NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Maths Part 2 (Business) Examination

2013-14

MARK SCHEME

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A3. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

If a student has answered more than the required number of questions, credit should only be given for the first n answers, in the order that they are written in the student's answer booklet (n being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1	
a)	The mean number of absences =(0x7+1x12+2x10+3x4) / (7+12+10+4) = 44 / 33 **M1** =1.3333 = 1.3 days **A1**
b)	The median number of absences Middle position is (33+1)/2 = 17th item **M1** Therefore median is 1 day of absence **A1** accept finding n/2 rather than (n+1)/2 and this would also give median of 1 day

Question A2	
a)	Range = 1.18-1.05 = 0.13 metres **M1**
b)	Ordered data: 1.05, 1.05, 1.10, 1.13, 1.14, 1.16, 1.18 **M1** Upper quartile 1.16 lower quartile 1.05 IQR=1.16-1.05 =0.11 metres **M1, A1**

Question A3

p(in a sample of seven adults at least three of them will own a mobile phone)
= 1-[P[0) + P(1) +p(2)] **M1**

$=1-[\left(\frac{1}{5}\right)^7+\binom{7}{1}\left(\frac{4}{5}\right)^1\left(\frac{1}{5}\right)^6+\binom{7}{2}(\frac{4}{5})^2(\frac{1}{5})^5\ ]$ **M2**

=1-0.004672
=0.995328 = 0.995 **(to 3SF)** **A1**

Alternatively may find [p(3) + p(4) + p(5) + p(6) + p(7)]
Or may consider the number of adults who do NOT own a mobile phone, $\overline{X}$,
$\overline{X}$ is B(0.2,7) **M1**
And $P(X \geq 3) = P(\overline{X} \leq 4)$ **M1**
=0.995 from cumulative binomial distribution tables **M1, A1** for using tables correctly

Question A4	
a)	$P(A\|B)=\frac{p(A\cap B)}{P(B)}$ therefore $P(B)=\frac{p(A\cap B)}{P(A\|B)}$ $=\frac{\frac{4}{10}}{\frac{4}{7}}=\frac{7}{10}$ **M1,A1**
b)	$p(\bar{B})=1-P(B)$ $=1-\frac{7}{10}=\frac{3}{10}$ **A1**
c)	If independent then $p(A\cap B)=p(A)p(B)$ Therefore $\frac{4}{10}=\frac{7}{10}p(A)$ $P(A)=\frac{\frac{4}{10}}{\frac{7}{10}}=\frac{4}{7}$ **M1,A1**

Question A5

a) The value of k:

$\frac{1}{4}+\frac{1}{4}+\frac{1}{20}+k+k+k=1$ **M1**

Therefore $k=\frac{9}{20}\div 3=\frac{3}{20}$ **A1**

b) $V[X]=E[X^2]-\mu^2$

$\mu=E[X]=1\times\frac{1}{4}+2\times\frac{3}{20}+3\times\frac{1}{20}+4\times\frac{3}{20}+5\times\frac{3}{20}+6\times\frac{1}{4}$ **M1**

$=3.55$

$E[X^2]=1^2\times\frac{1}{4}+2^2\times\frac{3}{20}+3^2\times\frac{1}{20}+4^2\times\frac{3}{20}+5^2\times\frac{3}{20}+6^2\times\frac{1}{4}$ **M1**

$=\frac{329}{20}=16.45$

$V[X]=E[X^2]-\mu^2$

$=16.45-3.55^2=3.8475$ **A1**

Question A6

$(3x-1)^2=(x-5)^2$

$9x^2-6x+1=x^2-10x+25$

$8x^2+4x-24=0$

$2x^2+x-6=0$

$(2x-3)(x+2)=0$

$\therefore x=\frac{3}{2}$ or $x=-2$

3 marks for any acceptable method

For example
M1 for attempting to sq both sides.

M1 for $8x^2+4x-24=0$
M1 for $(2x-3)(x+2)=0$
A1(mark for both answers.)

Question A7

$\frac{(x+7)}{(x+1)^2(x+3)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x+3)}$

$(x+7)=A(x+1)(x+3)+B(x+3)+C(x+1)^2$

$x=-1$

$6=2B\Rightarrow B=3$

$x=-3$

$4=4C\Rightarrow C=1$

Eq.coeff

$x^2:0=A+C\Rightarrow A=-1$

M1 for the split

M1 for a method

A2 marks for all correct answers **OR**

A1 mark for two correct answers.

