NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Mathematics
Part 2 (Business) Examination Mark Scheme

2012-13

Mark Scheme

<u>Notice to markers.</u>

Significant Figures:

All <u>correct</u> answers should be rewarded regardless of the number of significant figures used, with the exception of question A2. For this question, 1 discretionary mark is available which will <u>only</u> be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the candidate to calculate-or otherwise produce-a piece of information that is to be used later in the question, a marker should consider the possibility of error carried forward. A careless error early in the question may make it impossible for a candidate to answer the remainder of the question correctly. Where a candidate has been careless with initial data, but has gone on to demonstrate knowledge of the correct method, they should be awarded marks for the method only.

When this happens, write ECF next to the ticks.

M=Method
A=Answer

Question A1

$\bar{x} = \frac{\sum x}{n}$

$\bar{x} = \frac{245}{9}$ **1 mark**

$\bar{x} = 27.2$ (1dp) **1 mark**

$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$

$\sigma = \sqrt{\frac{6953}{9} - 27.2^2}$ **1 mark**

$\sigma = 5.6\ (1\ dp)$ **1 mark**

Question A2

a)i.

$hgf(x) = hg(x+3)$

$= h(e^{(x+3)})$ **1 mark**

$= (e^{(x+3)})^2$

$= e^{(2x+6)}$ **1 mark**

ii.

$hgf(0.5) = e^{(2(0.5)+6)}$

$= e^7$ **1 mark**

$= 1096.6$

$= 1100\ (3sf)$ **1 mark (only award mark if answer given to 3sf)**

Question A3

Let X be the number born on a Friday then X is Binomial (10,1/7)

$P(x < 3) = P(0) + P(1) + P(2)$ **1 mark**

$= \binom{10}{0}\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^{10} + \binom{10}{1}\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^9 + \binom{10}{2}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^8$ **2 marks**

=0.214058+0.356764+0.2675728

=0.838395=0.84 **1 mark**

Assumptions: A person equally likely to be born on any of the 7 week days and for any two individuals, the birthday is independent **1 mark**

Question A4

Using $AA^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or implied from eqs **1 mark**

$$\begin{bmatrix} 3 & -2 & 0 \\ -1 & 2 & 1 \\ -2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 2 \\ 1 & a & 3 \\ b & c & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Any acceptable method where working is shown which gives

a=−3, b=−1.and c=5 **3 marks**

Question A5

a)
E[3X+10]=45.6 then

E[X}=(45.6-10/3) = 11.8666 **1 mark**

=11.9 **1 mark**

b) Var[X]

V(3X+10)= 3.2

V(3X) =3.2 **1 mark**

9V(X)=3.2

V(X)=3.2/9

=0.35555…

=0.356 =0.36 **1 mark**

Question A6

$$\frac{x+5}{(x+2)^2(x-4)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x-4)}$$

1 mark for the split

Any method which gives

$$A = \frac{-1}{4}, B = \frac{-1}{2}, C = \frac{1}{4}$$

3 marks one for each correct answer

Question A7

$u = e^{4x}$ $\qquad v = x^3 cosx$

$u' = 4e^{4x}$ $\qquad v' = 3x^2 cosx - x^3 sinx$

2 marks one for each correct u' and v'

$$\frac{dy}{dx} = \frac{(x^3 cosx)4e^{4x} - e^{4x}(3x^2 cosx - x^3 sinx)}{(x^3 cosx)^2}$$

1 mark for putting it together

$$= \frac{e^{4x}((4x^3 cosx) - (3x^2 cosx - x^3 sinx))}{x^6 cos^2 x}$$

$$= \frac{x^2 e^{4x}(4xcosx - 3cosx + xsinx)}{x^6 cos^2 x}$$

$$= \frac{e^{4x}(4xcosx - 3cosx + xsinx)}{x^4 cos^2 x}$$

1 mark

Question A8

$\int_1^2 2xe^{x^2} dx = \left[e^{x^2}\right]_1^2$ **1 mark**

$= e^4 - e$ **1 mark**

$= 51.87$

$= 51.9\ (1dp)$ **1 mark**

Question A9

0.1x250+0.2x310+0.7x430 **2 marks**

=388 **1 mark**

therefore £390 to nearest £10 **1 mark**

Question A10

a) $\mu = 9.2 \pm 1.96\sqrt{\frac{16}{10}}$ **1 mark**

$\mu = [6.72, 11.7]$ **2 marks**

b) Statement B is true. **1 mark**

Question B1

a) Appropriate comment relating to whether scatter diagram indicates a linear model **1 mark**

