NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Examination

Examination Session	**Time Allowed**
Semester Two	2 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet and graph paper.

- **Answers must not be written during the first 10 minutes.**
- Write your NCUK ID Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No electronic devices** (e.g. mobile phones, tablets, iPads) are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Show **ALL** workings in your answer booklet. Marks will be awarded for correct workings.
- Examination materials must not be removed from the examination room.
- Write your name and candidate number on all loose sheets/diagrams.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

A curve has the equation $y = \frac{x^2}{1+x}$ $(x \neq -1)$

Find the values of x for which the curve has a turning point. **[4]**

Question A2

The function $f(x)$ is defined as $f(x) = |2x + 1|$

Sketch the graph of $y = f(x)$ showing clearly the coordinates of the points where the graph crosses or touches the x – and y – axes. **[3]**

Question A3

Angle *A* is obtuse and $\sin A = \frac{24}{25}$

Without working out the value of *A*, find the exact value of $\sqrt{}(1 - cosA)$ **[3]**

You must show each stage of your working.

Question A4

Use the substitution $u = 1 + x$ to find the exact value of: **[5]**

$$\int_0^1 \frac{x}{1+x}dx$$

Question A5

A line passes through the points with coordinates (4, -1, 3) and (-3, 4, 2).

Write down a vector equation of this line. **[3]**

Question A6

Find the equation of a curve that has gradient function $\frac{dy}{dx} = 1 - \frac{1}{x} - \frac{1}{x^2}$ and passes through the point (1, 0). **[5]**

Question A7

The function $f(x)$ is defined as $f(x) = \sin x - \cos x$

An approximate solution to the equation $f(x) = 0$ is $x_0 = 0.8$ radians.

Use the Newton-Raphson method **once** to find a better approximation. **Give your answer to four significant figures**. **[4]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A8

A curve *C* has equation $x^2 - x + 3y - 2y^2 = 0$

Find the value of $\frac{dx}{dy}$ at the point (2, 2). **[5]**

Question A9

Use integration by parts to evaluate: **[5]**

$$\int_0^1 5xe^{5x}\,dx$$

Question A10

A group has 8 male students and 12 female students. The mean age of the male students is 19 and the mean age of the female students is 18.

Find the mean age of the whole group. **[3]**

Section B begins on the following page

Section B
Answer <u>4</u> questions. This section carries 60 marks.

Question B1

a) i. Show that the equation $x^2 - 8 = 0$ has a root between $x = 2.8$ and $x = 2.9$ **[3]**

You do **not** need to find the actual value of the root.

A better approximation to $x^2 - 8 = 0$ can be obtained by using the iterative formula $x_{n+1} = \frac{1}{2}[x_n + (8 \div x_n)]$

ii. Starting with $x_0 = 2.9$, find x_1 and x_2 **[3]**

b) i. Use a trigonometric identity to show that $\tan^2\theta + 1 = \sec^2\theta$. **[2]**

ii. A curve is defined as $y = \tan^{-1} x$

Show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ **[4]**

c) Differentiate $y = 3^x e^{2x}$ **[3]**

Question B2

a) i. Given that $\cos 2\theta = \cos^2\theta - \sin^2\theta$, show that $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ **[3]**

ii. Making use of the result in part (i), evaluate:

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2\theta \, d\theta$$ **[5]**

b) Solve the equation $2\sin^2\theta - \cos\theta - 1 = 0$ for $0 \leq \theta \leq 2\pi$ **[7]**

Give your answers as exact multiples of π.

Question B3

a)

