THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYMB001 Mathematics
Part 2 (Business) Examination Mark Scheme

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Find the median, lower quartile and the upper quartile of the following set of data:

11, 12, 16, 16, 17, 18, 19, 20, 21, 22, 23 **[4]**

Ordering the data 1 mark
Lower quartile 16 1 mark
Upper quartile 21 1 mark
Median 18 1 mark

Question A2

Using the quotient rule, differentiate $y = \frac{x^2 + \sin(x)}{e^{3x} + 5x}$ **[3]**

Letting
$u = x^2 + \sin(x), v = e^{3x} + 5x$

$\frac{du}{dx} = 2x + \cos x,$ 1 mark

$\frac{dv}{dx} = 3e^{3x} + 5$ 1 mark

$y = \frac{(e^{3x} + 5x).(2x + \cos x) - (x^2 + \sin x).(3e^{3x} + 5)}{(e^{3x} + 5x)^2}$ 1 mark

Question A3

A random variable Y has mean 13 and variance 15. Find the mean and variance of $2Y + 5$ **[4]**

Mean = 2X13+5=31 M1 A1
Variance = $2^2 X15 = 60$ M1 A1

Question A4

Using matrices **only** determine the solution to the following simultaneous equations:

$5x + 6y = 9$
$3x - y = 10$ **[5]**

$$\begin{pmatrix} 5 & 6 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$$ 1 mark

$$A^{-1} = \frac{1}{-5-18}\begin{pmatrix} -1 & -6 \\ -3 & 5 \end{pmatrix} = \frac{1}{-23}\begin{pmatrix} -1 & -6 \\ -3 & 5 \end{pmatrix}$$ 2 marks

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-23}\begin{pmatrix} -1 & -6 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 9 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$ 1 mark

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} x = 3, y = -1$$ 1 mark

Marks only to be awarded if matrices are used.

Question A5

Find the values of A, B and C in the identity:

$$\frac{3+4x}{x(x+2)^2} \equiv \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$ **[5]**

$3+4x = A(x+2)^2 + Bx(x+2) + Cx$ 2 marks Cover-up Method acceptable for A and C.

Letting x=0, gives A=3/4 1 mark
Letting x=-2, gives C=5/2 1 mark
Letting x=1, gives B=-3/4 1 mark

Question A6

Rainfall records collected over many years in a certain holiday resort in the UK show that the probability of rain is 0.3. Minjun is going on holiday for seven days and wants to calculate the probability that it will rain on exactly two days of her holiday. Define the distribution Minjun should use to calculate the probability it will rain during her holiday. Calculate the probability that it will rain on exactly two days of her holiday.

[4]

$W \approx B(7,0.3)$ **1 mark for B and 1 mark for (7,0.3)**

$P(W=2) = \binom{7}{2}(0.3)^2(0.7)^5 = 0.3177 = 0.318$ **1 mark for formula sub.**

1 mark for ans. Either 4sf or 3sf is acceptable

Question A7

Use Integration by parts to determine the following indefinite integral: **[5]**

$\int x\cos(3x)dx$.

Defining

$$I = \int x\cos(3x)dx$$

And letting $u = x \Rightarrow \frac{du}{dx} = 1$ 1 mark

Letting

$\frac{dv}{dx} = \cos 3x, v = \frac{\sin 3x}{3}$ 1 mark

So that

$I = \int x\cos(3x)dx = \frac{x\sin 3x}{3} + \frac{\cos 3x}{9} + c$ 3 marks (1 mark for each term here)

Question A8

Determine the coefficient of linear correlation between the variables X and Y presented in the data below:

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

and comment on the meaning of the coefficient where: **[5]**

$$\sum X = 56$$
$$\left(\sum X\right)^2 = 3136$$
$$\sum X^2 = 524$$
$$\sum Y = 40$$
$$\left(\sum Y\right)^2 = 1600$$
$$\sum Y^2 = 256$$
$$\sum XY = 364$$

$$r = \frac{n\sum XY - \sum X \sum Y}{\left(\sqrt{n\sum Y^2 - \left(\sum Y\right)^2}\right)\left(\sqrt{n\sum X^2 - \left(\sum X\right)^2}\right)}$$

