NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Mark Scheme

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Differentiate $\frac{e^x \cos x}{x^2+1}$ with respect to x. **[3]**

(It is not necessary to simplify your answer).

Letting:

$$y = \frac{e^x \cos x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{(x^2+1)\frac{d}{dx}e^x \cos x - e^x \cos x.2x}{(x^2+1)^2}$$

1 mark

So that:

$$\frac{dy}{dx} = \frac{(x^2+1)[e^x \cos x - e^x \sin x] - 2xe^x \cos x.}{(x^2+1)^2}$$

2 marks

Question A2

The equation $x^3 + 8x^2 - 5 = 0$ is to be solved numerically using an iteration formula. One possible formula is: **[4]**

$$x_{n+1} = \frac{5}{x_n^2 + 8x_n}$$

Give **four** other iteration formulae for the given equation
(You do **not** need to find the actual value of the root).

Other rearrangements such as:

$$x_{n+1} = \sqrt{\frac{5 - x_n^3}{8}} \qquad x_{n+1} = \sqrt[3]{5 - 8x_n^2} \qquad x_{n+1} = \sqrt{\frac{5}{x_n + 8}} \qquad x_{n+1} = \frac{5}{x_n^2} - 8$$

1 mark each

Question A3

Given that $y = \ln\left(3x^2 - 5\right)$ where $x^2 > \frac{5}{3}$, find $\frac{dy}{dx}$

[2]

$$\frac{dy}{dx} = \frac{6x}{(3x^2 - 5)}$$ 2 marks

Question A4

[5]

Expand the function $f(x) = 3\cos(2x + \frac{\pi}{3})$ Hence show whether $f(x)$ is odd, even or neither. Find the period and amplitude of $f(x)$.

$$f(x) = 3\cos(2x + \frac{\pi}{3}) = \frac{3}{2}[\cos 2x - \sqrt{3}\sin 2x]$$

1 mark

$$f(-x) = \frac{3}{2}[\cos 2x + \sqrt{3}\sin 2x]$$

1 mark

$$f(x) \neq f(-x) \quad f(x) \neq -f(-x)$$

Thus the function is neither odd nor even.

1 mark

Period is $\frac{2\pi}{2} = \pi$

1 mark

Amplitude is 3.

1 mark

Question A5

If$sinA = \frac{3}{5}$, where $0° < A < 180°$, find the exact value of $\cos 2A$. **[3]**

Using

$$\cos 2A = 1 - 2\sin^2 A$$

1 mark

$$\cos 2A = 1 - 2(\frac{9}{25})$$

1 mark

$$\cos 2A = 7/25$$

1 mark

Question A6

Find the angle, in degrees, between the vectors **u** and **v** where $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}$. **[5]**

$u = i + j + k, v = i - j + k$

$u.v = 1 - 1 + 1 = 1$

1 mark

$1 = \sqrt{3}.\sqrt{3}\cos\vartheta$

2 marks

$\cos\vartheta = \frac{1}{3}$

1 mark

$\vartheta = 70.5$

1 mark

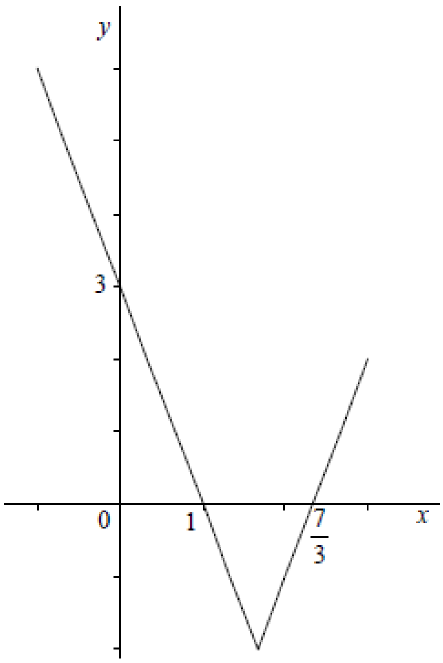
Question A7

[6]

The function *f(x)* is defined for all real values of x by:

$f(x) = |3x - 5| - 2$

Sketch the graph of y=*f(x)* indicating the coordinates of the points where the graph crosses the axes and state the range of .*f(x)*