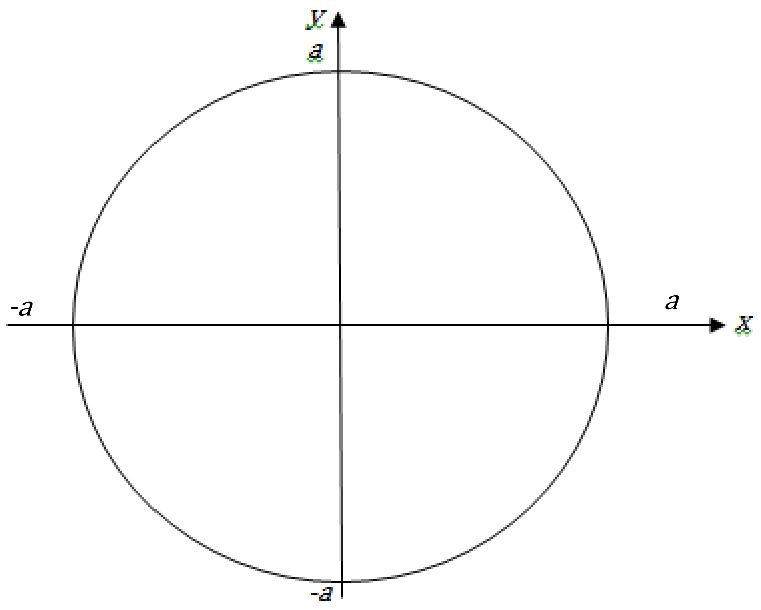

Figure 1

i. Figure 1 shows the curve $y = f(x)$ $(-a \leq x \leq a)$, which is a circle with centre at the origin and radius a units.

What type of function is $f(x)$ over this domain? Choose one of the following: **[1]**

I one-one

II many-one

III one-many

IV many-many

ii. On separate axes, sketch the following (on each sketch show clearly the coordinates where the curve crosses the x - and y – axes):

I $y = 2\,f(x)$ **[3]**

II $y = f(x - a)$ **[2]**

Question B3 continues on the following page

b) Functions $g(x)$and $h(x)$ are defined as follows:

$g(x) = x^2 - 9 \ (-2 \le x \le 3)$

$h(x) = 2x - 3 \ (x \ge 0)$

i. Write down the range of $g(x)$ **[2]**

ii. Write down an expression for $h^{-1}(x)$ and state its range. **[3]**

iii. Solve the equation $g(h(x)) = -8$ **[4]**

Question B4

a) The line l_1 has vector equation $\mathbf{r} = (7\mathbf{i} + 5\mathbf{j} - 15\mathbf{k}) + t(-2\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ where t is a scalar.

Show that the point $A\,(1, 2, \text{-}3)$ lies on the line l_1. **[3]**

b) The point B lies at $(p^2, 3p, 1)$

Write down an expression, in terms of p, for vector $\overrightarrow{AB}$. **[2]**

c) The vector $\overrightarrow{AB}$ is perpendicular to the line l_1. Form an equation in p and solve it. **[4]**

d) Given that $p < 0$, write down the coordinates of point B. **[1]**

e) The line l_2 has vector equation $\mathbf{r} = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + s(\mathbf{i} + 3\mathbf{k})$ where s is a scalar.

Show that the lines l_1 and l_2 **do not** intersect. **[5]**

Question B5

a) i. The function $f(x)$ is defined as $f(x) = \frac{3x^2+6x-5}{x^2+2x-3}$ **[3]**

Show that $f(x)$ can be written in the form $A + \frac{4}{(x+3)(x-1)}$

where A is an integer.

ii. Write $\frac{4}{(x+3)(x-1)}$ in the form $\frac{B}{x+3} + \frac{C}{x-1}$ **[3]**

where B and C are constants to be determined.

iii. Evaluate:

$$\int_2^3 f(x)\, dx$$

writing your answer in the form $p + \ln(\frac{q}{r})$ where p, q and r are integers. **[4]**

b) Solve the differential equation $\frac{dy}{dx} = 2xy$ given $y = e$ when $x = 2$.

Give your answer in the form $y = e^{g(x)}$ **[5]**

Section B continues on the following page

Question B6

a) i. The ages of 7 children are 7, 13, 15, 7, 8, 9 and 11.

Find the mean, mode and median. **[3]**

ii. Find the standard deviation. **[2]**

iii. Write down the mean and standard deviation of the ages of these children after exactly 3 years. **[1]**

b) 350 people took part in a long distance charity run and the times taken to complete the race are shown in the table below:

Time t (minutes)	**Frequency**
$50 \le t \le 70$	54
$70 < t \le 80$	45
$80 < t \le 90$	69
$90 < t \le 95$	46
$95 < t \le 100$	36
$100 < t \le 110$	58
$110 < t \le 130$	42

You may wish to copy and extend this table to help you to answer some of the questions below.

i. Estimate the mean. **[3]**

ii. Work out the frequency densities. **[2]**

iii. Draw a histogram using the graph paper provided to represent the data. **[3]**

iv. It was decided to award a certificate to all runners who completed the race in 76 minutes or less. **[1]**

Use your histogram to estimate how many runners received a certificate.