	May – Aug	15
	Sep – Dec	42
2005	Jan – Apr	36
	May – Aug	19
	Sep – Dec	44
2006	Jan – Apr	41
	May – Aug	22
	Sep – Dec	47
2007	Jan – Apr	45
	May – Aug	26
	Sep – Dec	52

(i) Plot the data in an appropriate manner. **[3]**

(ii) Calculate suitable moving averages and add these to your chart. **[7]**

(iii) Draw a suitable trendline for these moving averages on your chart. **[1]**

(iv) Calculate the mean seasonal deviation for the Jan – Apr periods. **[2]**

(v) Forecast the sales for Jan – Apr 2008. **[2]**

Question B4

A market gardener expects 65% of his tomato seeds to germinate and grow into healthy plants. He plants 15 seeds in a tray.

(i) Find the probability that exactly 3 will **fail** to grow. **[3]**

(ii) Find the probability that more than 5 will fail to grow. **[3]**

(iii) If he planted 4 trays each with 15 seeds, find the probability that in exactly 2 of the trays more than 5 of the seeds will fail to grow. **[3]**

(iv) The gardener buys a different variety of tomato seed which claims to have a higher germination rate. He plants 15 of these new seeds in a tray. How many of the seeds must germinate to convince the gardener that the new variety is really better than the old. (The null hypothesis and the alternative hypothesis should be clearly stated and you should use a 5% significance level). **[6]**

Question B5

The functions f and g are defined as follows:

$$f(x) = \tan\left(\frac{\pi x}{2}\right) \qquad -1 < x < 1$$

$$g(x) = |x| \qquad x \in \mathbb{R}$$

(i) Find the range of $f(x)$. **[1]**

(ii) Write down $gf(x)$ and find the value of $g(f(-1/3))$. **[3]**

(iii) Sketch the graphs of $f(x)$ and $gf(x)$ using the same axes. **[4]**

(iv) Find an expression for $f^{-1}(x)$.
Explain why $g(x)$ has no inverse. **[4]**

(v) One of f and g is an odd function.
State which one is odd, giving your reason.
Explain why the other function is not odd. **[3]**

Question B6

(a) Find the stationary values (turning points) of $y = x^2 \mathrm{e}^{-x}$ and decide if each is a maximum or a minimum. **[6]**

(b) Let $f(x) = 1 - \frac{1}{x^2}$.

(i) Sketch the curve $y = f(x)$ for $1 \le x \le 2$. **[2]**

(ii) Find the area under this curve in the given region. **[3]**

(iii) The area is rotated completely about the x-axis.
Find the exact value of the volume of the solid formed. **[4]**

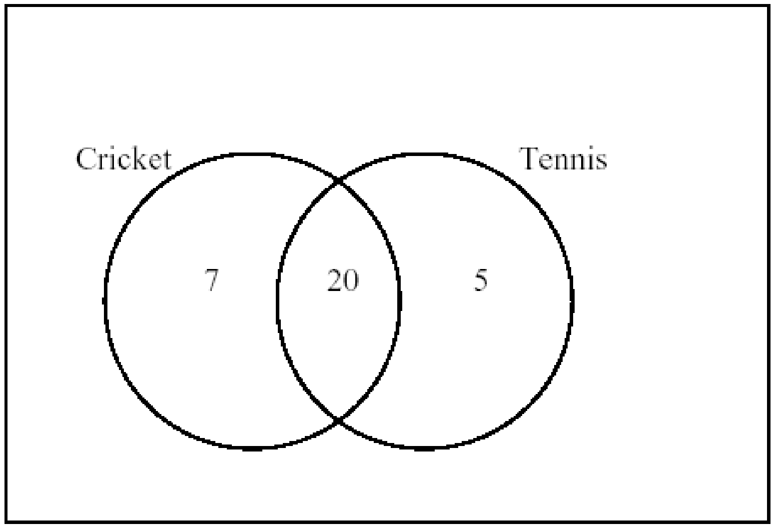
Practice Exam 1 Answers

Section A

A1 $\frac{dy}{dx} = 2x \ln x^2 + x^2 \frac{1}{x^2} 2x$ **[2]**

$= 2x \ln x^2 + 2x$ **[1]**

A2



Venn diagram drawn **[1]**
Numbers entered as above **[2]**
Total number in club = 7+20+5 = 32 **[1]**

A3 $\text{Mean}(3Y+4) = 3 \times 5 + 4 = 19$ **[M1A1]**

$\text{Variance}(3Y+4) = 3^2 \times 12 = 108$ **[M1A1]**

A4

x	p	xp	$x^2 p$
1	0.2	0.2	0.2
2	0.4	0.8	1.6
3	0.3	0.9	2.7
4	0.1	0.4	1.6
sum	1.0	2.3	6.1

