THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Examination

2012-13

Mark Scheme

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A1. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the candidate to calculate-or otherwise produce-a piece of information that is to be used later in the question, a marker should consider the possibility of error carried forward. A careless error early in the question may make it impossible for a candidate to answer the remainder of the question correctly. Where a candidate has been careless with initial data, but has gone on to demonstrate knowledge of the correct method, they should be awarded marks for the method only.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

Question A1 (Learning Outcomes H1 and H3)

$f(x) = \frac{10^x}{x}$

$f'(x) = \frac{(x)(10^x)' - (10^x)(x)'}{(x)^2}$ 1 M

$f'(x) = \frac{x \ln(10)10^x - 10^x}{x^2}$ 1 A

$f'(1) = \frac{1 \ln(10)10 - 10}{1}$ 1 A

$= 13.03$ (Accept only this) 1 A

Question A2 (Learning Outcome H2)

$x^3 + y^4 = 1$

$3x^2 + 4y^3 \frac{dy}{dx} = 0$ 1 M

$\frac{dy}{dx} = -\frac{3x^2}{4y^3}$ 1 A

$\frac{dy}{dx}(1,1) = -\frac{3}{4}$ (Accept also) -0.75 1 A

Question A3 (Learning Outcomes I5)

$f(x) = \frac{1+9x}{8+4x}$

$y = \frac{1+9x}{8+4x}$

$4xy - 9x = 1 - 8y$ 1 M

$x = \frac{1-8\,y}{4y-9}$ 1 A

$f^{-1}(x) = \frac{1-8x}{4x-9}$ (Accept also) $f^{-1}(y) = \frac{1-8y}{4y-9}$ 1 A

Question A4 (Learning Outcomes J1 and J2)

$\cos(2A) = \frac{7}{9}$

$1 - 2\sin^2(A) = \frac{7}{9}$ 1 M

$\sin(A) = \sqrt{\frac{1-\frac{7}{9}}{2}}$ 1 A

$\sin(A) = \frac{1}{3}$ 1 A

Question A5 (Learning Outcomes J1 and J3)

$\sin(2\theta) + \cos^2(\theta) = 0$

$2\sin(\theta)\cos(\theta) + \cos^2(\theta) = 0$ 1 A

$\cos(\theta)\,(2\sin(\theta) + \cos(\theta)) = 0$

$\cos(\theta) = 0\ OR\ (2\sin(\theta) + \cos(\theta)) = 0$ 1 M

$\cos(\theta) = \ 0\ OR \tan(\theta) = \ -\frac{1}{2}$ 1 A

$\theta = \cos^{-1} 0\ \ OR\ \theta = \tan^{-1}(-\frac{1}{2})$ 1 M

$\theta = 1.57, 2.68, 4.71\ OR\ 5.82$ (or equivalent) 1 A

Question A6 (Learning Outcome K6)

$u = 1 + x$

$x = u - 1$ and $du = dx$ 1 A

$\int_0^1 \frac{x}{\sqrt{1+x}} dx = \int_{x=0}^{x=1} \frac{u-1}{\sqrt{u}}\ du$ 1 M

$= \int_{x=0}^{x=1} u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$ 1 M

$= \int_{u=1}^{u=2} u^{\frac{1}{2}} - u^{-\frac{1}{2}} du$ 1 M

$= \left[\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^2$ 1 A

$= \frac{4}{3} - \frac{2}{3}\sqrt{2}$ (Accept also) 0.391 1 A

Question A7 (Learning Outcomes L6 and L7)

$3t\,\boldsymbol{i} + 6t\,\boldsymbol{j} + 6t\,\boldsymbol{k}$ is a unit vector

$|3t\,\boldsymbol{i} + 6t\,\boldsymbol{j} + 6t\,\boldsymbol{k}| = 1$ 1 M

$\sqrt{9t^2 + 36t^2 + 36t^2} = 1$ 1 M

$81\,t^2 = 1$

$t = \pm\frac{1}{9}$ (Accept also) ± 0.111 1 A

Question A8 (Learning Outcomes M5 and K1)

$\frac{dy}{dx} + x^2\ \frac{dy}{dx} - 2x = 0$

$\frac{dy}{dx}(1 + x^2) = 2x$

$\frac{dy}{dx} = \frac{2x}{1+x^2}$ 1 M

$y = \ln(1 + x^2) + C$ 1 A

$0 = \ln(1 + 0^2) + C$ 1 M

$C = 0$ 1 A

$y = \ln(1 + x^2)$ 1 A

Question A9 (Learning Outcome N1)

