NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Mathematics
Part 2 (Business) Examination

Examination Session	**Time Allowed**
Semester Two	2 Hours 10 minutes
	(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet and graph paper.

- **Answers must not be written during the first 10 minutes.**

- Write your NCUK ID Number clearly on the answer books in the space provided.

- Write the answers in the answer books provided. Additional sheets will be provided on request.

- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.

- **No** written material is to be brought into the examination room.

- **No electronic devices** (e.g. mobile phones, tablets, iPads) are allowed in the examination room.

- An approved calculator may be used in the examination.

- State the units where necessary.

- Show **ALL** workings in your answer booklet. Marks will be awarded for correct workings.

- Examination materials must not be removed from the examination room.

- Write your name and candidate number on all loose sheets/diagrams.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

The number of days off due to sickness taken by staff in a hotel are shown in the table below:

Number of days absent	Number of staff
0	7
1	12
2	10
3	4

Find:

a)   The mean number of absences.   **[2]**

b)   The median number of absences.   **[2]**

Question A2

The heights in metres of seven children are :

1.05, 1.13, 1.18, 1.14, 1.16, 1.05, 1.10

Find:

a)   The range of the heights.   **[1]**

b)   The interquartile range of the heights.   **[3]**

Question A3

Four in every five adults owns a mobile phone. What is the probability that in a sample of seven randomly selected adults, **at least** three of them will own a mobile phone? Give your answer to **three significant figures.**   **[4]**

In this question, 1 mark will be awarded for the correct use of significant figures.

Question A4

A and B are two events. Given that

$$p(A \cap B) = \frac{4}{10} \quad \text{and} \quad p(A|B) = \frac{4}{7}$$

Find:

a) $p(B)$ **[2]**

b) $p(B')$ **[1]**

c)   If A and B are independent find $p(A)$ **[2]**

Question A5

A cubical die is biased so that it has a probability distribution as shown:

$$p(x = r) = \frac{1}{4} \quad \text{for r} = 1, 6$$

$$\frac{1}{20} \quad \text{for r} = 3$$

$$k \quad \text{for r} = 2, 4, 5$$

Find:

a)   The value of k.   **[2]**

b)   V[X]   **[3]**

Question A6

Solve

$$|3x - 1| = |x - 5|$$ **[4]**

Question A7

Express as partial fractions $\dfrac{(x+7)}{(x+1)^2(x+3)}$ **[4]**

Question A8

A curve C is described by the equation

$$6 = 4x + 2y + 8xy$$

Find the gradient of the curve at the point $\left(-1, \dfrac{-5}{3}\right)$. **[4]**

Question A9

The function f is defined by $f :\rightarrow 6 + 4x - x^2, x \in R, x \geq 2$.

a)   Express $f(x)$ in the form $a - (x + b)^2$ where a and b are integers. **[2]**

b)   Find $f^{-1}(x)$. **[3]**

c)   State the range for which $f^{-1}(x)$ exists. **[1]**

Section B

Answer <u>4</u> questions. This section carries 60 marks.

Question B1

a) Company A buys a machine which produces batteries. 1% of the batteries produced during Period A do not work.

After Period A the machine is serviced by an engineer. Following this service, only 0.5% of the batteries produced do not work.

Company B buys 200 batteries from company A. Forty five of these batteries were produced during Period A, the rest were produced after the machine was serviced.

    i.   **In your answer booklet**, copy and complete the tree diagram below: **[2]**
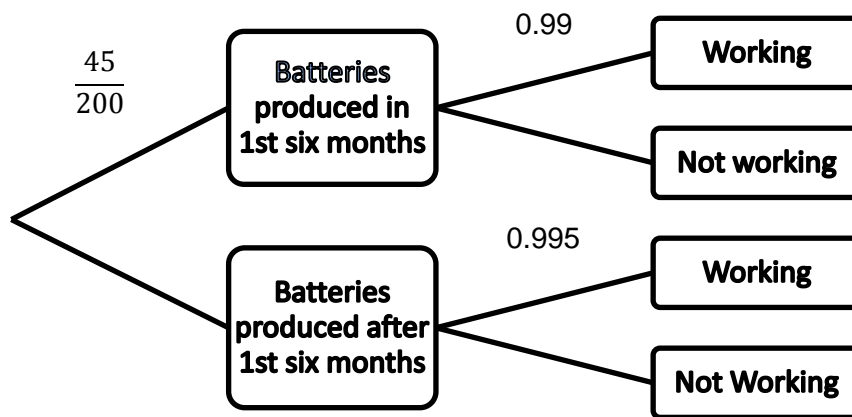