Question A8

$$6 = 4x + 2y + 8xy$$

$$0 = 4 + 2\frac{dy}{dx} + 8y + 8x\frac{dy}{dx}$$

$$0 = 4 + 2\frac{dy}{dx} + 8\left(\frac{-5}{3}\right) + 8(-1)\frac{dy}{dx}$$

$$0 = 4 + 2\frac{dy}{dx} + \left(\frac{-40}{3}\right) - 8\frac{dy}{dx}$$

$$0 = \left(\frac{12}{3}\right) + \left(\frac{-40}{3}\right) - 6\frac{dy}{dx}$$

$$\frac{28}{3} = -6\frac{dy}{dx}$$

$$\frac{28}{3 \times -6} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{14}{-9}$$

M2 for ALL correct diff.
M1 mark for 2 correct diff.

M1 for sub and take away

A1

Question A9

a)

$$6+4x-x^2 = -(x^2-4x)+6$$
$$= -[(x-2)^2-4]+6$$
$$= -(x-2)^2+4+6$$
$$= -(x-2)^2+10$$
$$= 10-(x-2)^2$$

A1 for 10, **A1** for -2

b)

$$f(x) = 10-(x-2)^2$$
$$y = 10-(x-2)^2$$
$$y-10 = -(x-2)^2$$
$$(x-2)^2 = 10-y$$
$$(x-2) = \sqrt{10-y}$$
$$-x = -2 \pm \sqrt{10-y}$$
$$x = 2 \mp \sqrt{10-y}$$
$$y = 2 \mp \sqrt{10-x}$$
$$f^{-1}(x) = 2 \mp \sqrt{10-x}$$
$$\text{As } x \geq 2$$
$$f^{-1}(x) = 2+\sqrt{10-x}$$

M1

M1 for sq root

A1

c)

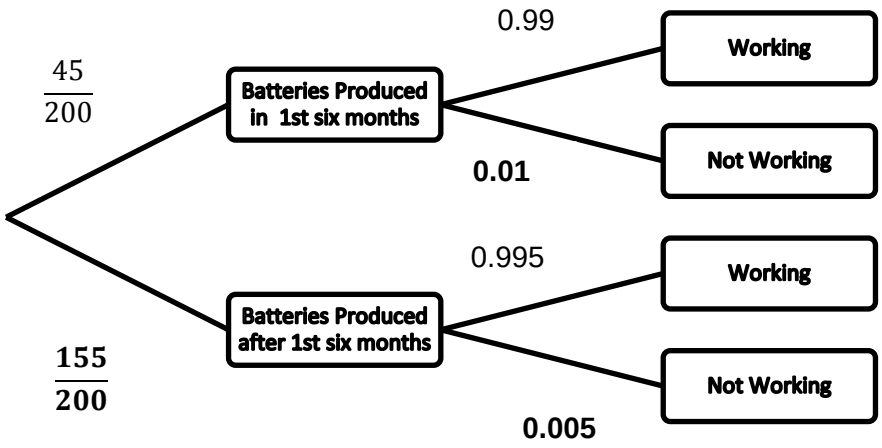
$$x : x \in R, x \leq 10$$
$$f^{-1}(x) : f^{-1}(x) \in R, f^{-1}(x) \geq 2$$

A1 for ≥ 2

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

a) i. Completed tree diagram:



if all **three** correct,(-1 for each error or omission until you get to 0) **A2**

ii. P(found to not work)=

$\frac{45}{200} \times 0.01 + \frac{155}{200} \times 0.005 = 0.006125$ **M1,A1**

iii. P(that the battery was produced during period A, when the machine was found to be defective)

=P(produced during period A **AND** found to not work) / P(found to not work)