$E(X) = \sum xp = 2.3$ **[M1A1]**

$Var(X) = \sum x^2 p - \left(\sum xp\right)^2 = 6.1 - 2.3^2 = 0.81$ **[M1A1]**

A5 The means are, respectively,
25, 65, 635, 4609, 1678 **[1]**

$S_T = 3.162$ **[1]**

$S_n = 19.60$ **[1]**

$S_{Tn} = 53.43$ **[1]**

$r = 0.862$ **[1]**

A6 $A^{-1}A = I$

$$\begin{pmatrix} -33 & 2 & a \\ b & 1 & -3 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 & 4 \\ 2 & -3 & 11 \\ -3 & 6 & -11 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$-33 + 4 - 3a = 1,\ a = -10$ **[1]**

$b + 2 + 9 = 0,\ b = -11$ **[1]**

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -33 & 2 & -10 \\ -11 & 1 & -3 \\ 3 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -73 \\ -25 \\ 6 \end{pmatrix}$$

$x = -73$ **[1]**

$y = -25$ **[1]**

$z = 6$ **[1]**

A7 $\dfrac{dy}{dx} = \dfrac{-5}{(2x-3)^6}2 = \dfrac{-10}{(2x-3)^6}$ **[1]**

$= -10$ at the point $(2,1)$. **[1]**

The equation of the tangent is $y = -10x + c$. **[1]**

$1 = -10 \times 2 + c,\ c = 21.$ **[1]**

$y = -10x + 21$ **[1]**

A8 $\displaystyle\int_0^1 x^2 e^x \, dx = \left[x^2 e^x\right]_0^1 - \int_0^1 2x e^x \, dx$ **[1]**

$\displaystyle = \left[x^2 e^x - 2x e^x\right]_0^1 + \int_0^1 2 e^x \, dx$ **[1]**

$= \left[x^2 e^x - 2x e^x + 2 e^x\right]_0^1$ **[1]**

$= e - 2e + 2e - 0 + 0 - 2$ **[1]**

$= e - 2.$ **[1]**

A9 Let $u = \ln x$ **[1]**

So $du = \dfrac{1}{x}dx$ **[1]**

Hence $\displaystyle\int_1^2 \frac{\ln x}{x} dx = \int_{\ln 1}^{\ln 2} u \, du$ **[1]**

$$= \left[\frac{u^2}{2}\right]_{\ln 1}^{\ln 2} = \frac{(\ln 2)^2 - (\ln 1)^2}{2} = \frac{1}{2}(\ln 2)^2 \approx 0.240$$ **[2]**

Section B

B1(i)

		x	f	xf
65.5	80.5	73	20	1460
80.5	85.5	83	30	2490
85.5	90.5	88	55	4840
90.5	95.5	93	50	4650
95.5	100.5	98	35	3430
100.5	115.5	108	10	1080
			200	17950

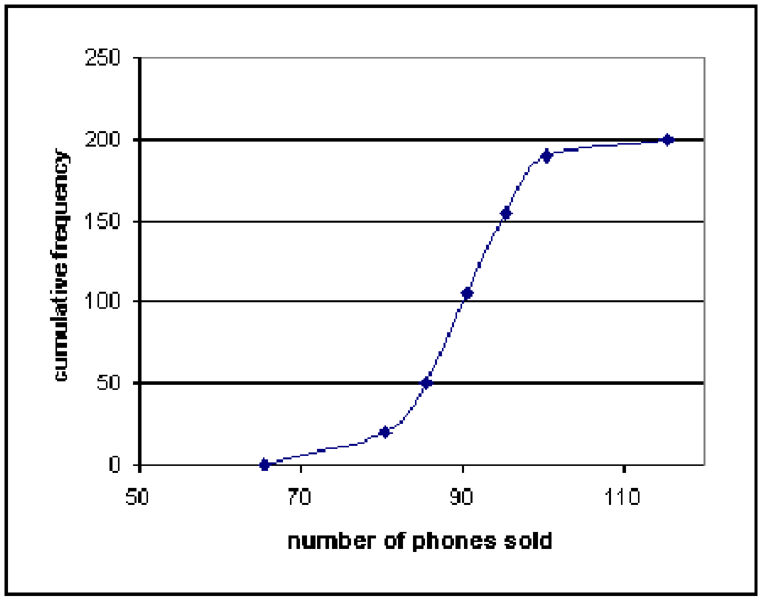
x column **[1]**

xf column **[2]**

mean $= \dfrac{\sum xf}{\sum f} = \dfrac{17950}{200} = 89.75$. **[1]**

(ii)

