



THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Examination

Examination Session
Semester Two

Time Allowed
2 Hours 10 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 40% of the exam marks.

SECTION B Answer 4 questions. This section carries 60% of the exam marks.

The marks for each question are indicated in square brackets [].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Full marks will only be given for **full and detailed answers**.

Section A

Answer ALL questions. This section carries 40 marks.

Question A1

Find the value of $f'(1)$ to **four significant figures** if $f(x) = \frac{10^x}{x}$ [4]

In this question one mark will be awarded for the correct use of significant figures.

Question A2

Find the value of the derivative $\frac{dy}{dx}$ from the implicit equation $x^3 + y^4 = 1$ [3]

at the point (1,1).

Question A3

What is the inverse function f^{-1} of the function f defined by $f(x) = \frac{1+9x}{8+4x}$ [3]

given that $x \neq -2$?

Question A4

If $\cos 2A = \frac{7}{9}$ then show that $\sin A = \frac{1}{3}$ [3]

(to be shown without finding the acute positive angle A .)

Question A5

Find **all** values of θ (in radians) from the equation below: [5]

$$\sin 2\theta + \cos^2\theta = 0$$

where $0 \leq \theta < 2\pi$.

Question A6

Calculate: [6]

$$\int_0^1 \frac{x}{\sqrt{1+x}} dx$$

with the help of the substitution $u = 1 + x$, where $0 \leq x \leq 1$.

Question A7

The vector $\mathbf{r} = (3t)\mathbf{i} + (6t)\mathbf{j} + (6t)\mathbf{k}$ is a unit vector, find the possible values of t . **[3]**

Question A8

Find the exact solution y to the given differential equation: **[5]**

$$\frac{dy}{dx} + x^2 \frac{dy}{dx} - 2x = 0$$

where $y = 0$ when $x = 0$.

Question A9

Show that the curve defined by: **[3]**

$$y = x^2 - x - 1 + \ln(1 + x^2)$$

has a root between $x = 0$ and $x = 2$. (You do **not** need to find the root.)

Question A10

Find the mean, median and mode of the following set of data 4,6,2,4,5,15,4,4,1. **[5]**

Section B

Answer 4 questions. This section carries 60 marks.

Question B1

- a) The curve C is defined by the equation $4y^2 + 3y - 1 = x^2 - x + 3xy$.
- i. The point $A(1, a)$ and the point $B(1, b)$ are on the curve C . Find the values of a and b , where $a < b$. [3]
 - ii. What is the value of $\frac{dy}{dx}$ at the point A and the point B ? [4]
- b) The function $f(x)$ is defined as $e^{-x} \cos 2x$.
- i. Find an expression for $f'(x)$. [3]
 - ii. Find an expression for $f''(x)$. [3]
 - iii. Show that $f''(x) + 2f'(x) + 5f(x) = 0$ for all real values of x . [2]

Question B2

- a)
 - i. The real function f is defined as follows $f(x) = \sqrt{2x+1}$ where $x > 1$. What is the range for this function f ? [1]
 - ii. Find the inverse function f^{-1} for this function f . [3]
 - iii. The real functions g and h are defined as follows:

$$g(x) = 2x^2 - 3x + 4$$

$$h(x) = 3x - 2$$
 Find an expression for the function $g(h(x))$. [3]
 - iv. Is the function gh one-one or many-one if the domain of the function $g(h(x))$ is $0 \leq x \leq 2$? [2]
- b)
 - i. Plot $y = 2|4x + 2| - 3$ showing clearly where the curve crosses the X axis and the Y axis. [5]
 - ii. Solve $2|4x + 2| - 3 < 0$ for real x . [1]

Question B3

- a) i. Find all real values of x (in radians) that satisfy $3 - 5 \sin x - \cos 2x = 0$, where $0 \leq x < 2\pi$. [4]
- ii. Show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(2 \sin x - 1) \cos x}{3 - 5 \sin x - \cos 2x} dx = \ln\left(\frac{4 - \sqrt{3}}{4 - \sqrt{2}}\right)$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$. [5]
- b) i. The line L is made by drawing a line from the origin to the point (h, r) , where h and r are positive non-zero real constants. Find the equation of this line. [2]
- ii. The line L is rotated around the X -axis between $x = 0$ and $x = h$ and a cone is generated. By using integration **only** show that the volume of this cone will be $\frac{1}{3}\pi r^2 h$. [4]

Question B4

- a) The line L passes through the point $2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and is parallel to the vector $2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$. Find an expression for the line L . [2]
- b) Consider the line K defined by $\mathbf{r} = (2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) + s(2\mathbf{i} + 4\mathbf{k})$ given that $-\infty < s < \infty$. Where does the line L intersect the line K ? [5]
- c) Find the acute angle the line L makes with the line K . [4]
- d) Consider the non-zero non-equal vectors \mathbf{a} and \mathbf{b} . If $(\mathbf{b} + \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a}) = 0$, then show that $|\mathbf{a}| = |\mathbf{b}|$. [4]

Question B5

- a) i. Express the (real) fraction $\frac{1}{x-x^2}$ as a partial fraction. **[3]**
- ii. Find the exact solution to the first order ordinary differential equation $\frac{dP}{dt} = P - P^2$, where $P = \frac{1}{2}$ when $t = 0$. **[7]**
- (Express your answer in the form $P = f(t)$).
- b) By using the Newton-Raphson method twice and starting with, $x_0 = -1.5$ find an approximation for the root of the cubic equation: **[5]**

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1.$$

(The root is between -1 and -2 .)

Question B6

A survey is carried out on the height of 20 men. The result of the survey is presented in the frequency table below.

Class Interval	Frequency
$145 \leq \text{Height} < 150$	2
$150 \leq \text{Height} < 155$	4
$155 \leq \text{Height} < 160$	6
$160 \leq \text{Height} < 165$	3
$165 \leq \text{Height} < 170$	2
$170 \leq \text{Height} < 175$	2
$175 \leq \text{Height} < 180$	1

- a) Use mid-interval estimates to calculate the mean height of the men. **[4]**
- b) Find the standard deviation of the data. **[5]**
- c) Draw a histogram for the data. **[4]**
- d) Is this data bimodal? **[1]**
- e) Is this data positively skewed or negatively skewed? **[1]**

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