b) Any formula or formulae appropriate to linear regression

$W - \bar{W} = \frac{S_{WT}}{S_{T^2}}(T - \bar{T})$ **1 mark**

(alternatively $b = \frac{S_{WT}}{S_{T^2}}$ and $a = \bar{W} - b\bar{T}$) do not penalise if use X and Y instead of T and W

$W - \frac{1025}{10} = \frac{\left[\frac{20103}{10} - \frac{190}{10}\frac{1025}{10}\right]}{\frac{3940}{10} - (\frac{190}{10})^2}(T - \frac{190}{10})$ **2 marks**

$W - 102.5 = \frac{62.8(T-19)}{33}$ **1 mark**

$W = 1.9T + 66.3$ **1 mark**

c) T is the independent variable because its position in the regression equation indicates the regression of 'W on T' (or any relevant comment)
1 mark

d) r=0.87 indicates strong positive correlation between W and T **1 mark for 'positive' and 1 mark for 'strong'**

e)

i. When T=19, W=1.9(19)+66.3
=102.4 **2 marks**

ii. When T=40, W= 1.9(40)+66.3
=142.3 **2 marks**

iii. Comments: The results for T=19 are reliable because T=19 is within the range of the data collected **1 mark**

The results for T=40 are not reliable as T=40 lies outside the range of the available data **1 mark**

Question B2

a)

i. $P(X = 3) = 0.4$ **1 mark**

ii. $P(X > 3) = 0.2 + 0.1 = 0.3$ **1 mark**

iii. $E[X] = 2x0.3 + 3x0.4 + 4x0.2 + 5x0.1$ **1 mark**

=3.1 **1 mark**

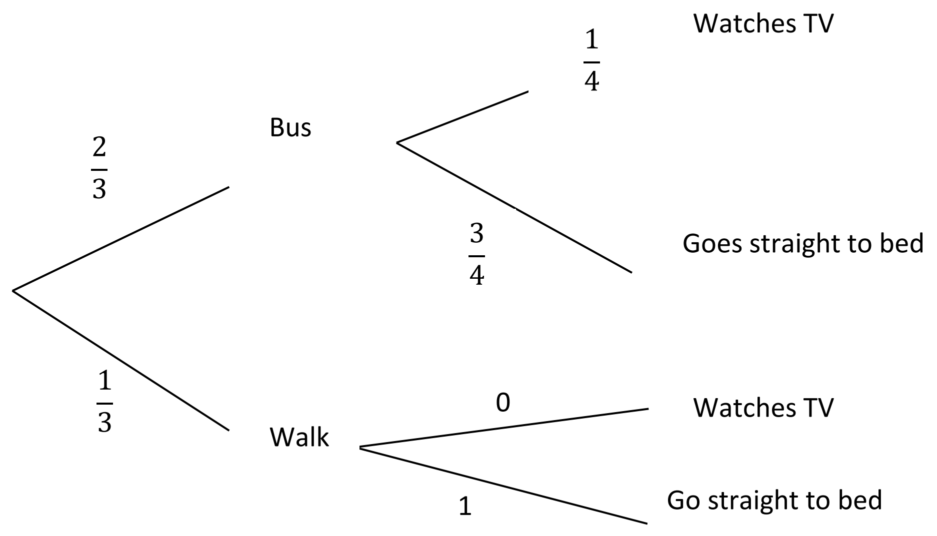
iv. $P(X = 3 \text{ given that } X > 2) = \frac{P(X=3 \cap X>2)}{P(X>2)}$

$= \frac{P(X=3)}{P(X>2)}$ **1 mark**

$= \frac{0.4}{0.4+0.2+0.1}$ **1 mark**

$= \frac{4}{7}$ (=0.571) **1 mark**

b) i.



