NCUK

# THE NCUK INTERNATIONAL FOUNDATION YEAR

# IFYME001 Maths Part 2 (Science & Engineering) Examination

## 2013-14

## MARK SCHEME

**Notice to markers.**

**Significant Figures:**
**All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A7. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.**

**Error Carried Forward:**
**Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks.**

**When this happens, write ECF next to the ticks.**

**M=Method**
**A=Answer**

If a student has answered more than the required number of questions, credit should only be given for the first $n$ answers, in the order that they are written in the student's answer booklet ($n$ being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

## Section A
## Answer ALL questions. This section carries 40 marks.

| | |
|---|---|
| **Question A1** | |
| Correct use of Quotient Rule | **[ M1 ]** |
| $\frac{dy}{dx} = \frac{2x(1+x) - x^2}{(1+x)^2}$ | **[ A1 ]** |
| $=\ldots\ldots\ldots = \frac{x(2+x)}{(1+x)^2}$ | |
| $x = 0$ or -2. (1 mark each) | **[ A2 ]** |

| | |
|---|---|
| **Question A2** | |
| Correct shape. There must be a V-shape which crosses the $y-$ axis and touches the negative $x-$ axis. | **[ A1 ]** |
| Touches the $x-$ axis at (-½, 0) (1 mark) and crosses the $y-$ axis at (0, 1) (1 mark) | **[ A2 ]** |

| | |
|---|---|
| **Question A3** | |
| A correct method to find cos A | **[ M1 ]** |
| Cos A = $-\frac{7}{25}$ | **[ A1 ]** |
| $= \surd[1 - ( - \frac{7}{25})]$ | |
| $= \surd(\frac{32}{25})$ (or $\frac{\sqrt{32}}{5}$ or $\frac{4\sqrt{2}}{5}$ or equivalent). | **[ A1 ]** |

| | |
|---|---|
| **Question A4** | |
| $\frac{du}{dx} = 1$ | **[ A1 ]** |
| Integral becomes $\int_1^2 \frac{u-1}{u}\,du$ (Attempts to transform integral completely into $u$, with no $x-$ terms). | **[ M1 ]** |
| $= \left[ u - \ln u \right]_1^2$ | **[ A1 ]** |
| $= (2 - \ln 2) - (1 - \ln 1)$ (Substitutes in limits and subtracts the right way round) | **[ M1 ]** |
| $= 1 - \ln 2$. | **[ A1 ]** |

| | |
|---|---|
| **Question A5** | |
| Attempt to write equation in form $\mathbf{r} = (\mathrm{a}\mathbf{i} + \mathrm{b}\mathbf{j} + \mathrm{c}\mathbf{k}) + t(\mathrm{d}\mathbf{i} + \mathrm{e}\mathbf{j} + \mathrm{f}\mathbf{k})$ | **[ M1 ]** |
| $\mathbf{r} = (4\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + t(-7\mathbf{i} + 5\mathbf{j} - \mathbf{k})$ or $\mathbf{r} = (-3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) + t(7\mathbf{i} - 5\mathbf{j} + \mathbf{k})$<br>or any other correct form (1 mark for each part). | **[ A2 ]** |

| **Question A6** | |
|---|---|
| Attempts to integrate the right-hand side | **[ M1 ]** |
| $y = x - \ln x + \frac{1}{x} + c$.  (1 mark for any two parts correct,<br>2 marks for fully correct and + $c$) | **[ A2 ]** |
| Substitutes in $x = 1$ and $y = 0$ to find $c$. | **[ M1 ]** |
| c = -2,  so $y = x - \ln x + \frac{1}{x} - 2$. | **[ A1 ]** |

| **Question A7** | |
|---|---|
| $f'(x) = \cos x + \sin x$ | **[ A1 ]** |
| $x_1 = 0.8 - \frac{\sin 0.8 - \cos 0.8}{\cos 0.8 + \sin 0.8}$  (Correct use of formula) | **[ M1 ]** |
| $= 0.785397125$....... | **[ A1 ]** |
| $= 0.\,7854$ to four significant figures. | **[ A1 ]** |