$r = \frac{8X364 - 56X40}{\left(\sqrt{8X256 - 1600}\right)\left(\sqrt{8X524 - (56)^2}\right)}$ 2 marks

$$r = \frac{672}{21.1660 X 32.496}$$ 1 mark

$$r = 0.977$$ 1 mark

So there is strong +ve correlation 1 mark

Question A9

A module has three separate parts in the assessment: a final examination, coursework and a presentation. The final examination is worth three times more than each of the other two. A student has a final examination grade of 85, a coursework grade of 70 and a presentation grade of 90. Find the student's overall module grade. **[5]**

Exam: Coursework: Presentation
3 : 1 : 1 2 marks
0.6X85+0.2X70+0.2X90 2 marks
=83% 1 mark

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

The following table shows the frequency distribution of the daily wages to the nearest pound of 73 employees at a particular company:

Wages (£)	Number of Employees
55-60	9
61-70	12
71-80	15
81-90	16
91-100	12
101-110	6
111-120	3

a) Calculate an estimate for the mean daily wage of the employees to the nearest pound. **[4]**

b) Find the standard deviation of the data. **[5]**

c) By copying and extending the table in an appropriate manner, draw the graph of the cumulative frequency polygon. **[4]**

d) Using your graph, estimate the median of the data. **[2]**

Solutions

a)

Wages (£)	Number of Employees	$x_{1/2}$	$x_{1/2}f$	$x_{1/2}^2 f$	True Ends	Cumulative frequency
					54.5	0
55 - 60	9	57.5	517.5	29756.25	60.5	9
61 - 70	12	65.5	786	51483	70.5	21
71- 80	15	75.5	1132.5	85503.75	80.5	36
81 - 90	16	85.5	1368	116964	90.5	52
91 - 100	12	95.5	1146	109443	100.5	64
101 -110	6	105.5	633	66781.5	110.5	70
111 -120	3	115.5	346.5	40020.75	120.5	73
Totals	73		5929.5	499952.3		

Correct midpoint column 1 mark

Correct sum of midpoints X f 2 marks

$\overline{X} = £81.23$ 1 mark or £81.2 if student has given answer to 3sf

(If the mean has been calculated using a calculator you must see $\bar{x} = \frac{\sum fx}{\sum f} =$ and the sums for $\sum fx$ and $\sum f$ substituted for the 2 marks, followed by A1 for the correct $\overline{X}$)

b) Correct sum of $x_{1/2}^2 f$ 2 marks or correct sum from calculator.

$$\sigma = \sqrt{\frac{499952.3}{73} - (81.23)^2}$$ 2 marks

$\sigma = £15.84$ 1 mark for 4sf ans or if student has given answer to 3sf

c) Correct cumulative frequency column 1 mark

Correctly plotted and labelled graph 3 marks suggest: - plot 1 mark,
labels on axes 1 mark,
and first class coordinate must be (?,**0**)
1 mark.

d) Arrow to show Median marker 1 mark
Median £(80 to 82) acceptable 1 mark

Question B2

In the USA, a study of 890 college students classified the students by whether they play tennis and at the same time classified their parents whether they played football and tennis. The results were:

	Students playing of tennis			Total
	Never	Occasional	Regular	
Parents played neither football nor tennis	282	108	80	470
Parents played one	196	88	102	326
Parents played both football and tennis	34	22	38	94
Total	452	218	220	890

a) Estimate the probability that a randomly chosen student:

(i) Never plays tennis. **[1]**
(ii) Has parents who played both football and tennis. **[2]**
(iii) Regularly plays tennis **and** has parents who played both football and tennis. **[2]**
(iv) Never plays tennis **or** neither parents played football or tennis. **[2]**

b)
(i) Find the probability that a student regularly plays tennis. **[1]**

(ii) Find the probability that a student regularly plays tennis **given that** his/her parents play both football and tennis. **[2]**

(iii) Find the probability that parents play at least one of the sports. **[2]**

(iv) Comment on whether the two following events are likely to be independent: **[3]**

Student regularly plays tennis.