$= \frac{\frac{45}{200} \times 0.01}{0.006125}$

$= \frac{0.00225}{0.006125}$

$= 0.3673$ ($= \frac{18}{49}$) **M1,A1**

b) i. $p(X > 14) = 0.063$ then

$p\left(Z > \frac{14-11}{\sigma}\right) = 0.063$ **M1**

$p\left(Z \leq \frac{14-11}{\sigma}\right) = 0.937$

from tables $p(Z \leq 1.53) = 0.937$ **A1** for correct use of tables

therefore $\frac{14-11}{\sigma} = 1.53$

$\sigma = \frac{14-11}{1.53} = 1.961$ or anything rounding to 1.96 **A1**

	ii.	$p(9.5 < X < 12.5) = p(\frac{9.5-11}{1.961} < Z < \frac{12.5-11}{1.961})$ **M1** $= p(-0.76 < Z < 0.76)$ $= p(Z < 0.76) - p(Z < -0.76)$ $= 2p(Z < 0.76) - 1$ $= 2 \times 0.7764 - 1 \ = 0.5528$ **M1,A1** (or answer appropriate for value of σ used) As a percentage $0.5528 \times 100 = 55.3\%$ **A1**
	iii.	Using $\bar{x} \pm 1.96\frac{s}{\sqrt{n}}$ $11.5 \pm 1.96\frac{4}{\sqrt{50}}$ correct substitution of values **M1** (10.4,12.6) or any suitable rounding of limits **A1**

Question B2	
a)	Using $y-\bar{y}=\frac{s_{xy}}{s_{x^2}}(x-\bar{x})$ where $s_{xy}=\frac{\sum xy}{n}-\bar{x}\bar{y}$ $s_{x^2}=\frac{\sum x^2}{n}-\bar{x}^2$ $s_{xy}=\frac{49825}{9}-\left(\frac{485}{9}\right)\left(\frac{776}{9}\right)=\frac{72065}{81}$ $s_{x^2}=\frac{31475}{9}-\left(\frac{485}{9}\right)^2=\frac{48050}{81}$ $y-\frac{776}{9}=\frac{72065}{48050}(x-\left(\frac{485}{9}\right))$ **M3** $y=1.5x+5.4$ or any appropriately rounded coefficients **A1**
b)	The value of b represents the gradient of the line. **A1** **Or** The value of b represents the rate of change of y for a unit change in x.
c)	The value x=140 is quite far beyond the range of the recorded values of x, hence the equation would not provide a reliable estimate of y for a value of x = 140. **A1** **Or any other well-justified response.**
d)	Using the equation, when x= 45 $y=1.5(45)+5.4=72.9$ **A1**
e)	For calculating a difference between the estimate of y and the observed value of y $eg.$ 70-72.9= -2.9 **A1** For any appropriate comment which relates to the suitability (or otherwise) of the regression equation found such as "There is only a small difference therefore the regression equation is a good fit" **A1**
f)	$r=\frac{s_{xy}}{s_x s_y}$ where $s_{y^2}=\frac{\sum y^2}{n}-y^2$ $=\frac{78976}{9}-\left(\frac{776}{9}\right)^2$ $r=\frac{889.69136}{\sqrt{593.2098765}\sqrt{1340.839506}}$ **M2** =0.998 **A1**
g)	The value of r indicates strong positive correlation, **A1** as indicated by the appearance of the points in the scatter diagram lying very close to a straight line which has positive gradient **A1**
h)	The two variables may both have a dependence upon another (a third) variable such as time**.** **A1**

Question B3		
a)	"The peaks and troughs are decreasing over time and each set of quarterly variations within a year they appear to decrease proportionally." **A1**for recognising the change of size of the seasonal variations over time and **A1** for recognising a proportional change.	
b)	A=(30+35+92+79)/4=59 **A1** B=(62.25+57.75)/2=60 **A1** C=92/58=1.59 **A1**	
c)	D=0.5 **A1** E=(0.48+0.43+0.5)/3=0.47 **A1** F=(1.14+1.18)/2=1.16 **A1**	
d)	A seasonal effect of 1.72 for Quarter 1 means that the sales in Quarter 1 are on average 72 percent above the general trend and 0.6 for Quarter 4 means that the sales in Quarter 4 are on average 40 percent below the general trend. (or are only equivalent to 60 percent of the general trend value) Only award the full **A3** if the proportionality is accounted for in the description of the figures.	
e)	i.	quarter 4 of year 4: x=16 Trend $y = -2.86(16) + 91.7 = 45.94$ **M1** Seasonal Ratio 0.6 Projected value $45.94 \times 0.6 = 27.6$ (£27,600) **A1**
	ii.	quarter 1 of year 5 x=17 Trend $y = -2.86(17) + 91.7 = 43.08$ (£43,080) **M1** Seasonal Ratio 1.72 Projected value $43.08 \times 1.72 = 74.1$ (£74,100) **A1**