3 marks

ii. $P(\text{Bus} \cap \text{TV}) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$ **2 marks**

iii. $P(\text{Bus given goes straight to bed}) = \frac{P(\text{Bus} \cap \text{Straight to bed})}{P(\text{Straight to bed})}$ **1 mark**

$= \frac{\frac{2}{3} \times \frac{3}{4}}{\frac{2}{3} \times \frac{3}{4} + \frac{1}{3} \times 1}$

$= \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3}} = \frac{3}{5}$ **2 marks**

Question B3

a) i.

Day	Moving average	y-a
Wed	69/5=13.8	12.4-13.8=-1.4

2 marks (1 mark for each of the 2 missing values)

ii. Table 2

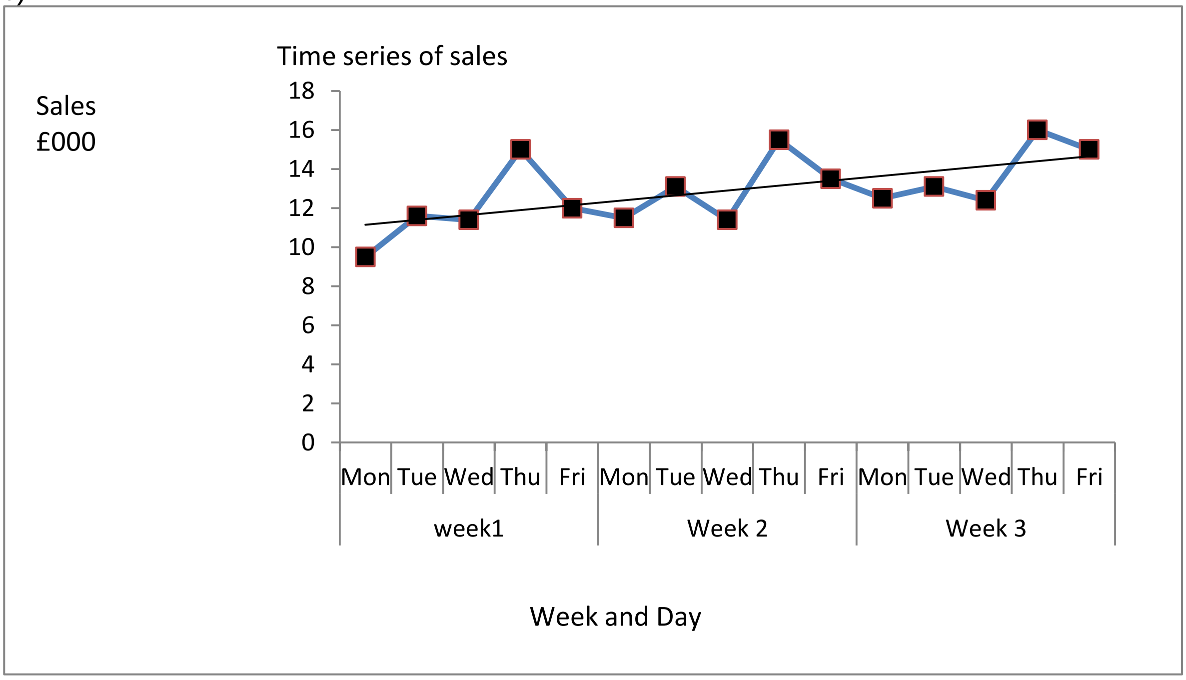
Seasonal Deviations

	Mon	Tue	Wed	Thu	Fri
Week 1	-	-	-0.5	2.7	-0.6
Week 2	-1.1	0.4	-1.6	2.3	0.3
Week 3	-0.9	-0.4	-1.4	-	-
Total	-2.0	0.0	-3.5	5.0	-0.3
Average	-1.0	0.0	-1.2	2.5	-0.1

Or any appropriate rounding

2 marks for all 3 missing values (only 1 mark if less than 3 completed)

b) i.