Parents play both football and tennis.

Solutions

a)

(i) P(Never plays tennis) = 452/890=0.508 (3sf) . [A1]

(ii) P(Has parents who played both football and tennis)=94/890=0.106 [M1A1]

(iii) P(Regularly plays tennis and has parents who played both football and tennis)=38/890=0.0427(3sf) [M1A1]

(iv) P(Never plays tennis or neither parents played football or tennis)=(452+470-282)/890 =0.719(3sf) or 0.72 [M1A1]

b)

(i) P(a student regularly plays tennis)=220/890=0.247(3sf). [A1]

(ii) P(a student regularly plays tennis given that his/her parents play both football and tennis)=38/94=0.404 (3sf) [M1A1]

(iii) P(parents play at least one of the sports)=(326+94)/890=0.472 [M1A1].

(iv) Comment on whether the two following events are likely to be independent:

Student regularly plays tennis.

Parents play both football and tennis.

If student regularly playing is independent of parents both playing football and tennis then we would expect

P(student plays | both parents play)=P(student regularly plays),

i.e. the probability would be unaffected by parents' behaviour M1

but $0.247 \neq 0.404$, hence seems as if the students behaviour is NOT independent of parents behaviour. [M1 for 0.247 ≠0.404, A1 for conclusion]

Question B3

a)

If $A=\{1,3,5,7,9\}, B=\{2,6,8\}, C=\{1,2,5,7\}, U=\{1,2,3,4,5,6,7,8,9,10\}$

Find

i. $(A\cup C)\cap B$ [2]

ii $(A\cup B)\cap(A\cup C)$ [2]

iii. $\overline{A\cup B}$ [1]

iv. $\overline{A} \cap \overline{B}$ [3]

v. $\overline{A \cap B}$ [2]

vi $\overline{A} \cup \overline{B}$ [1]

b) If a coin is thrown 3 times find the following probabilities

i. $P(0tails)$ [1]

ii. $P(1head)$ [1]

iii. $P(2tails)$ [1]

Solutions

i. $(A \cup C) = \{1,2,3,5,7,9\}$ 1 mark

$(A \cup C) \cap B = \{1,2,3,5,7,9\} \cap \{2,6,8\} = \{2\}$ 1 mark

ii. $(A \cup B) = \{1,2,3,5,6,7,8,9\}$ 1 mark

$(A \cup C) = \{1,2,3,5,7,9\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,5,6,7,8,9\}$ 1 mark

iii. $\overline{A \cup B} = \{4,10\}$ 1 mark

iv. $\overline{A} = \{2,4,6,8,10\}$, $\overline{B} = \{1,3,4,5,7,9,10\}$ 2 mark

so that $\overline{A} \cap \overline{B} = \{4,10\}$ (as expected form part(iii) 1st De Morgan law.
1 mark

v. $A \cap B = \Phi, empty$ 1 mark

So that $\overline{A \cap B} = \overline{\Phi} = U, universe$ 1 mark

vi. $\overline{A} = \{2,4,6,8,10\}$

$\overline{B} = \{1,3,4,5,7,9,10\}$

So that
$\overline{A} \cup \overline{B} = U$ 1 mark (as expected, 2nd De Morgan law) 1 mark

b)
using Pascal's triangle

0H	1H	2H	3H	4H	5H	
1						Total
1	1					
1	2	1				

1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	32

So looking at the 5th row we have

i. $P(0 tails) = 1/32$ [1]

ii. $P(1 head) = 5/32$ [1]

iii. $P(2 tails) = 10/32$ [1]

iv. $P(3 tails) = 10/32$ [1]

Students may use tree diagrams or the Binomial distribution formula, all correct methods are acceptable.