lcb	ucb	cf
	65.5	0
65.5	80.5	20
80.5	85.5	50
85.5	90.5	105
90.5	95.5	155
95.5	100.5	190
100.5	115.5	200



see above

labels on vertical axis either absolute or as percentage. **[1]**

values plotted on upper class boundaries **[1]**

points accurately found and plotted **[2]**

smooth curve **[1]**

(iii) On graph look at the 50% line, labelled 100, and read down. **[2]**

The value is 90. (Allow variation according to graph drawn.) **[1]**

(iv) The lower quartile is 85.5, (Allow variation according to graph drawn.) **[1]**

the upper quartile is 95, (Allow variation according to graph drawn.) **[1]**

the difference is 9.5. (Allow variation according to graph drawn.) **[1]**

B2(a)

(i) $P(A) = 0.7,$ **[1]**

$P(A \cap B) = P(A)P(B) = 0.28$ **[1]**

(ii) $P(A') = 0.3,\ P(B') = 0.6,$ **[1]**

$P(A' \cap B') = 0.18$ **[1]**

(iii) If A and B are mutually exclusive, $P(A \cap B) = 0 \neq 0.28.$ **[2]**

(iv)

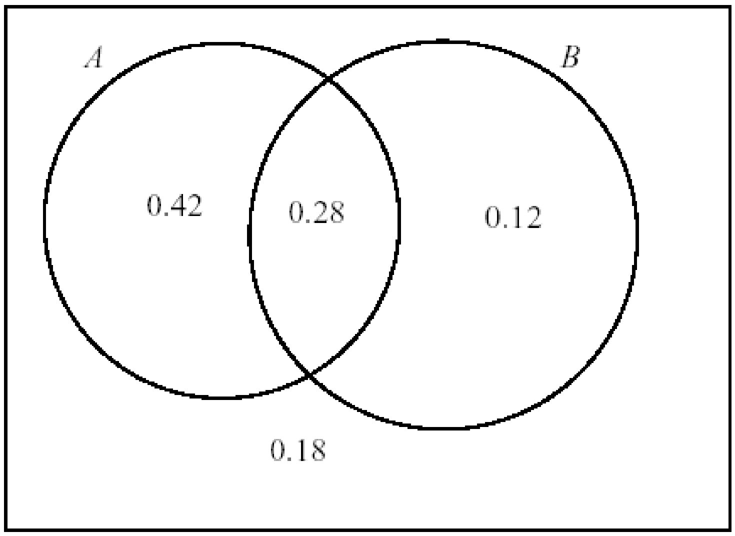
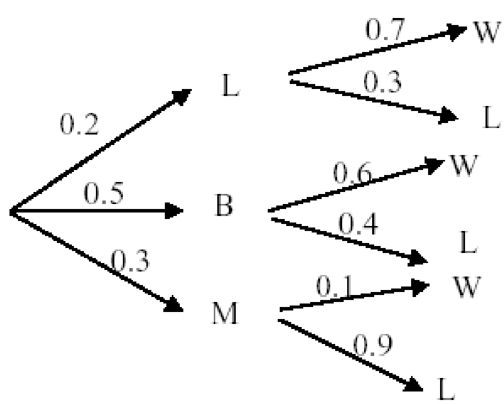


Diagram **[1]**

All 4 probabilities shown **[1]**

(b)(i)



[2]

(ii) P(J wins) = P(J in L and W) + P(J in B and W) + P (J in M and W) **[1]**

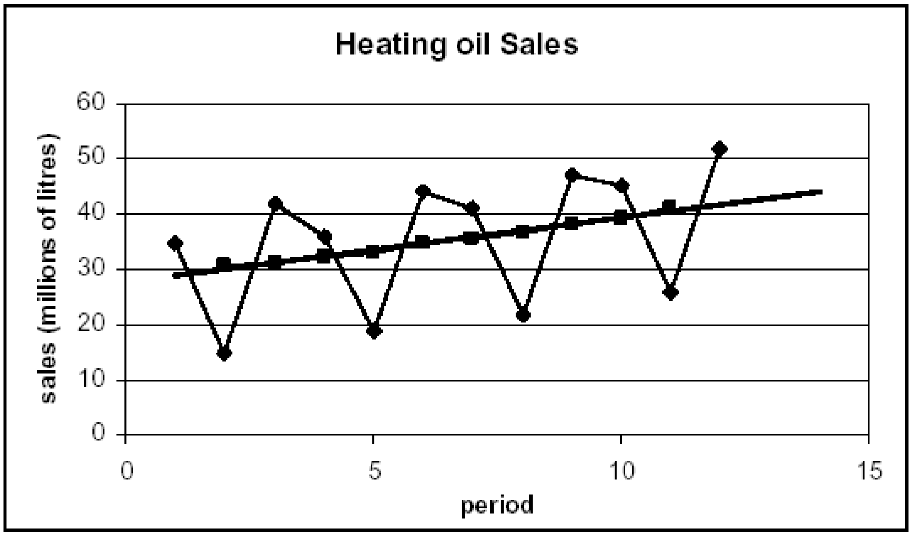
$= 0.2 \times 0.7 + 0.5 \times 0.6 + 0.3 \times 0.1$ **[1]**