Shape of **f(x)**

2 marks

Passes through (0,3) 1 mark

Passes through (1,0) 1 mark

Passes through $\left(\frac{7}{3},0\right)$ 1 mark

(ii) The range is **y≥-2** 1 mark

Question A8

Use the substitution $u=1+\sin x$ to evaluate **[6]**

$$\int_0^{\frac{\pi}{2}} \cos x\sqrt{1+\sin x}\,dx$$

$u=1+\sin x, du=\cos xdx$ 1 mark

$$=\int_{x=0}^{\frac{\pi}{2}} u^{1/2}du$$

1 mark

Changing limits

$$=\int_{u=1}^{2} u^{1/2}du$$

1 mark

$$=\left[\frac{2}{3}u^{3/2}\right]_1^2$$

1 mark

$$=\frac{2}{3}\left[2^{3/2}-1\right]$$

1 mark

$$=\frac{2}{3}\left[\sqrt{8}-1\right]$$

1 mark

Question A9

Find the range, the median and the interquartile range of the following set of data: **[6]**

32,57,25,82,54, 37, 29, 47, 65, 56, 32

Range = 82 -25 = 58 1 mark

25,29,32,32,37,47,54,56,57,65,82 1 mark

Lower quartile 32 1 mark

Upper Quartile 57	1 mark
Median 47	1 mark
Interquartile range 57-32=25	1 mark

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

a)

i) Given that a curve has equation $y^2 + 3xy + 4x^2 = 37$ find the value of $\frac{dy}{dx}$ at the point (4,-3) **[4]**

ii) Find the equation of the normal to $y^2 + 3xy + 4x^2 = 37$ at the point (4,-3). **[3]**

b) Given that $y = x\sin 3x$, show that $\frac{d^2y}{dx^2} + 9y = 6\cos 3x$. **[4]**

c) Find the general solution of the differential equation: **[4]**

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\ln(y)}$$

a)

Commencing with:

i) $y^2 + 3xy + 4x^2 = 37$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y + 8x = 0$$ 1 mark

$$(2y + 3x)\frac{dy}{dx} = -3y - 8x$$ 1 mark

$$\frac{dy}{dx} = \frac{-3y - 8x}{2y + 3x}$$ 1 mark

At (4,-3)

$$\frac{dy}{dx} = -23/6$$ 1 mark

ii) At (4,-3)

$$\frac{y+3}{x-4} = \frac{dy}{dx}\Big|_{(4,-3)} = \frac{6}{23}$$ 2 marks

So that:

$23y = 6x - 93$ 1 mark

b) $y = x\sin 3x$

Therefore:

$\frac{dy}{dx} = \sin 3x + 3x\cos 3x$ 1 mark

And:

$\frac{d^2y}{dx^2} = -9x\sin 3x + 6\cos 3x$ 1 mark

So that:

$\frac{d^2y}{dx^2} + 9y = -9x\sin 3x + 6\cos 3x + 9\sin 3x = 6\cos 3x$

as required. 2 marks

c)

$\int (1)\ln(y) = \int xdx$ 1 mark

$u = \ln(y), \frac{dv}{dy} = 1, v = y, \frac{du}{dy} = \frac{1}{y}$ 1 mark

$y\ln(y) - \int y.\frac{1}{y}dy = \frac{x^2}{2} + c$ 1 mark

$y\ln(y) - y = \frac{x^2}{2} + c$ 1 mark

Question B2

a) Figure 1 shows the curve with equation $y = f(x)$.

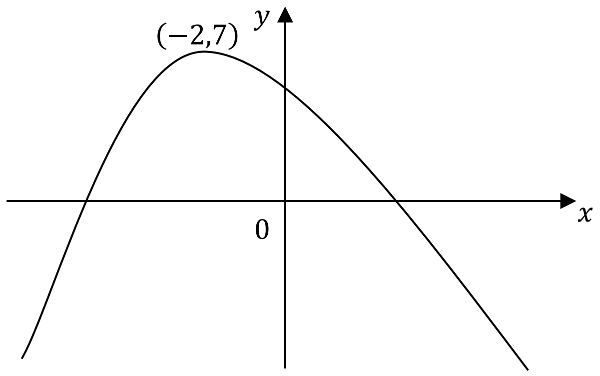


Figure 1

The only maximum point of the curve is $A\ (-2,7)$.