    ii.   Show that if a battery is selected at random from the 200 batteries purchased by Company B, the probability that it does not work is 0.006125.   **[2]**

    iii.   A manager in company B finds that a randomly selected battery from the 200 batteries does not work. What is the probability that the battery was produced during Period A?   **[2]**

b) A variety of plant is expected to grow to a height that is Normally distributed with mean 11 cm and variance σ^2.

    i. If the probability that a randomly selected plant grows to a height of more than 14 cm is 0.063, find the value of the standard deviation, σ. **[3]**

    ii. Find the percentage of plants that would be expected to grow to a height between 9.5 cm and 12.5 cm. **[4]**

    iii. Another variety of plant has a height which is Normally distributed with unknown mean and variance. In a random sample of fifty of these plants the mean height is 11.5 cm and the standard deviation is 4 cm. Use these sample estimates to find a 95% confidence interval for the population mean height, μ, of plants of this variety. **[2]**

Question B2

In a statistical investigation observations were obtained on 9 pairs of values (x,y) and the data was illustrated in a scatter diagram as in figure 1 below:

Figure 1
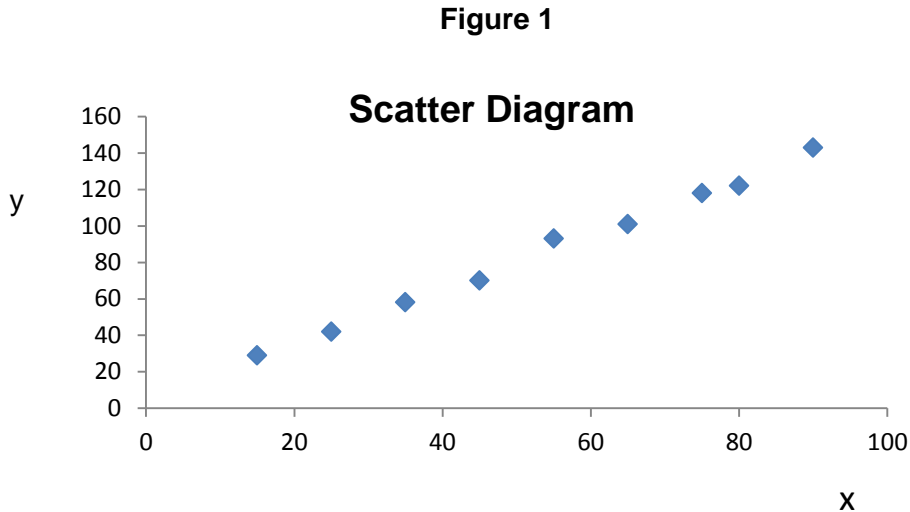
A set of totals was obtained; these are given in the table below:

n	$\sum x$	$\sum y$	$\sum x^2$	$\sum y^2$	$\sum xy$
9	485	776	31475	78976	49825

a) By using the appropriate totals, find the Ordinary Least Squares regression equation $y = a + bx$. **[4]**

b) Explain what the value of b represents in this equation. **[1]**

c) State, giving a reason, whether the equation would provide a reliable estimate of y for a value of x = 140. **[1]**

d) Use the equation which you found in part (a) to estimate the value of y when x = 45. **[1]**

e) One of the observed data pairs is x = 45 and y = 70. Calculate the difference between this value of y and the estimated value of y from part (d) and comment on what this difference indicates. **[2]**

f) Calculate the value of Pearson's Product Moment Correlation Coefficient, r. **[3]**

g) Interpret the value of r which you have found in relation to the scatter diagram in Figure 1. **[2]**

h) Explain briefly why a high value of a correlation coefficient r does not "prove" that y depends upon x. **[1]**