= 0.14 + 0.3 + 0.03 = 0.47 **[1]**

(iii) P(J in B | J loses) = P(J in B and J loses)/P(J loses) **[1]**

= 0.2 / 0.53 = 0.377. **[1]**

B3



period	sales	moving average	deviation
1	35		
2	15	30.67	
3	42	31.00	
4	36	32.33	3.67
5	19	33.00	
6	44	34.67	
7	41	35.67	5.33
8	22	36.67	
9	47	38.00	
10	45	39.33	5.67
11	26	41.00	
12	52		

(i) Chart (data)
label axes **[1]**
scales on axes **[1]**
plot points correctly **[1]**

(ii) use 3-point average **[1]**
correct positions in table **[1]**
correct formula **[1]**
correct calculations **[2]**
add correctly to chart **[2]**

(iii) add trendline by eye **[1]**

(iv) calculate deviations **[1]**
mean deviation $= \dfrac{3.67 + 5.33 + 5.67}{3} = 4.89$ **[1]**

(v) read value from trendline for period 13 (42.8) **[1]**
forecast is $42.8 + 4.89 = 47.7$ millions of litres **[1]**

B4(i) Looking in the binomial distribution tables in the formula booklet, the probability that the seeds fail to grow is 35%,
so the probability that less than or equal to 3 will fail is 0.1727, **[1]**
that less than or equal to 2 will fail is 0.0617, **[1]**
and the difference is 0.111. **[1]**

(ii) The probability that less than or equal to 5 will fail is 0.5643, **[M1A1]**
so the probability that more than 5 will fail is 0.4357. (Accept 0.436.) **[1]**

(iii) We need to work out ${}^4C_2 p^2 q^2$ **[1]**

$= \frac{4\times 3}{1\times 2} 0.4357^2 \times 0.5643^2$. **[1]**

$= 0.363$ **[1]**

(iv) $H_0 : P(\text{failure}) = 0.35$ **[1]**

$H_1 : P(\text{failure}) < 0.35$ **[1]**

X is the number of seeds which fail.

$X \sim B(15, 0.35)$ **[1]**

$P(X = 0) = 0.0016$

$P(X \le 1) = 0.0142$ **[1]**

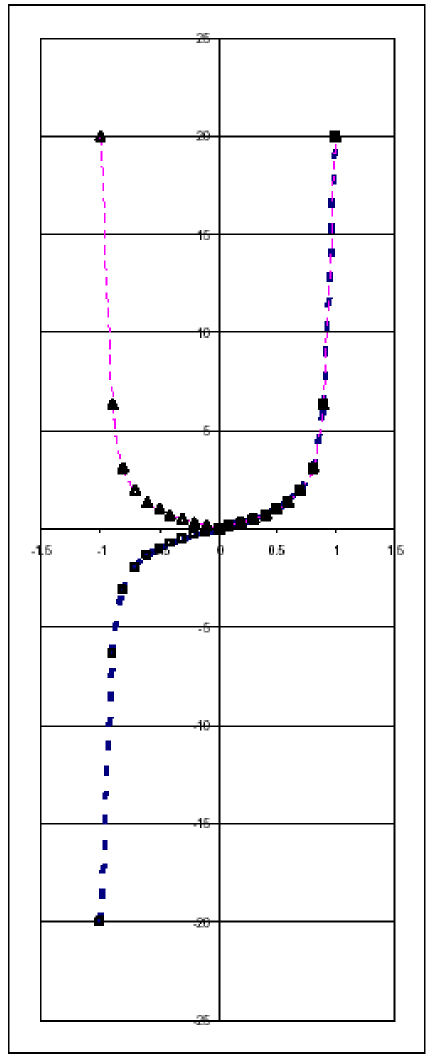
$P(X \le 2) = 0.0617 > 5\%$ **[1]**

so if we are to accept H_1 we must have at most one failure,
that is at least 14 seeds must germinate. **[1]**