$y = x^2 - x - 1 + \ln(1 + x^2)$

$y(0) = -1 < 0$ 1 M

$y(2) = 1 + \ln(5) > 0$ 1 M

Sign Changes ⇒ A Root Exists 1 A

(Accept equivalent statements or explanatory graph)

Question A10 (Learning Outcome O4)

Data = 1,2,4,4,4,4,5,6,15 1 M

Mode is 4 1 A

Median is 4 1 A

Mean is $\frac{1+2+4+4+4+4+5+6+15}{9} = \frac{45}{9}$ 1 M

= 5 1 A

Question B1 (Learning Outcomes H1, H2 and M2)

(a)(i)

$4y^2 + 3y - 1 = x^2 - x + 3xy$

$4y^2 + 3y - 1 = (1)^2 - (1) + 3(1)y$ 1 M

$4y^2 = 1$ 1 A

$a = -\frac{1}{2} AND\ b = \frac{1}{2}$ 1 A

(a)(ii)

$4y^2 + 3y - 1 = x^2 - x + 3xy$

$8y\frac{dy}{dx} + 3\frac{dy}{dx} = 2x - 1 + \frac{d}{dx}(3xy)$ 1 M

$8y\frac{dy}{dx} + 3\frac{dy}{dx} = 2x - 1 + 3y + 3x\frac{dy}{dx}$ 1 M

$\frac{dy}{dx} = \frac{2x+3y-1}{8y-3x+3}$ 1 A

$\frac{dy}{dx}\left(1, -\frac{1}{2}\right) = \frac{1}{8} AND\ \frac{dy}{dx}\left(1, \frac{1}{2}\right) = \frac{5}{8}$ 1 A

(Accept also) 0.125 and 0.625

(b)(i)

$f(x) = e^{-x}\cos(2x)$

$f'(x) = (e^{-x})'(\cos(2x)) + (e^{-x})(\cos(2x))'$ 1 M

$f'(x) = -e^{-x}\cos(2x) - 2e^{-x}\sin(2x)$ 2 A

(Accept also) $-e^{-x}(\cos(2x) + 2\sin(2x))$ or equivalent

(b)(ii)

$f'(x) = -e^{-x}(\cos(2x) + 2\sin(2x))$

$f''(x) = (-e^{-x})'(\cos(2x) + 2\sin(2x)) + (-e^{-x})(\cos(2x) + 2\sin(2x))'$ 1 M

$f''(x) = -e^{-x}(3\cos(2x) - 4\sin(2x))$ 2 A

(Accept equivalent expression)

(b)(iii)

$f''(x) + 2f'(x) + 5f(x)$

$= -e^{-x}(3\cos(3x) - 4\sin(x)) - 2e^{-x}(\cos(2x) + 2\sin(2x)) + 5e^{-x}\cos(2x)$ 1 M

$= e^{-x}(0\cos(2x) + 0\sin(2x)) = 0$ 1 A

Question B2 (Learning Outcomes I1, I2, I3, I5 and I7)

(a)(i)

$f(x) = \sqrt{2x+1}$

$f(1) = \sqrt{3}$

Range = $(\sqrt{3}, \infty)$ 1 A

(a)(ii)

$f(x) = \sqrt{2x+1}$

$y = \sqrt{2x+1}$

$y^2 = 2x + 1$ 1 M

$\frac{y^2 - 1}{2} = x$ 1 M

$f^{-1}(x) = \frac{x^2-1}{2}$ or equivalent (Accept also) $f^{-1}(y) = \frac{y^2-1}{2}$ 1 A

(a)(iii)

$gh(x) = g(h(x))$

$gh(x) = 2(3x-2)^2 - 3(3x-2) + 4$ 1 M

$gh(x) = 18x^2 - 24x + 8 - 9x + 6 + 4$ 1 M

$gh(x) = 18x^2 - 33x + 18$ 1 A

(a)(iv)

It is many-one since $f(0) = f\left(\frac{7}{6}\right) = 18$ 2 A

(Accept equivalent statements EG: It is symmetric and the minimum $x = \frac{7}{12}$ is in the domain) (b)(i)