| | |
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| **Question A8** | |
| Correct use of implicit differentiation (sight of $3\frac{dy}{dx}$ or - $4y\frac{dy}{dx}$ is sufficient here) | **[ M1 ]** |
| $2x - 1 + 3\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$ | **[ A1 ]** |
| $\frac{dy}{dx} = \frac{1-2x}{3-4y}$ (Makes $\frac{dy}{dx}$ the subject) | **[ M1 ]** |
| Substitutes in $x$ = 2 and $y$ = 2, and inverts ($\frac{dy}{dx} = \frac{3}{5}$) | **[ M1 ]** |
| $\frac{dx}{dy} = \frac{5}{3}$. | **[ A1 ]** |
| Alternatively: (Sight of $2x\frac{dx}{dy}$ or - $\frac{dx}{dy}$) **[First M1]** | |
| $2x\frac{dx}{dy} - \frac{dx}{dy} + 3 - 4y = 0$ **[First A1]** | |
| $\frac{dx}{dy} = \frac{4y-3}{2x-1}$ (Makes $\frac{dx}{dy}$ the subject) **[Second M1]** | |
| Substitutes in $x$ = 2 and $y$ = 2 **[Third M1]** $\frac{dx}{dy} = \frac{5}{3}$ **[Final A1]** | |

| | |
|---|---|
| **Question A9** | |
| Uses integration by parts in right direction | **[ M1 ]** |
| $= \left[5x \times \frac{1}{5}e^{5x}\right]_0^1 - \int_0^1 5 \times \frac{1}{5}e^{5x}\,dx$ | |
| $= [e^5 - 0] - \left[\frac{1}{5}e^{5x}\right]_0^1$ (1 mark for correct first integration; 2 marks for both integrations carried out correctly) | **[ A2 ]** |
| $= e^5 - [\frac{1}{5}e^5 - \frac{1}{5}]$ (Substitutes in limits and subtracts the right way round) | **[ M1 ]** |
| $= \frac{4}{5}e^5 + \frac{1}{5}$ ($\approx 119$) | **[ A1 ]** |

| | |
|---|---|
| **Question A10** | |
| Total age of male students is 152 | **[ A1 ]** |
| Total age of female students is 216 | **[ A1 ]** |
| Mean of whole group is (152 + 216) ÷ 20 = 18.4 | **[ A1 ]** |

# Section B
## Answer 4 questions. This section carries 60 marks.

| **Question B1** | | | |
|---|---|---|---|
| a) | i. | (Substitutes in 2.8 and 2.9) | **[ M1 ]** |
| | | If $x = 2.8$, $2.8^2 - 8 = -0.16$<br>If $x = 2.9$, $2.9^2 - 8 = 0.41$ (Both correct) | **[ A1 ]** |
| | | Change of sign, so root lies between 2.8 and 2.9. (Reason & conclusion) | **[ A1 ]** |
| | ii. | Substitutes in $x = 2.9$ | **[ M1 ]** |
| | | $x_1$ = answer rounding to 2.829; | **[ A1 ]** |
| | | $x_2$ = answer rounding to 2.828. | **[ A1 ]** |
| b) | i. | Uses $\cos^2\theta + \sin^2\theta = 1$ | **[ M1 ]** |
| | | Divides by $\cos^2\theta$ to obtain result. | **[ A1 ]** |
| | ii. | $x = \tan y$ | **[ A1 ]** |
| | | $\frac{dx}{dy} = \sec^2 y$ | **[ A1 ]** |
| | | $= \tan^2 y + 1$ | **[ A1 ]** |
| | | $\frac{dy}{dx} = \frac{1}{x^2+1}$ (Replaces tan $y$ with $x$ and inverts) | **[ A1 ]** |
| c) | Correct use of Chain Rule. | | **[ M1 ]** |
| | $\frac{dy}{dx} = (3^x \ln 3)e^{2x} + 3^x(2e^{2x})$. | | **[ A2 ]** |
| | (1 mark for each correct part, but if the candidate then writes the second part as $6^x e^{2x}$, the final mark is lost.) | | |