Scales **2 marks**, Axes labels **1 mark**, plot of sales data **4 mark**

ii. Appropriate line drawn by eye **1 mark**

iii. For the prediction: a value extracted from trend line **1 mark**
(or any other reasonable approach such as finding the gradient of line)

the correct seasonal adjustment added ie. for Wednesday of week 4 (ie. -1.2) **1 mark**

Reasonable final prediction (= their trend value +(- 1.2) **1 mark**

Question B4

a)

Boundaries	C.F.
0	0
3	10
6	29
9	40
12	56
15	68
18	77
21	80

1 mark for the correct class boundaries which should include 0.
1 mark for the CF column

b)

Scales	**1 mark**
Axes labels (both no half marks)	**1 mark**
Plot of CF <u>6 or more correct</u>. FOR FULL MARKS. But only if GRAPH PAPER IS USED.	**2 marks**
Polygon straight lines only	**1 mark**

c)

Median	9	**1 mark**	**Award marks according to students graph**
Lower Quartile	4.6	**1 mark**	
Upper Quartile	12.6	**1 mark**	
IQR	8	**1 mark**	

d)

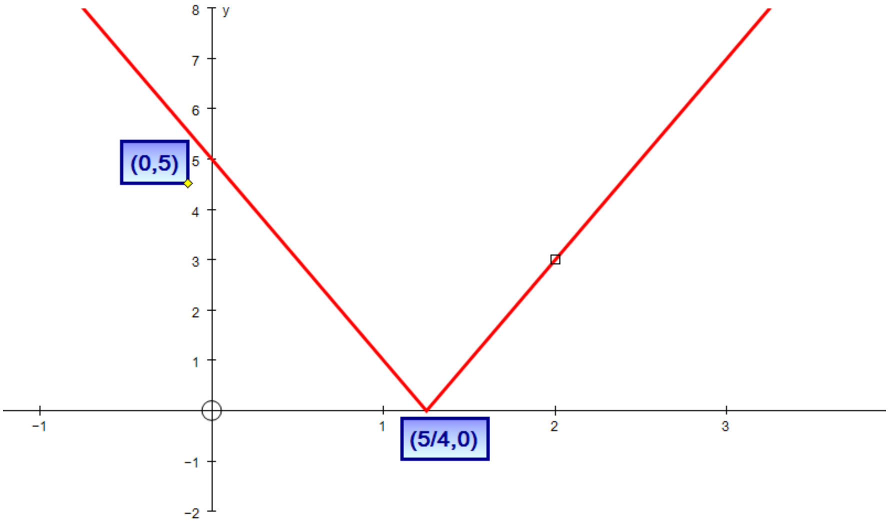
Females	km		**Males**	km	**Any reasonable comments award the marks. Please give 1 mark if students actually look at the raw data as well as the Median, and quartiles. If they come up with any reasonable explanation for the data award a mark.** 4 marks
Min. Distance	0.5		Min. Distance	0.6	
Max Distance	15		Max Distance	19.3	
Median	7.6		Median	9	
Lower Quartile	3.5		Lower Quartile	4.6	
Upper Quartile	10.9		Upper Quartile	12.6	
IQR	7.4		IQR	8	

Question B5

a)

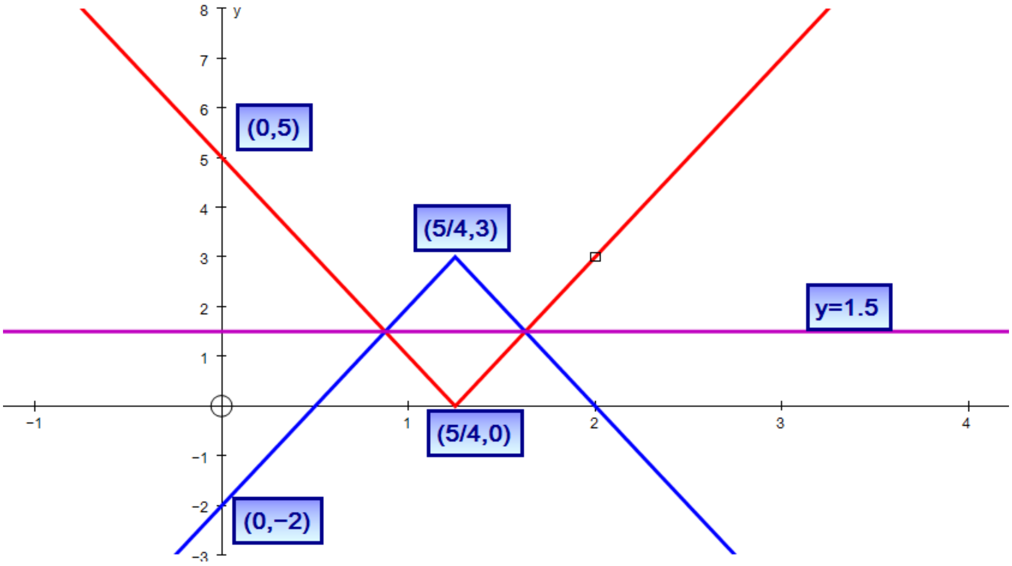
i. 1 – 1	**1 mark**
ii. n-1	**1 mark**
iii.1-n or 1-2	**1 mark**
iv. n-n	**1 mark**

b)i.