Describe the transformation and write down the coordinates of the maximum point for the curves with each of the following equations:

i. $y = f(x) - 2$ **[2]**

ii. $y = f(3x) + 7$ **[3]**

iii. $y = 3f(x+2)$ **[3]**

b) Functions p and q are defined as follows:

$$p(x) = x + 2$$
$$q(x) = x^2 + 3x - 4$$

i. Evaluate $p\big(q(-3)\big)$and $q\big(p(-3)\big)$. **[2]**

ii. Solve $p\big(q(x)\big) = q\big(p(x)\big)$for x. **[4]**

iii. Find an expression for $p^{-1}(x)$. **[1]**

Solutions:

a)

i. Translate the whole curve -2 units parallel to y-axis. 1 mark

new minimum @ (-2,5). 1 mark

ii. Translate the whole curve 7 units parallel to y-axis 1 mark
stretch it by a factor of 1/3 parallel to x-axis, 1 mark
new max at (-2/3,14). 1 mark

iii. Translates the whole curve 2 units to the left 1 mark
and stretches by scale factor 3 parallel to y-axis 1 mark

new max at (-4,21). 1 mark

b)

i q(-3) = -4, pq(-3) = -2, p(-3) = -1, qp(-3) = -6 2 marks

ii $x^2 + 3x - 2$ = x^2 + 4x + 4 +3x + 6 – 4 2 marks

-8 = 4x, x = -2 2 marks

Iii $p^{-1}(x)$ = x – 2 1 mark

Question B3

a) Use integration by parts to find the exact value of: **[5]**

$$\int_0^1 x^2 e^x dx$$

b) i. Express: $f(x) = \frac{2}{(2-x)(1+x)^2}$ **[5]**

in the form:

$$f(x) = \frac{A}{(2-x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

where A, B and C are constants to be determined.

ii. Use your results from b)(i) above to show that: **[5]**

$$\int_0^1 f(x)dx = \frac{1}{9}\ln(16) + \frac{1}{3}$$

a) Define

$$I = \int_0^1 x^2 e^x dx$$

Letting

$$u = x^2, \frac{du}{dx} = 2x$$

$$\frac{dv}{dx} = e^x, v = e^x$$

1 mark

So that

$$I = [x^2 e^x]_0^1 - 2\int_0^1 xe^x dx$$

1 mark

Letting

$$u = x, \frac{du}{dx} = 1$$

$$\frac{dv}{dx} = e^x, v = e^x$$

1 mark

So that:

$$I = [x^2e^x - 2xe^x + 2e^x]_0^1 = e - 2$$

2 marks

b) i)

$$f(x) = \frac{A}{(2-x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

$$\therefore 2 = A(1+x)^2 + B(2-x)(1+x) + C(2-x)$$

2 marks

Letting: $x = 2 \Rightarrow A = 2/9$

1 mark

Letting: $x = -1 \Rightarrow C = 2/3$

1 mark

Letting: $x = 0 \Rightarrow B = 2/9$

1 mark

If the student equates coefficients, one mark is to be awarded for each correct determination of the constants.

ii)

$$\int_0^1 f(x)dx = \frac{2}{9}\int_0^1 \frac{1}{2-x}dx + \frac{2}{9}\int_0^1 \frac{1}{1+x}dx + \frac{2}{3}\int_0^1 \frac{1}{(1+x)^2}dx$$

So that:

$$\int_0^1 f(x)dx = -\frac{2}{9}[\ln(2-x)]_0^1 + \frac{2}{9}[\ln(1+x)]_0^1 - \frac{2}{3}\left[\frac{1}{1+x}\right]_0^1$$

2 marks

Giving:

$$\int_0^1 f(x)dx = \frac{2}{9}\ln(2) - \frac{1}{3} + \frac{2}{9}\ln(2) + \frac{2}{3}$$