Question B3

Sales data was collected and analysed. The graph in Figure 1 below shows the sales data in a time series graph and tables 1 and 2 show the time series analysis for a **multiplicative model**.
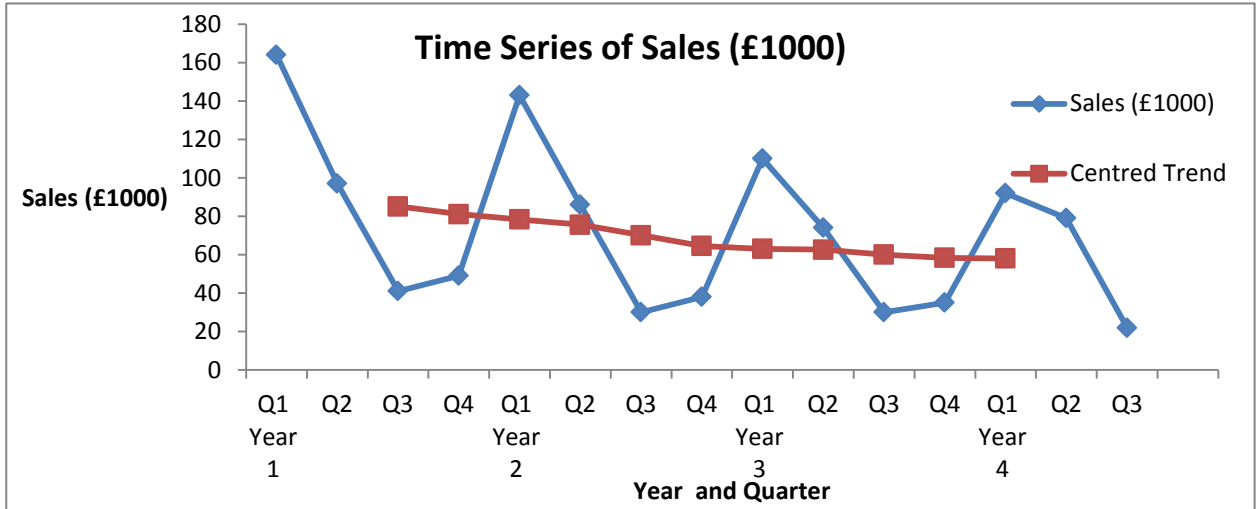
Figure 1

Table 1

	Sales (£1000)	4 Point Moving Average	Centred Trend	Ratio
Year 1 Q1	164			
Q2	97	87.75		
Q3	41	82.5	85.125	0.48
Q4	49	79.75	81.125	0.60
Year 2 Q1	143	77	78.375	1.82
Q2	86	74.25	75.625	1.14
Q3	30	66	70.125	0.43
Q4	38	63	64.5	0.59
Year 3 Q1	110	63	63	1.75
Q2	74	62.25	62.625	1.18
Q3	30	57.75	**B**	0.50
Q4	35	**A**	58.375	0.60
Year 4 Q1	92	57	58	**C**
Q2	79			
Q3	22			

Table 2
Seasonal Effects

	Q1	**Q2**	**Q3**	**Q4**
Year 1			0.48	0.60
Year 2	1.82	1.14	0.43	0.59
Year 3	1.75	1.18	**D**	0.60
Year 4	1.59			
Average	1.72	**F**	**E**	0.60

Question B3 continues on the following page.

a) Explain why the graph indicates that a multiplicative model may be appropriate to describe the time series. **[2]**

b) Find the **three** missing values A, B and C in table 1 and write them down in your answer booklet. **[3]**

c) Find the **three** missing values D, E and F in table 2 and write them down in your answer booklet. **[3]**

d) Describe the meaning of values of the seasonal effects of 1.72 for quarter 1 and 0.6 for quarter 4. **[3]**