$y = 2|4x + 2| - 3$

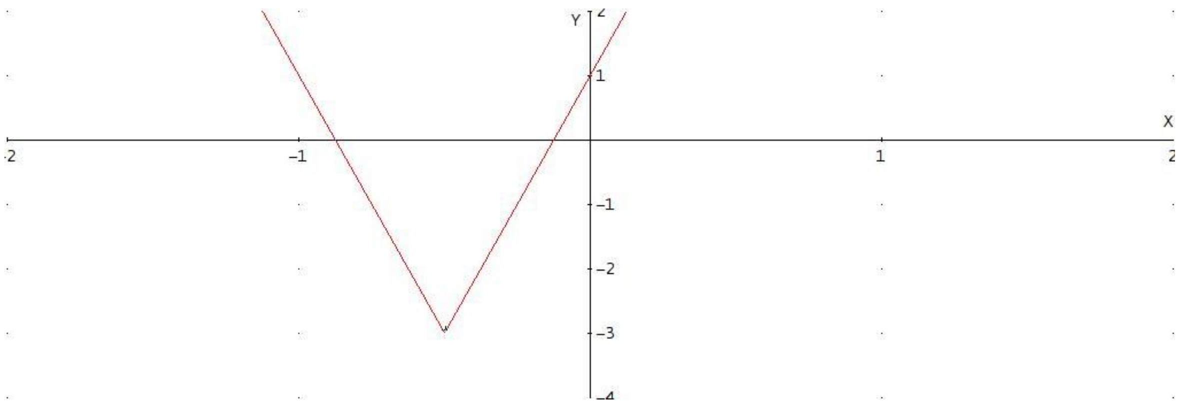
Roots: $4x + 2 = \pm\frac{3}{2}$ 1 A

Points are $(-\frac{1}{2} \pm \frac{3}{8}, 0)$ = $(-\frac{7}{8}, 0)$ AND $(-\frac{1}{8}, 0)$ 1 A

Y-intercept: $y = 2|4(0) + 2| - 3$ 1 A

Point is (0,1) 1 A

The positive Two indicates a V Shape 1 A



(b)(ii)

The graph implies $-\frac{7}{8} < x < -\frac{1}{8}$ 1 A

Question B3 (Learning Outcomes J1, J3, J4, K1 and K5)

(a)(i)

$3 - 5\sin(x) - \cos(2x) = 0$

$2\sin^2(x) - 5\sin(x) + 2 = 0$ 1 M

$(\sin(x) - 2)(2\sin(x) - 1) = 0$

$\sin(x) = \frac{1}{2}$ 1 M

$x = \sin^{-1}\left(\frac{1}{2}\right)$ 1 A

$x = \frac{\pi}{6}\ OR\ \frac{5\pi}{6}$ (Accept also 0.524 OR 2.62) 1 A

(a)(ii)

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(2\sin(x) - 1)\cos(x)}{3 - 5\sin(x) - \cos(2x)}\, dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(2\sin(x) - 1)\cos(x)}{(\sin(x) - 2)(2\sin(x) - 1)}\, dx$ 1 M

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos(x)}{\sin(x) - 2}\, dx$ 1 M

$= [\ln(\sin(x) - 2)]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ 1 M

$= \left[\ln\left(\frac{\sqrt{3} - 4}{2}\right) - \ln\left(\frac{\sqrt{2} - 4}{2}\right)\right]$ 1 A

$= \ln\left(\frac{4 - \sqrt{3}}{4 - \sqrt{2}}\right)$ 1 A (b)(i)

Passing though the origin implies c is 0

Contains (0,0) and (h,r) implies m is $\frac{r}{h}$ 1 A

The equation of the line is $y = \frac{r}{h}x$ 1 A

(b)(ii)

$V = \pi \int_0^h (\frac{r}{h}x)^2 dx$ 1 M

$= \pi \int_0^h \frac{r^2}{h^2} x^2 dx$ 1 M

$= \pi \left[\frac{r^2}{3h^2}x^3\right]_0^h$ 1 M

$= \frac{1}{3}\pi r^2 h$ 1 A

Question B4 (Learning Outcomes L6, L11, L12, L13 and L14)

(a)

$\boldsymbol{r} = (2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) + t(2\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k})$ 2 A

(b)

$(2\boldsymbol{i} + 6\boldsymbol{j} - 3\boldsymbol{k}) + s(2\boldsymbol{i} + 4\boldsymbol{k}) = (2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}) + t(2\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k})$ 1 M

$(2 + 2s)\boldsymbol{i} + 6\boldsymbol{j} + (-3 + 4s)\boldsymbol{k} = (2 + 2t)\boldsymbol{i} + (3 + 3t)\boldsymbol{j} + (-2 + 3t)\boldsymbol{k}$ 1 M

