NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME001 Mathematics Part 2 (Science & Engineering) Examination

Examination Session
Semester Two
2010-11

Time Allowed
2 Hours 10 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**
SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for **full and detailed answers**.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Differentiate $\frac{e^x \cos x}{x^2+1}$ with respect to x. **[3]**

(It is **not** necessary to simplify your answer).

Question A2

The equation $x^3+8x^2-5=0$ is to be solved numerically using an iteration formula. **[4]**
One possible formula is:

$$x_{n+1} = \frac{5}{x_n^2+8x_n}.$$

Give **four** other iteration formulae for the equation.
(You do **not** need to find the actual value of the root).

Question A3

Given that $y = \ln(3x^2-5)$ where $x^2 > \frac{5}{3}$, find $\frac{dy}{dx}$. **[2]**

Question A4

Expand the function $f(x) = 3\cos(2x+\frac{\pi}{3})$ and hence show whether $f(x)$ is odd, even or neither. Find the period and amplitude of $f(x)$. **[5]**

Question A5

If $sinA = \frac{3}{5}$, where $0° < A < 180°$, find the exact value of $\cos 2A$. **[3]**

Question A6

Find the angle, in degrees, between the vectors $\mathbf{u}$ and $\mathbf{v}$ where $\mathbf{u} = \mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{v} = \mathbf{i}-\mathbf{j}+\mathbf{k}$. **[5]**

Question A7

The function $f(x)$ is defined for all real values of x by: **[6]**

$$f(x) = |3x - 5| - 2$$

Sketch the graph of $y=f(x)$ indicating the coordinates of the points where the graph crosses the axes and state the range of $f(x)$.

Question A8

Use the substitution $u = 1 + \sin x$ to evaluate: **[6]**

$$\int_0^{\frac{\pi}{2}} \cos x \sqrt{1 + \sin x} dx$$

Question A9

Find the range, the median and the interquartile range of the following set of data: **[6]**

$$32,57,25,82,54,37,29,47,65,56,32$$

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

a) i. Given that a curve has equation $y^2 + 3xy + 4x^2 = 37$ find the value of $\frac{dy}{dx}$ at the point (4,-3). **[4]**

ii. Find the equation of the normal to $y^2 + 3xy + 4x^2 = 37$ at the point (4,-3). **[3]**

b) Given that $y = x\sin 3x$, show that $\frac{d^2y}{dx^2} + 9y = 6\cos 3x$. **[4]**

c) Find the general solution of the differential equation: **[4]**

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{\ln(y)}$$

Question B2

a) Figure 1 shows the curve with equation $y = f(x)$:

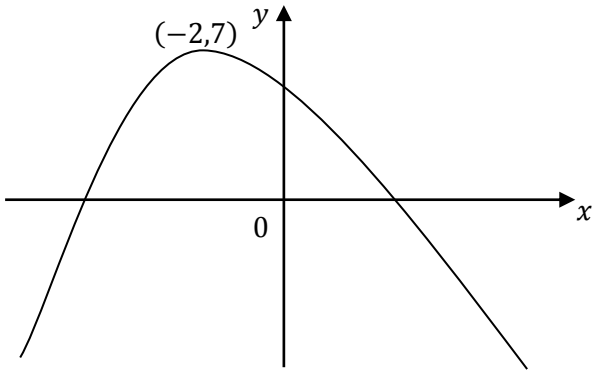

Figure 1

The only maximum point of the curve is $A\ (-2,7)$.

Describe the transformation and write down the coordinates of the maximum point for the curves with each of the following equations:

i. $y = f(x) - 2$ **[2]**

ii. $y = f(3x) + 7$ **[3]**

iii. $y = 3f(x + 2)$ **[3]**

b) Functions p and q are defined as follows:

$$p(x) = x + 2$$
$$q(x) = x^2 + 3x - 4$$

i. Evaluate $p\big(q(-3)\big)$ and $q\big(p(-3)\big)$. **[2]**

ii. Solve $p\big(q(x)\big) = q\big(p(x)\big)$for x. **[4]**

iii. Find an expression for $p^{-1}(x)$. **[1]**

Question B3

a) Use integration by parts to find the exact value of: **[5]**

$$\int_0^1 x^2 e^x dx$$

b) i. Express: $f(x) = \dfrac{2}{(2-x)(1+x)^2}$ **[5]**

in the form:

$$f(x) = \frac{A}{(2-x)} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

where A, B and C are constants to be determined.

ii. Use your results from b) i. above to show that: **[5]**

$$\int_0^1 f(x)dx = \frac{1}{9}\ln(16) + \frac{1}{3}$$

Question B4

Referred to an origin O, the lines l_1 and l_2 have equations:

$l_1 : r = i + 2j + 6k + \lambda(i + j - 9k)$
$l_2 : r = 4i - 2j - 8k + \mu(i - 6j + 4k)$

respectively, where λ and μ are scalars to be determined.

a) i. Prove that the lines intersect and find the position vector of the point of intersection P. **[5]**

ii. Find the distance between O and P. **[2]**

iii. Find the vector equation of the line which passes through the points (2,1,9) and (4,-1,8). **[3]**

b) i. Find the value of λ given that the given vectors are perpendicular: **[2]**

$9i - 3j + 5k$
$\lambda i + \lambda j + 3k.$

ii. Simplify as far as possible, given that b is perpendicular to c: **[3]**

$\underline{a}.(\underline{b} + \underline{c}) + \underline{b}.(\underline{a} - \underline{c}).$

Question B5

a) i. Show that the function $f(x) = 2x^2 - \ln(x + 2)$ has a root between x = 0 and $x = 1.$ **[3]**

ii. Starting with initial value $x_0 = 0.5$ use the Newton-Raphson method **twice**, to give a better approximation to the root of the equation in i. Give your final answer correct to **three** decimal places. **[6]**

b) Find the volume generated when the area between the curve $y = e^{2x} + 3$, the x-axis, the y-axis and the line x=1 is rotated through one revolution about the x-axis. **[6]**

Question B6

The grouped frequency table below shows the number of mobile phones sold per week in a store during a period of 110 weeks:

Number of mobile phones sold	Frequency
$0 - 19$	6
$20 - 29$	13
$30 - 39$	16
$40 - 49$	27
$50 - 59$	28
$60 - 69$	13
$70 - 89$	7

a) Calculate the mean number of mobile phones sold per week. **[4]**

b) Find the standard deviation of the data. **[5]**

c) By copying and extending the table in an appropriate manner, draw the graph of the cumulative frequency polygon. **[4]**

d) Using your graph, estimate the median of the data. **[2]**