e) A line of best fit to estimate the trend line has been found to be, $y = -2.86x + 91.7$, where $x = 1$ represents quarter 1 of year 1, $x = 2$ represents quarter 2 of year 1 etc. and where y represents the corresponding trend value of sales.
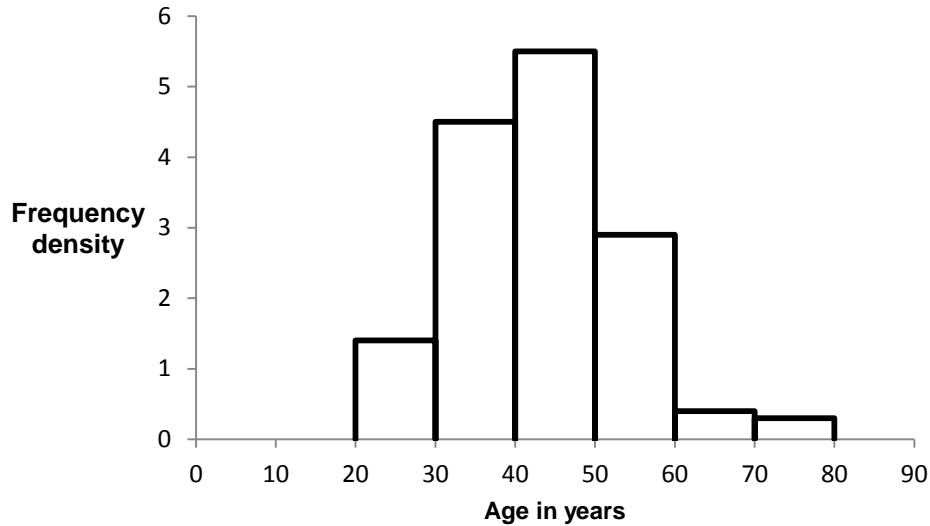
Use the equation and an appropriate seasonal adjustment to estimate the sales in:

i. Quarter 4 of year 4. **[2]**

ii. Quarter 1 of year 5. **[2]**

Section B continues on the following page.

Question B4

Sarah has gathered data on the ages of people who sing in choirs. The histogram of the ages of the people in one of the choirs is shown below.

a)   Comment on the shape of the histogram. **[1]**

b)   Sarah has lost the file names for her data collections. Below is the frequency table for one of the choirs in her study. Construct the cumulative frequency table in your answer booklet for this data. **[2]**

Age in complete years	Frequency
20 to 29	16
30 to 39	19
40 to 49	13
50 to 59	28
60 to 69	33
70 to 79	41

c)   Plot the cumulative frequency polygon for this data on the graph paper provided. **[5]**

d)   From the cumulative frequency polygon calculate the median, the lower quartile and the upper quartile. **[3]**

e)   By calculating Q2 - Q1 and Q3 - Q2 decide if these results support Sarah's belief that the cumulative frequency data and the histogram data are from the same choir. Justify your answer. **[2]**

f)   Using your cumulative frequency polygon estimate the number of choir members whose ages are greater than 55 years. **[2]**

Question B5

a)   Given the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & -2 \end{bmatrix}$ and its inverse $A^{-1} = \frac{1}{\Delta}\begin{bmatrix} -8 & 4 & 0 \\ 16 & a & 4 \\ b & c & -2 \end{bmatrix}$

Find:

    i.   The determinant of A.   **[3]**

    ii.   The values of a, b and c.   **[4]**

    iii.   Hence solve the following set of simultaneous equations: **[4]**

$$x + y + 2z = 1$$
$$4x + 2y + 4z = 6$$
$$2x + y - 2z = -1$$

b)   i.   Sketch the graph of $y = \sin^{-1}x$, clearly labelling the values of the domain and range of the function.   **[3]**

    ii.   Find the value of the $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ in terms of π.   **[1]**

Section B continues on the following page.

Question B6

a)   Integrate   **[4]**

$$\int e^x \sin x \, dx$$

b)   Use the substitution $u = 4 - x$ to find   **[6]**

$$\int_1^4 (x+2)(4-x)^{\frac{1}{2}} \, dx$$

c)   The graph below shows part of the curve $y = \frac{4}{(7-3x)}$. Calculate the volume   **[5]**
       generated when the shaded area is rotated through one complete turn about the
       x-axis. Give your answer in terms of π.