| **Question B2** | | | |
|---|---|---|---|
| a) | i. | $\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$ (Replaces $\sin^2\theta$ with 1 - $\cos^2\theta$) | **[ M1 ]** |
| | | $\cos 2\theta = 2\cos^2\theta - 1$ | |
| | | (Makes $\cos^2\theta$ the subject) | **[ M1 ]** |
| | | $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ | **[ A1 ]** |
| | ii. | $\frac{1}{2}\int_{\pi/4}^{\pi/2}(\cos 2\theta + 1)\, d\theta$ (Uses previous result) | **[ M1 ]** |
| | | $= \left[ \frac{1}{2}(\frac{1}{2}\sin 2\theta + \theta) \right]_{\pi/4}^{\pi/2}$ (1 mark for each correct integrated part) | **[ A2 ]** |
| | | (Substitutes in the limits and subtracts the right way round) | **[ M1 ]** |
| | | $= \frac{1}{2}(\frac{\pi}{4} - \frac{1}{2})$ or $\frac{\pi}{8} - \frac{1}{4}$ $(\approx 0.143)$ | **[ A1 ]** |
| b) | $2(1 - \cos^2\theta) - \cos\theta - 1 = 0$ (Replaces $\sin^2\theta$ with $1 - \cos^2\theta$) | | **[ M1 ]** |
| | $2\cos^2\theta + \cos\theta - 1 = 0$ (Forms a 3-term quadratic equation set equal to 0) | | **[ M1 ]** |
| | $(2\cos\theta - 1)(\cos\theta + 1) = 0$ (Attempts to factorise) | | **[ M1 ]** |
| | $\cos\theta = \frac{1}{2}$ or -1 (1 mark for each correct answer) | | **[ A2 ]** |
| | $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$, π. (1 mark for any two correct, 2 marks for all three correct) | | **[ A2 ]** |

| **Question B3** | | | | |
|---|---|---|---|---|
| a) | i. | IV | | **[ A1 ]** |
| | ii. | I | Correct elliptical shape | **[ A1 ]** |
| | | | $(0, -2a)$ and $(0, 2a)$ shown. | **[ A1 ]** |
| | | | $(-a, 0)$ and $(a, 0)$ shown. | **[ A1 ]** |
| | | II | Circle translated in the positive $x$ – direction. Position is unimportant for this mark, but the $x$ – axis must form a line of symmetry. | **[ A1 ]** |
| | | | $(2a, 0)$ and $(0, 0)$ shown. If the circle clearly goes through the origin, then the latter coordinates do not need to be indicated. | **[ A1 ]** |
| b) | i. | $-9 \le \mathrm{g}(x) \le 0$. (1 mark for each correct limit). | | **[ A2 ]** |
| | ii. | Writes $y = 2x - 3$, makes $x$ the subject and exchanges $x$ and $y$. | | **[ M1 ]** |
| | | $\mathrm{h}^{-1}(x) = \dfrac{x+3}{2}$ (If answer appears on its own, 2 marks can be given). | | **[ A1 ]** |
| | | Range is $\mathrm{h}^{-1}(x) \ge 0$. | | **[ A1 ]** |
| | iii. | $(2\mathrm{x} - 3)^2 - 9 = -8$ (Writes down composite function the right way round). | | **[ M1 ]** |
| | | $(2\mathrm{x} - 3)^2 = 1$ (Takes the squared term on to one side) | | **[ M1 ]** |
| | | $2\mathrm{x} - 3 = \pm 1$ (Square roots both sides) | | **[ M1 ]** |
| | | $x = 2$ or $1$ | | **[ A1 ]** |
| | | <u>**Or**</u> | | |
| | | $(2\mathrm{x} - 3)^2 - 9 = -8$ (Writes down composite function the right way round). | | **[ M1 ]** |
| | | $4x^2 - 12x + 8 = 0$ (Forms 3-term quadratic equation set equal to zero). | | **[ M1 ]** |
| | | $x^2 - 3x + 2 = 0$ giving $(x-1)(x-2) = 0$ (Factorises) | | **[ M1 ]** |
| | | $x = 1$ or $2$. (Both correct). | | **[ A1 ]** |