Question B4

The heights of adult males in the UK are normally distributed with mean 1.75m and standard deviation 0.2m.

a) Find the probability that a man chosen at random will have a height:
(i) More than 1.80m. **[4]**
(ii) Less than 1.71m. **[3]**
(iii) Between 1.73m and 1.76m. **[3]**

b) A shop has 150 visitors in a particular week.
(i) How many of these visitors would be expected to have heights between 1.73m and 1.76m? **[1]**
(ii) Calculate a 95% Confidence Interval for the average height of a shopper. **[4]**

Solutions

$H \sim N(1.75, 0.2^2)$

a) (i) $Z = \dfrac{1.8 - 1.75}{0.2} = 0.25$ 1 mark

$P(Z > 0.25) = 1 - \Phi(0.25)$ 1 mark

$P(Z > 0.25) = 1 - 0.5987$ 1 mark

So that

$P(H > 1.8) = 0.4013$ or 0.401 (3sf) 1 mark

(ii) $P(H < 1.71) = P(Z < \frac{1.71 - 1.75}{0.2})$ 1 mark

$P(Z < -0.2)$

$\Phi(0.2) = 0.5793$ 1 mark

so we require 1-0.5793 = 0.4207 or 0.421 (3sf) 1 mark

$P(1.73 < H < 1.76)$

(iii) $Z_1 = \frac{1.73 - 1.75}{0.2} = -0.1$

$Z_2 = \frac{1.76 - 1.75}{0.2} = 0.05$

$\Phi(0.05) = 0.5199$ 1 mark
$\Phi(0.1) = 0.5398$ 1 mark
1-0.5398=0.4602

Required probability 0.5199-0.4602=0.0597 1 mark

b) (i) 150X0.0597=8.955 (9 approx) 1 mark
(ii)

Confidence interval $(1.75 \pm 1.96 * \frac{0.2}{\sqrt{150}})$ 2 marks

Which gives (1.75 ± 0.032) =(1.718,1.782) 1 mark
$(1.72, 1.78)$ (3sf) 1 mark

Question B5

a) If $f(x)=x^2-3$ and $g(x)=x+3$

Find:

(i) $f(g(x))$ **[2]**
(ii) $f(f(x))$ **[2]**
(iii) $g(g(x))$ **[1]**
(iv) $g(f(x))$ **[1]**
(v) $g^{-1}(x)$ **[2]**

b) If $f(x)=x^2+1, x\geq 0$,

i) Write down the range for $f(x)$ **[2]**
ii) Obtain an expression for $f^{-1}(x)$ **[3]**
iii) Write down the domain and range for $f^{-1}(x)$ **[2]**

Solutions

a)