Question B4

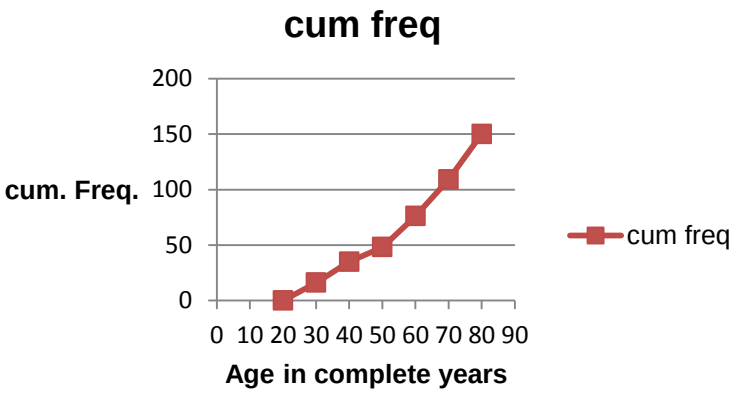
a) Accept either symmetrical or positive skewness **A1**

b)

Age in complete years	Frequency	Upper bounds	cum freq
		20	0
20 to 29	16	30	16
30 to 39	19	40	35
40 to 49	13	50	48
50 to 59	28	60	76
60 to 69	33	70	109
70 to 79	41	80	150
Total	150		

A1 for the upper bounds
A1 for the cum. Freq. Column. (at least 4 correct to get mark)

c)



M1 for scale
M1 for both labels, **M1** for joining points with straight lines
M2 for the plots (1 mark **must be** for plotting the first point at their lower class bound on the horizontal axis and 1 mark if at least 6 points are plotted correctly based on upper bounds found by the student).
[**-1** if graph paper is not used].

d)

Median =	60
Q1=	40
Q3=	72

M3 [1 for each from answer from their graph.]

e)

Q2-Q1=	20
Q3-Q2=	12

M1 for the calculation. **A1**for comment Negative skew and therefore histogram and cumulative frequency chart are not from the same choir.

f) About 90 people. **M1** for use of graph. **A1** for answer **FROM THEIR GRAPH**

Question B5

a)i

$\Delta = 1\begin{vmatrix}2 & 4\\ 1 & -2\end{vmatrix} - 1\begin{vmatrix}4 & 4\\ 2 & -2\end{vmatrix} + 2\begin{vmatrix}4 & 2\\ 2 & 1\end{vmatrix}$ **M1**

$\Delta = 1(-4-4) - 1(-8-8) + 2(4-4)$ **M1**

$\Delta = -8 + 16 + 0 = 8$ **A1**

Must show working, if no working then **M1,M0, A0** they have used their calculator.

ii.

Using $A^{-1}A=I$

$$\frac{1}{8}\begin{bmatrix}-8 & 4 & 0\\ 16 & a & 4\\ b & c & -2\end{bmatrix}\begin{bmatrix}1 & 1 & 2\\ 4 & 2 & 4\\ 2 & 1 & -2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$ **M1**

$\frac{1}{8}(16 + 4a + 8) = 0$

$24 + 4a = 0 \rightarrow a = -6$ **A1**

$\frac{1}{8}(b + 4c - 4) = 0$

$b + 4c = 4$ eq1

$\frac{1}{8}(b + 2c - 2) = 0$

$b + 2c = 2$ eq2

eq1-eq2 gives $2c = 2 \rightarrow c = 1 \ \therefore b = 0$ **M1, A1** for both answers

iii.