| | | |
|---|---|---|
| **Question B4** | | |
| a) | Writing $7 - 2t = 1$<br>$5 - t = 2$<br>$-15 + 4t = -3$<br>Showing that $t = 3$ for any one equation<br>Showing that $t = 3$ for other two equations.<br><u>Or</u> Solving any one equation to find $t$; (Second mark)<br>Showing that $t$ satisfies both other equations (Third mark)<br><u>Or</u> Any other clear and valid method | **[ A1 ]**<br>**[ A1 ]**<br>**[ A1 ]** |
| b) | Subtracts vector components<br>$(p^2 - 1)\mathbf{i} + (3p - 2)\mathbf{j} + 4\mathbf{k}$. | **[ M1 ]**<br>**[ A1 ]** |
| c) | $-2(p^2 - 1) - (3p - 2) + 16 = 0$ (Uses the scalar product on the answer to (b) and the correct directional vector)<br>$2p^2 + 3p - 20 = 0$ (Forms a 3-term quadratic equation set equal to zero).<br>$(2p - 5)(p + 4) = 0$ (Factorises).<br>$p = \frac{5}{2}$ or $-4$. | **[ M1 ]**<br>**[ M1 ]**<br>**[ M1 ]**<br>**[ A1 ]** |
| d) | B lies at (16, -12, 1) | **[ A1 ]** |
| e) | Writes **1.** $2 + s = 7 - 2t$<br>**2.** $1 = 5 - t$<br>**3.** $4 + 3s = -15 + 4t$<br>From **2**, $t = 4$.<br><u>Either</u> substitutes $t = 4$ into **1** **[M1]** and finds $s = -3$ **[A1]**<br><u>Or</u> substitutes $t = 4$ into **3** **[M1]** and finds $s = -1$ **[A1]**<br>Confirms that values of $t$ and $s$ do not satisfy their third equation.<br>[Candidates may carry out both '<u>either</u>' and '<u>or</u>' operations above **[M1]**; find both $s$ values **[A1]** and comment that they are different **[Final A1]**<br>**Or** Some candidates may solve **1** and **3** **[M1]** ($t = \frac{17}{5}$ and $s = -\frac{9}{5}$)<br>**[A1]** for each correct answer; then confirms **2** is not satisfied **[Final A1]** | **[ A1 ]**<br>**[ A1 ]**<br>**[ M1 ]**<br>**[ A1 ]**<br>**[ A1 ]** |