B5(i) The range is $\mathbb{R}$ or $-\infty < x < \infty$. **[1]**

(ii) $f(-1/3) = \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}} (\approx -0.577)$, $g(f(-1/3)) = \frac{1}{\sqrt{3}} (\approx 0.577)$. **[1]**

(iii)



Shape of $f(x)$ **[1]**

Passes through origin **[1]**

tends to $\pm\infty$ at ± 1 **[1]**

shape of $g(f(x))$ **[1]**

(iv) $f^{-1}(x) = \frac{2}{\pi}\tan^{-1} x$ **[2]**

$g(x)$ has no inverse because it is not one-one. $g(-x) = g(x)$ for all x. **[2]**

(v) f is the odd function **[1]**

since $f(-x) = -f(x)$ for all x. **[1]**

g is not odd since $g(-x) = g(x) \neq -g(x)$ except when $x = 0$. **[1]**

B6(a) $y = x^2 e^{-x}$

$$\frac{dy}{dx} = 2xe^{-x} - x^2e^{-x}$$

$$= xe^{-x}(2-x) = 0 \text{ at stationary points}$$ **[1]**

$e^{-x} \neq 0$, so $x = 0$ or $x = 2$. **[1]**

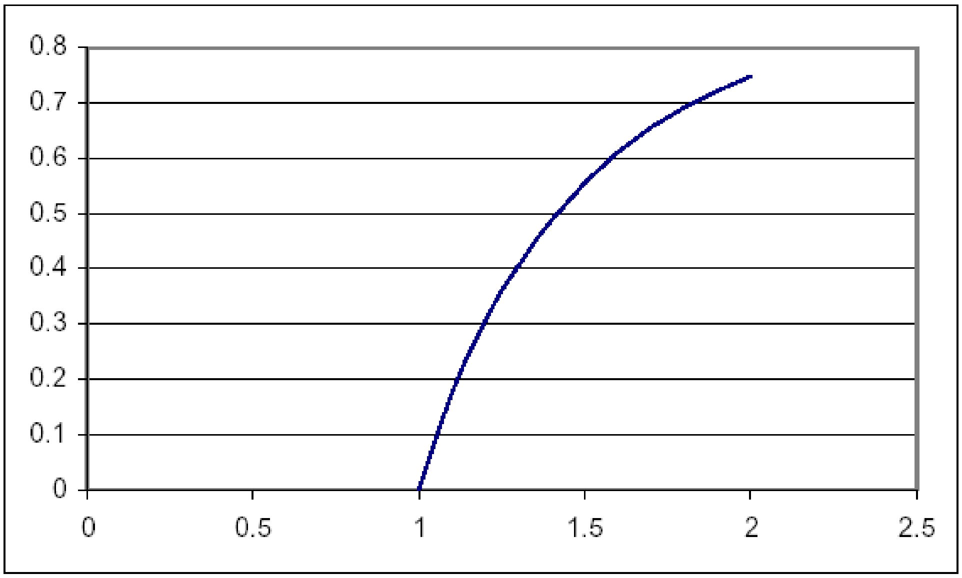
When $x = 0, y = 0$ and when $x = 2, y = 4e^{-2}$ (≈ 0.541) **[1]**

$$\frac{d^2y}{dx^2} = 2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2e^{-x} = 2e^{-x} - 4xe^{-x} + x^2e^{-x}$$ **[1]**

$= 2 > 0$ when $x = 0$ and $= -2e^{-2} < 0$ when $x = 2$ **[1]**

hence we have a minimum at $x = 0$ and a maximum when $x = 2$. **[1]**

(b)(i)



Shape **[1]**
End limits **[1]**

(ii) $$\int_1^2 \left(1 - \frac{1}{x^2}\right)dx = \left[x + \frac{1}{x}\right]_1^2$$ **[1]**

$$= \left[2 + \frac{1}{2} - 1 - 1\right]$$ **[1]**

$$= \frac{1}{2}$$ **[1]**

(iii) $$\pi\int_1^2 \left(1 - \frac{1}{x^2}\right)^2 dx = \pi\int_1^2 \left(1 - \frac{2}{x^2} + \frac{1}{x^4}\right)dx$$ **[1]**

$$= \pi\left[x + \frac{2}{x} - \frac{1}{3x^3}\right]_1^2$$ **[1]**

$$= \pi\left[2 + 1 - \frac{1}{24} - 1 - 2 + \frac{1}{3}\right]$$ **[1]**

$$= \frac{7\pi}{24}.$$ **[1]**