Hence,

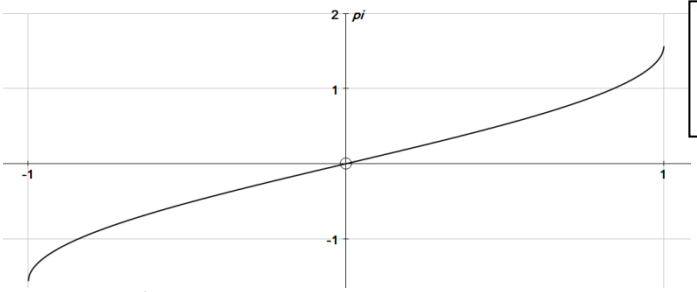
$$\begin{bmatrix}1 & 1 & 2\\ 4 & 2 & 4\\ 2 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 6\\ -1\end{bmatrix}$$ **M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{8}\begin{bmatrix}-8 & 4 & 0\\ 16 & -6 & 4\\ 0 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\ 6\\ -1\end{bmatrix}$$ **M1**

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{8}\begin{bmatrix}16\\ -24\\ 8\end{bmatrix}$$ **M1**

$x = 2, y = -3, z = 1$ **A1 ALL 3 correct**

b) i.



M1 shape
M1 domain -1 to 1
M1 range $\frac{-\pi}{2}$ *to* $\frac{\pi}{2}$

ii. $sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{-\pi}{3}$ **A1**

Question B6

a)

$$\int e^x sinx\, dx$$

$u = e^x$ $\qquad$ $v' = sinx$

$u' = e^x$ $\qquad$ $v = -cosx$ $\qquad$ **A1** for u', **A1** for v

$$\int e^x sinx\, dx = -e^x cosx - \int e^x(-cosx)dx$$

$$\int e^x sinx\, dx = -e^x cosx + \int e^x(cosx)dx \qquad (1)$$

$$\int e^x(cosx)dx$$

$u = e^x$ $\qquad$ $v' = cosx$

$u' = e^x$ $\qquad$ $v = sinx$

$\int e^x(cosx)dx = e^x(sinx) - \int e^x(sinx)dx$ $\qquad$ sub into (1) $\qquad$ **M1** for 2nd integ.

$$\int e^x sinx\, dx = -e^x cosx + e^x(sinx) - \int e^x(sinx)dx$$

$$2\int e^x sinx\, dx = -e^x cosx + e^x(sinx) = \frac{1}{2}e^x(sinx - cosx) + c \qquad \textbf{A1}$$

b)

$u = 4 - x \rightarrow x = 4 - u \therefore x + 2 = 6 - u$

$\frac{du}{dx} = -1 \rightarrow \frac{dx}{du} = -1$

x	*u=4-x*
4	0
1	3

$$\int_3^0 (6-u)u^{\frac{1}{2}}(-1)du = \int_3^0 (u-6)u^{\frac{1}{2}}du$$

$$= \int_3^0 u^{\frac{3}{2}} - 6u^{\frac{1}{2}}du$$

$$= \left[\frac{2}{5}u^{\frac{5}{2}} - 6\times\frac{2}{3}u^{\frac{3}{2}}\right]_3^0$$

$$= 0 - \left(\frac{2}{5}\left(3^{\frac{5}{2}}\right) - 4\left(3^{\frac{3}{2}}\right)\right)$$

$$= 14.5492 = 14.55$$

M1 for $x + 2 = 6 - u$

M1 for $\frac{dx}{du} = -1$

M1 for change of limits

M1 for $\int_3^0 (u-6)u^{\frac{1}{2}}du$

M1 for the integration

A1 for any answer which rounds to 15 will be acceptable.

c)

$$V = \pi\int_0^2 y^2\,dx = \pi\int_0^2\left(\frac{4}{7-3x}\right)^2 dx = \pi\int_0^2 \frac{4^2}{(7-3x)^2}\,dx$$

$$= \pi\int_0^2 16(7-3x)^{-2}\,dx$$

$$= \frac{16}{-3}\pi\int_0^2 -3(7-3x)^{-2}\,dx$$

$$= \frac{16\pi}{-3}\left[\frac{-1}{(7-3x)}\right]_0^2$$

$$= \frac{16}{3}\pi\left[\frac{1}{(7-3x)}\right]_0^2$$

$$= \frac{16}{3}\pi\left(1-\frac{1}{7}\right) = \frac{32}{7}\pi \text{ cubic units}$$

or $4.57\pi\ \text{u}^3$

M1 $\pi\int_0^2 \frac{4^2}{(7-3x)^2}\,dx$

M1 for the integration

M1 for substitution of limits
A1 for answer **IN TERMS OF π**
A1 for cubic units