2 marks

So that:

$$\int_0^1 f(x)dx = \frac{1}{9}\ln(16) + \frac{1}{3}$$

1 mark

Question B4

Referred to an origin O, the lines l_1 and l_2 have equations:

$$l_1 : r = i + 2j + 6k + \lambda(i + j - 9k)$$
$$l_2 : r = 4i - 2j - 8k + \mu(i - 6j + 4k)$$

respectively, where λ and μ are scalars to be determined.

a)

i. Prove that the lines intersect and find the position vector of the point of intersection P. [5]

ii. Find the distance between O and P. [2]

iii. Find the vector equation of the line which passes through the points (2,1,9) and (4,-1,8). [3]

b) i. Find the value of λ given that the given vectors are perpendicular

$$9i - 3j + 5k$$
$$\lambda i + \lambda j + 3k$$

[2]

ii. Simplify as far as possible, given that b is perpendicular to c

$$\underline{a}.(\underline{b} + \underline{c}) + \underline{b}.(\underline{a} - \underline{c})$$

[3]

Solutions:

a) equating components:

$$i : 1 + \lambda = 4 + \mu$$
$$j : 2 + \lambda = -2 - 6\mu$$
$$k : 6 - 9\lambda = -8 + 4\mu$$

1 mark

Using the i and j equations gives:

$$7\mu = -7$$

1 mark (If students eliminate λ then similarly award 1 mark)

$$\lambda = 2$$
$$\mu = -1$$

2 marks (1 for each correct answer)

Students must show that the remaining equation is now an identity.1 mark

$\overrightarrow{OP} = 3i + 4j - 12k$

1 mark

ii. $!\overrightarrow{OP}! = \sqrt{3^2 + 4^2 + (-12)^2}$

1 mark

$!\overrightarrow{OP}! = \sqrt{169} = 13$ 1 mark

iii.

$r = \underline{a} + \lambda(\underline{b} - \underline{a})$

$r = 2i + j + 9k + \lambda(4i - j + 8k - (2i + j + 9k))$

2 marks

$r = 2i + j + 9k + \lambda(2i - 2j + -k)$

1 mark

b)

i. $9\lambda - 3\lambda + 15 = 0$

1 mark

$\lambda = -16/6 = -5/2$

1 mark

ii. $\underline{a}.(\underline{b} + \underline{c}) + \underline{b}.(\underline{a} - \underline{c}) = \underline{a}.\underline{b} + \underline{a}.\underline{c} + \underline{b}.\underline{a} - \underline{b}.\underline{c}$

$\underline{a}.(\underline{b} + \underline{c}) + \underline{b}.(\underline{a} - \underline{c}) = \underline{a}.\underline{b} + \underline{a}.\underline{c} + \underline{b}.\underline{a} - \underline{b}.\underline{c}$

1 mark

$\underline{b}\underline{c} = 0$ 1 mark

$\underline{a}.(\underline{b} + \underline{c}) + \underline{b}.(\underline{a} - \underline{c}) = 2\underline{a}.\underline{b} + \underline{a}.\underline{c}$

1 mark

Question B5

a) i. Show that the function $f(x) = 2x^2 - \ln(x+2)$ has a root between x=0 and $x = 1$ **[3]**

ii. Starting with initial value $x_0 = 0.5$ use the Newton-Raphson method **twice**, to give a better approximation to the root of the equation in i. Give your final answer correct to **three** decimal places. **[6]**

b) Find the volume generated when the area between the curve $y = e^{2x} + 3$, the x-axis, the y axis and the line x=1 is rotated through one revolution about the x-axis. **[6]**

a) i.

$f(x) = 2x^2 - \ln(x+2)$

$f(0) = 2.0^2 - \ln(2) = -\ln(2) < 0$ 1 mark

$f(1) = 2.1^2 - \ln(3) = 0.901 > 0$ 1 mark

Thus there has been a change of sign hence a root exists between x=0 and x=1.
1 mark

ii.