| **Question B5** | | | |
|---|---|---|---|
| a) | i. | (Dividing)<br>$\begin{array}{r} 3 \\ x^2+2x-3\,\overline{)\,3x^2 + 6x - 5} \\ \underline{3x^2 + 6x - 9} \\ 4 \end{array}$<br>(Correct quotient and remainder)<br>So f$(x) = 3 + \frac{4}{x^2+2x-3} = 3 + \frac{4}{(x+3)(x-1)}$ (factorised bottom line)<br>Alternatively: Some candidates may realise $A = 3$ by looking at the coefficient of $x^2$ and work backwards<br>$3 + \frac{4}{(x+3)(x-1)} = \frac{3(x+3)(x-1)+4}{(x+3)(x-1)}$<br>**[M1]** for using common denominator; **[A1]** for correct expression<br>Simplifies top line to give $\frac{3x^2+6x-5}{x^2+2x-3}$ **[A1]** | **[ M1 ]**<br>**[ A1 ]**<br>**[ A1 ]** |
| | ii. | $4 = B(x-1) + C(x+3)$<br>$B = -1$ and $C = 1$ (1 mark each) | **[ M1 ]**<br>**[ A2 ]** |
| | iii. | $\int_2^3 3 - \frac{1}{x+3} + \frac{1}{x-1}\ dx = \Big[\, 3x - \ln(x+3) + \ln(x-1) \,\Big]_2^3$<br>(1 mark for any two parts correct; 2 marks for completely correct)<br>Substitutes in limits and subtracts the right way round.<br>$= 3 + \ln(\frac{10}{6})$ or equivalent. | **[ A2 ]**<br>**[ M1 ]**<br>**[ A1 ]** |
| b) | $\frac{1}{y}\,dy = 2x\,dx$ (Separates the variables)<br>$\ln y = x^2 + c$ (1 mark for $\ln y$, 1 mark for $x^2$ and + $c$ on either side)<br>Substitutes in values (giving $c = -3$)<br>$\ln y = x^2 - 3$<br>$y = e^{x^2-3}$ | | **[ M1 ]**<br>**[ A2 ]**<br>**[ M1 ]**<br>**[ A1 ]** |

<table>
<tr><td colspan="3">**Question B6**</td><td></td></tr>
<tr><td>a)</td><td>i.</td><td>Mean = 70 ÷ 7 = 10; Mode = 7; Median = 9 (1 mark each)</td><td>**[ A3 ]**</td></tr>
<tr><td></td><td>ii.</td><td>Sum of squares comes to 758.<br>Standard deviation = $\sqrt{(758 \div 7 - 10^2)} \approx 2.88$</td><td>**[ A1 ]**<br>**[ A1 ]**</td></tr>
<tr><td></td><td>iii.</td><td>Mean = 13; standard deviation = 2.88 (Both correct)</td><td>**[ A1 ]**</td></tr>
<tr><td>b)</td><td></td><td>
<table>
<tr><th>Mid-value (x)</th><th>Frequency (f)</th><th>fx</th><th>interval width</th><th>freq. density</th></tr>
<tr><td>60</td><td>54</td><td>3240</td><td>20</td><td>2.7</td></tr>
<tr><td>75</td><td>45</td><td>3375</td><td>10</td><td>4.5</td></tr>
<tr><td>85</td><td>69</td><td>5865</td><td>10</td><td>6.9</td></tr>
<tr><td>92.5</td><td>46</td><td>4255</td><td>5</td><td>9.2</td></tr>
<tr><td>97.5</td><td>36</td><td>3510</td><td>5</td><td>7.2</td></tr>
<tr><td>105</td><td>58</td><td>6090</td><td>10</td><td>5.8</td></tr>
<tr><td>120</td><td>42</td><td>5040</td><td>20</td><td>2.1</td></tr>
<tr><td></td><td>350</td><td>31375</td><td></td><td></td></tr>
</table>
</td><td></td></tr>
<tr><td></td><td>i.</td><td>Correct mid-values (1 mark);<br>fx column correct (1 mark);<br>Mean = 31375 ÷ 350 ≈ 89.6</td><td>**[ A1 ]**<br>**[ A1 ]**<br>**[ A1 ]**</td></tr>
<tr><td></td><td>ii.</td><td>Divides frequency by interval width<br>Correct frequency densities</td><td>**[ M1 ]**<br>**[ A1 ]**</td></tr>
<tr><td></td><td>iii.</td><td>1 mark lost (up to a maximum of 3) for each incorrect rectangle</td><td>**[ A3 ]**</td></tr>
<tr><td></td><td>iv.</td><td>$54 + \frac{3}{5}$ of 45 = 81</td><td>**[ A1 ]**</td></tr>
</table>