i) $f(g(x))=g^2-3=(x+3)^2-3$ 2 marks

ii) $f(f(x))=f(x^2-3)=(x^2-3)^2-3$ 2 marks

III) $g(g(x))=g(x+3)=(x+3)+3=x+6$ 1 mark

Iv $g(f(x))=f(x^2-3)=(x^2-3)+3=x^2$ 1 mark

v) $g^{-1}(x)=g-3 \Rightarrow g^{-1}(x)=x-3$ 2 marks

b) i) $f(x)=x^2+1$ so the range is $f(x)\geq 1$ 2 marks

ii) $f(x)=x^2+1 \Rightarrow x^2=f-1$ 1 mark

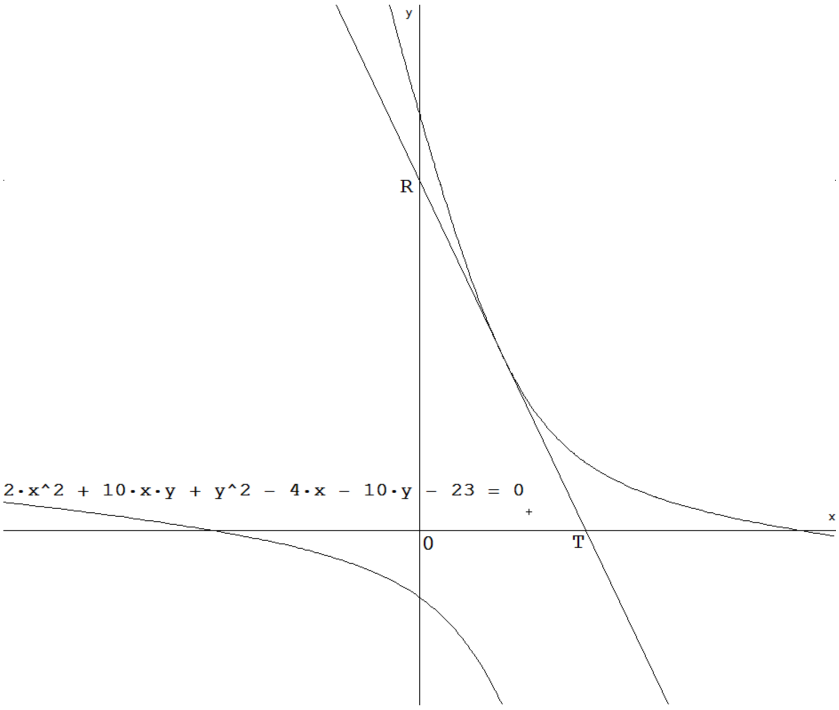
$x=\pm\sqrt{f-1}$ 1 mark

$f^{-1}(x)=\sqrt{x-1}$ taking positive root 1 mark

iii) Domain of $f^{-1}(x)$ is $f^{-1}(x)\geq 1$, range of $f^{-1}(x)$ is $f^{-1}(x)\geq 0$ 2 marks (one for each correct statement)

Question B6

Below is a plot of $2x^2 + 10xy + y^2 - 4x - 10y - 23 = 0$



a) Differentiate $2x^2 + 10xy + y^2 - 4x - 10y - 23 = 0$. **[2]**

b) Find the x coordinate when the curve has a gradient of -5. **[2]**

c) Find the corresponding y coordinates. **[3]**

d) Find the tangent to the curve with the positive y coordinate as shown in the above plot. **[2]**

e) Find the coordinates of T and R where the tangent crosses the x and y axes respectively. **[2]**

f) Find the volume generated when section TR of the tangent is rotated about the x-axis. Give an exact answer. **[4]**

Solutions

a) $4x + 10x\frac{dy}{dx} + 10y + 2y\frac{dy}{dx} - 4 - 10\frac{dy}{dx} = 0$ 2 marks [-1 each error or omission (eeoo)]

b)

$$4x + 10x \times -5 + 10y + 2y \times -5 - 4 - (10 \times -5) = 0$$
$$4x - 50x + 10y - 10y - 4 + 50 = 0$$
$$-46x + 46 = 0 \quad \text{M1 A1}$$
$$-46x = -46$$
$$x = 1$$

$$2x^2 + 10xy + y^2 - 4x - 10y - 23 = 0$$

$$2 + 10y + y^2 - 4 - 10y - 23 = 0$$

c) $y^2 - 25 = 0$ M2 A1

$y = \pm 5$

$y = -5x + c$

d) $5 = -5(1) + c \Rightarrow c = 10$ M1A1

$y = -5x + 10$

e) R is $(0,10)$ and T is $(2,0)$ A2

$$V = \pi\int_0^2 (-5x+10)^2 dx$$

$$= \pi\int_0^2 25x^2 - 100x + 100dx$$

f) $$= \pi\left[\frac{25x^3}{3} - \frac{100x^2}{2} + 100x\right]_0^2$$

$$= \pi\left[\frac{25\times 8}{3} - 200 + 200\right] = \frac{200}{3}\pi \text{ cubic units}$$

M1 for use of formula
M1 integration
A1 ans
A1 cubic units