2+2s = 2 + 2t, 6 = 3 + 3t AND -3 + 4s = -2 + 3t 1 A

s=t=1 1 A

$\boldsymbol{r} = 4\boldsymbol{i} + 6\boldsymbol{j} + \boldsymbol{k}$ (Accept also) $\begin{pmatrix}4\\6\\1\end{pmatrix}$ 1 A

(c)

$\cos(\theta) = \frac{\boldsymbol{r_1}.\boldsymbol{r_2}}{|\boldsymbol{r_1}||\boldsymbol{r_2}|}$ 1 M

$\cos(\theta) = \frac{(2\boldsymbol{i}+4\boldsymbol{k}).(2\boldsymbol{i}+3\boldsymbol{j}+3\boldsymbol{k})}{|(2\boldsymbol{i}+4\boldsymbol{k})||(2\boldsymbol{i}+3\boldsymbol{j}+3\boldsymbol{k})|}$ 1 M

$\cos(\theta) = 0.762770071$ 1 A

$\theta = 40.3^O$ 1 A

(d)

(**b**+**a**).(**b**-**a**) = 0

b.(**b**-**a**) + **a**.(**b**-**a**) = 0 1 A

b.b – **b.a** + **a.b** – **a.a** = 0 1 A

b.b – **a.b** + **a.b** – **a.a** = 0 1 A

$|\mathbf{b}|^2 = |\boldsymbol{a}|^2$

$|\boldsymbol{a}| = |\boldsymbol{b}|$ 1 A

Question B5 (Learning Outcomes K3, M5 and N3)

(a)(i)

$\frac{1}{x-x^2} = \frac{A}{x} + \frac{B}{1-x}$ 1 A

$1 = \mathrm{A(1\text{-}x) + Bx}$ 1 M

Consider x tending to 1, this means B = 1 and likewise as x tends towards 0 this means A = 1. 1 A

Award the full 3 marks if the student provides the correct value of A and B but does not show any workings.

(a)(ii)

$\frac{dP}{dt} = P - P^2$

$\frac{dP}{P-P^2} = dt$ 1 M

$t = \int \frac{1}{P} + \frac{1}{1-P} \ dP$

$t = \ln(P) - \ln(1-P) + C$ 1 M

$t = \ln\left(\frac{P}{1-P}\right) + C$ 1 M

The point $\left(0, \frac{1}{2}\right)$ implies $0 = \ln\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) + \mathrm{C}$ 1 A

And so $\mathrm{C=0}$ 1 A

$t = \ln\left(\frac{P}{1-P}\right)$

$e^t = \frac{P}{1-P}$ 1 A

$P = \frac{e^t}{1+e^t}$ 1 A

(b)

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ 1 M

$x_{n+1} = x_n - \frac{(\frac{1}{6}x_n^3+\frac{1}{2}x_n^2+x_n+1)}{(\frac{1}{6}x_n^3+\frac{1}{2}x_n^2+x_n+1)'}$ 1 M

$x_{n+1} = x_n - \frac{(\frac{1}{6}x_n^3+\frac{1}{2}x_n^2+x_n+1)}{\frac{1}{2}x_n^2+x_n+1}$ 1 M

$x_1 = -\frac{3}{2} - \frac{\frac{1}{6}\left(-\frac{3}{2}\right)^3+\frac{1}{2}\left(-\frac{3}{2}\right)^2-\frac{3}{2}+1}{\frac{1}{2}\left(-\frac{3}{2}\right)^2-\frac{3}{2}+1} = -\frac{8}{5}$ 1 M

$x_2 = -\frac{8}{5} - \frac{\frac{1}{6}\left(-\frac{8}{5}\right)^3+\frac{1}{2}\left(-\frac{8}{5}\right)^2-\frac{8}{5}+1}{\frac{1}{2}\left(-\frac{8}{5}\right)^2-\frac{8}{5}+1} = -\frac{407}{255} \approx$ -1.596078431 1 A

(Accept the answer in the form of a fraction of a decimal of 2sf or higher)

Question B6 (Learning Outcomes O3, O7 and O10)

(a)

Mean height $= \frac{\sum f_i x_i}{\sum f_i}$ 1 M

$= \frac{2(147.5)+4(152.5)+6(157.5)+3(162.5)+2\,(167.5)+2(172.5)+177.5}{20}$ 2 M

$= \frac{3195}{20} = 159.75\ cm$ 1 A

(b)