1 mark for the shape, 1 for labelling the vertex, 1 mark for the intercept.

ii.



1 mark for the shape, 1 for labelling the vertex, 1 mark for the intercept

iii.

y=4x-5 know y=1.5 **1 mark for knowing y = 1.5** 1.5=4x-5 therefore x=6.5/4=13/8 **1 mark for x** 13/8 -5/4 =3/8 therefore 5/4-3/8=7/8 **2 marks** (7/8,1.5) and (13/8,1.5) **1 mark**	**Alternative:** 4x-5=-4x+8 8x=13 x=13/8 **2 marks** put into y=4x-5 gives y=13/2-5=1.5 **1 mark** -4x+5=4x-2 -8x=-7➔x=7/8 **1 mark** therefore((7/8,1.5) and (13/8,1.5) **1 mark**

Question B6

a)i. A is on the x-axis, therefore y=0

$$x^{\left(\frac{2}{3}\right)} = 6$$

$x = \sqrt{6^3} = 6\sqrt{6}$ **1 mark**

Co-ordinates of A are ($6\sqrt{6}$,0) **1 mark**

ii) $x^{\left(\frac{2}{3}\right)} + y^{\left(\frac{2}{3}\right)} = 6$

$$y^{\left(\frac{2}{3}\right)} = 6 - x^{\left(\frac{2}{3}\right)}$$

$y^2 = \left(6 - x^{\left(\frac{2}{3}\right)}\right)^3$ **1 mark**

$$V = \pi \int_0^{6\sqrt{6}} y^2 dx$$

$V = \pi \int_0^{6\sqrt{6}} \left(6 - x^{\left(\frac{2}{3}\right)}\right)^3 dx$ using Pascal's Triangle

$$V = \pi \int_0^{6\sqrt{6}} (6^3 + 3.(6)^2 \left(-x^{\left(\frac{2}{3}\right)}\right) + \frac{3.2}{2}(6)(-x^{\frac{2}{3}})^2 + \left(-x^{\frac{2}{3}}\right)^3)dx$$

2 marks (1 mark for each pair)

$V = \pi \int_0^{6\sqrt{6}} \left(-x^2 + 18x^{\left(\frac{4}{3}\right)} - 108x^{\left(\frac{2}{3}\right)} + 216\right) dx$ **1 marks**

$$V = \pi \left[\frac{-x^3}{3} + \frac{3}{7}.18x^{\frac{7}{3}} - \frac{3}{5}.108x^{\frac{5}{3}} + 216x\right]_0^{6\sqrt{6}}$$

2 marks(1 mark for each pair)

V=1519.711 = 1520 **1 mark**

b)

i) $y^2 + 2y - 1 = 0$ **1 mark**

$$y = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2}$$

$y = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$ **1 mark**

ii) $2y\frac{dy}{dx} + 2\frac{dy}{dx} - 9x^2 siny - 3x^3 cosy\frac{dy}{dx} = 2$ **1 mark**

(If the student realises that they don't need the x terms then there is no need to see the diff.)

When x=0, and y = -1+√2

$2y\frac{dy}{dx} + 2\frac{dy}{dx} = 2$ **1 mark**

$$y\frac{dy}{dx} + \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}(y + 1) = 1$$

$$\frac{dy}{dx}(-1 + \sqrt{2} + 1) = 1$$

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ **1 mark**

Tangent equation

$$y = \frac{1}{\sqrt{2}}x + c$$

$$-1 + \sqrt{2} = \frac{1}{\sqrt{2}}.0 + c$$

$$-1 + \sqrt{2} = c$$

$y = \frac{1}{\sqrt{2}}x - 1 + \sqrt{2}$ **1 mark**