$f'(x) = 4x - \dfrac{1}{x+2}$ 1 mark

So that:

$x_{n+1} = x_n - \left[\dfrac{2x^2(x+2) - (x+2)\ln(x+2)}{4x(x+2) - 1}\right]$ o.e. 1 mark

$x_0 = 0.5$

$x_1 = 0.760181707$ 2 marks

$x_2 = 0.70774209 = 0.708(3dp)$ 2 marks

b) $V = \int_0^1 \pi y^2 dx$

c) So that:

$V = \pi \int_0^1 e^{4x} + 6e^{2x} + 9dx$ 1 mark

Giving:

$V = \pi \left[\dfrac{e^{4x}}{4} + 3e^{2x} + 9x\right]_0^1$ 2 marks

$V = \dfrac{\pi}{4}(\dfrac{e^4}{4} + 3e^2 + 9 - \dfrac{1}{4} - 3)$ 2 marks

$V = \dfrac{\pi}{4}(\dfrac{e^4}{4} + 3e^2 + \dfrac{23}{4})$ 1 mark

Question B6

The grouped frequency table below shows the number of mobile phones sold per week in a store during a period of 110 weeks:

Number of mobile phones sold	Frequency
0 – 19	6
20 – 29	13
30 – 39	16
40 – 49	27
50 – 59	28
60 – 69	13
70 – 89	7

a) Calculate the mean number of mobile phones sold per week. **[4]**

b) Find the standard deviation of the data. **[5]**

c) By copying and extending the table in an appropriate manner, draw the graph of the cumulative frequency polygon. **[4]**

d) Using your graph, estimate the median of the data. **[2]**

Solutions:

Number of cell phones sold	Frequency, f	$x_{midpoint}$	$x_{midpoint} f$	$x^2_{midpoint} f$	Cumulative frequency
0 – 19	6	9.5	57	541.5	6
20 – 29	13	24.5	318.5	7803.25	19
30 – 39	16	34.5	552	19055	35
40 – 49	27	44.5	1201.5	53466.75	62
50 – 59	28	54.5	1526	83167	90
60 – 69	13	64.5	838.5	54083.24	103
70 – 89	7	79.5	556.5	44241.75	110
	Sum=110		Sum=5050	Sum=262347.5	

a) Correct midpoint column. 1 mark

Correct $x_{midpoint} f$ column 1 mark

Correct calculation of mean=5050/110=45.91 2 marks

b) Correct $x^2_{midpoint} f$ column and sum. 2 marks

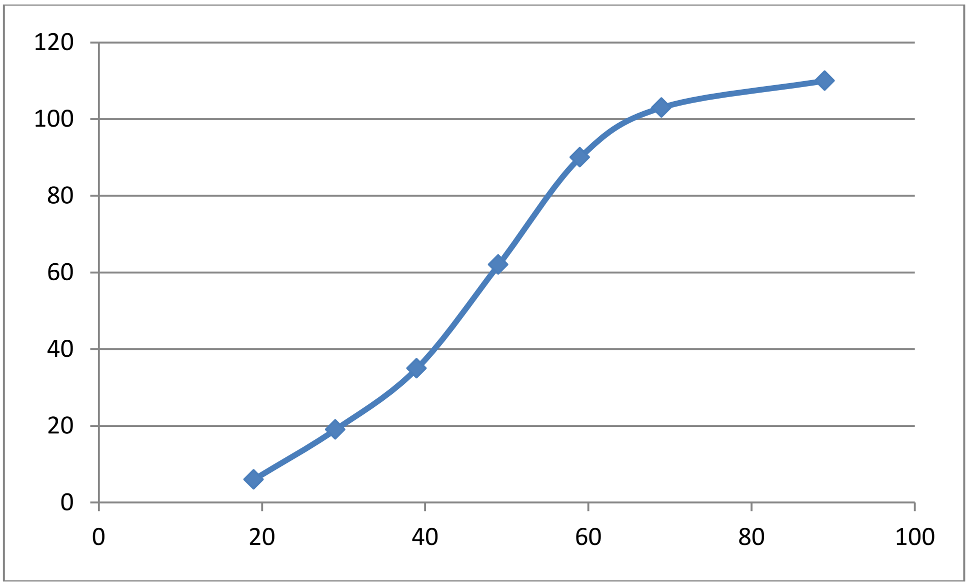
$$\sigma = \sqrt{\frac{262347.5}{110} - (45.91)^2}$$

2 marks

$$\sigma = 16.65 = 16.7$$

1 mark

c) correct cumulative frequency column. 1 mark
end points used 1 mark
Graph 2 marks



d) Indication of median read from cf 55 1 mark
Median45-48 acceptable 1 mark