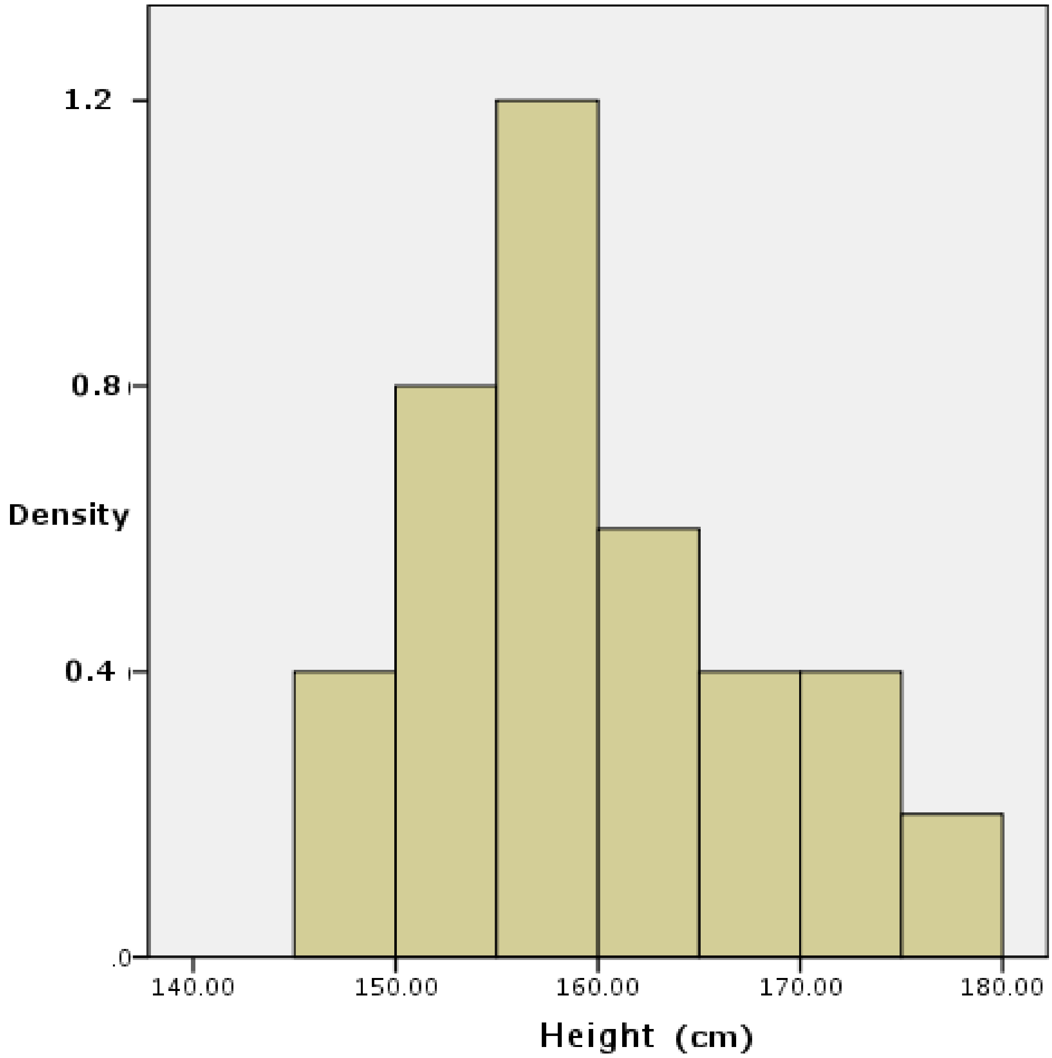
Standard Deviation $= \sqrt{\left(\frac{\sum f_i x_i^2}{\sum f_i}\right) - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$ 1 M

$\frac{2(147.5)^2+4(152.5)^2+6(157.5)^2+3(162.5)^2+2(167.5)^2+2(172.5)^2+177.5^2}{20} = \frac{511725}{20}$ 2 M

Standard Deviation $= \sqrt{\frac{511725}{20} - 159.75^2}$ 1 M

Standard Deviation = 8.14 1 A

(c)



Correct Heights (alternative Y-units acceptable, proportionality must be preserved) 1 A

Correct Class Limits 1 A

Bars touching 1 A

Labelled X and Y axis 1 A

(d)

No, this data is not bimodal. 1 A

(e)

Positively Skewed 1 A

