THE NCUK INTERNATIONAL FOUNDATION YEAR

IFY Mathematics Part 1

EXEMPLAR END OF SEMESTER 1 TEST
MARK SCHEME

Notice to markers.

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A8. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks.

When this happens, write ECF next to the ticks.

M=Method
A=Answer
B = Answer independent of method

If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Gradient = - 3 **(B1)**

Substitutes in $x = 7 \text{ and } y = 9$ **(M1)**

$y = -3x + 30.$ **(A1)** **[3]**

Question A2

$\frac{4}{9} \times \frac{3}{8}$ **(M1)** $\frac{5}{9} \times \frac{4}{8}$ **(M1)**

Adds their two answers together **(M1)**

= $\frac{4}{9}$ or equivalent **(A1)** **[4]**

Question A3

Recognises the quadratic equation which does not factorise, and applies the formula or completes the square. **(M1)**

$x = \frac{7 \pm \sqrt{[(-7)^2 - 4 \times 3 \times 3]}}{2 \times 3}$ or $3[(x - \frac{7}{6})^2 - \frac{49}{36} + 1] = 0$ **(M1)**

$x = \frac{7 + \sqrt{13}}{6} \text{ or } \frac{7 - \sqrt{13}}{6}$ **(A1)** **+ (A1)** or **(A2)** for $x = \frac{7 \pm \sqrt{13}}{6}$ **[4]**

Question A4

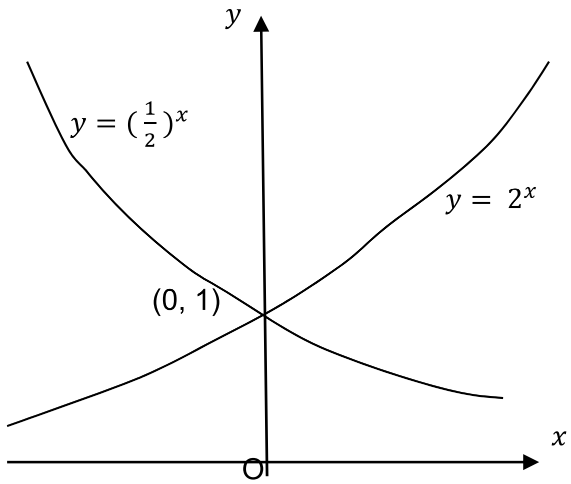
$2^8 + {}^8C_1 \times 2^7 \times (-5x) + {}^8C_2 \times 2^6 \times (-5x)^2 + {}^8C_3 \times 2^5 \times (-5x)^3$

(B1) for any two unsimplified correct; **(B2)** for correct unsimplified expression

$= 256 - 5120x + 44800x^2 - 224000x^3$

(B1) for any two correct; (**B2)** all correct. **[4]**

Question A5



(B1) for each correct curve

(B1) for (0, 1)

[3]

Question A6

$\frac{AC}{\sin 63} = \frac{15}{\sin 40}$ **(M1)** = any answer rounding to 20.8 (cm) **(A1)**

Area = ½ × 15 × their AC × sin 77 **(M1)** = anything rounding to 152 (cm^2) **(A1)** **[4]**

Question A7

$f'(x) = 15x^2 + \cos x - \frac{1}{x}$ **(M1)** for any one term correct; **(A1)** correct answer

Substitutes in $x = 0.2$ **(M1)** = - 3.419933...... **(A1)**

= - 3.42 to 3 significant figures **(A1)*** (*allow follow through). **[5]**

Question A8

$(x-5)^2$ **(B1)** + 2 **(B1)**

Correct curve which must be the right way up and above the $x -$ axis **(B1)**

Minimum at (5, 2) and crossing $y -$ axis at (0, 27) **(B1)** **[4]**

Question A9

$3t - 3\ln t - \frac{3}{t} - \frac{t^3}{9} + c$

(B1) for any one correct term; **(B2)** for two correct terms; **(B3)** for three correct terms; **(B4)** all terms correct and $+c$. **[4]**

Question A10

$\left[x^2 - 3x \right]_0^a$ **(M1)** for either term correct

Substitutes limits into their integrated expression and subtracts the right way round. **(M1)**

Reaches a three-term quadratic equation set equal to 0. **(M1)** $[\, a^2 - 3a - 28 = 0]$

Factorises **(M1)** $[\,(a-7)(a+4) = 0\,]$

$a = 7 \text{ or } -4$ **(A1)** **[5]**

Section B

Question B1

a) i. Correct method with either the x term on the RHS or the symbol reversed. **(M1)**

$x \geq \frac{1}{2}$ **(A1)** **[2]**

ii. $x > 2$ **(B1)** or $x < -2$ **(B1)** **[2]**

b) i. Substitutes $x = 2$ into expression **(M1)**

= 12 **(A1)** **[2]**

ii.

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x+2 & x^3 + 2x^2 - x - 2 \\
 & \underline{x^3 + 2x^2} \\
 & \qquad\quad -x - 2 \\
 & \qquad\quad \underline{-x - 2} \\
 & \qquad\quad \underline{\ldots\ldots\ldots\ldots}
\end{array}
$$

(M1) Correct divisions

(A1) Correct answer

[2]

iii. $f(x) = (x+2)(x^2 - 1)$ **(M1)** $= (x+2)(x+1)(x-1)$ **(A1)** **[2]**

c) Substitutes $y = x - 4$ into second equation **(M1)**

Reaches a three-term quadratic equation set equal to 0 **(M1)** **[5]**

[$2x^2 - 13x + 15 = 0$]

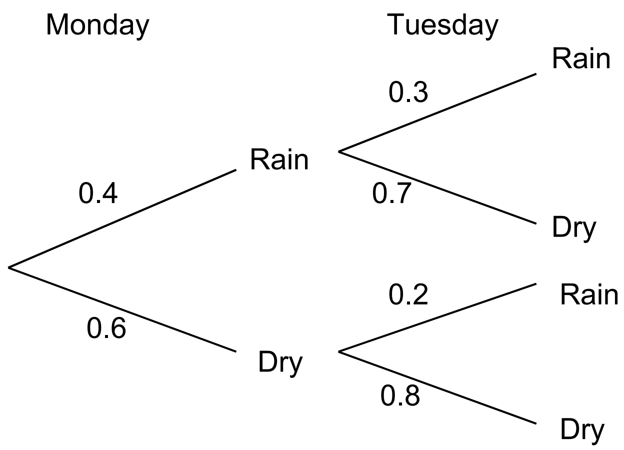
Factorises or uses the quadratic formula **(M1)**

[$(2x - 3)(x - 5) = 0$ or $x = \frac{13 \pm \sqrt{[(-13)^2 - 4 \times 2 \times 15]}}{2 \times 2}$]

Solutions are (5, 1) **(A1)** or ($\frac{3}{2}$, $-\frac{5}{2}$) **(A1)**

Question B2

a) i.



(B1) any pair of branches correct; **(B2)** all correct. **[2]**

ii. Their 0.6 × 0.8 **(M1)** = 0.48 **(A1)** **[2]**

b) i. $d = -4$ **(B1)** $a = 25$ **(B1)** **[2]**

ii. U_{48} = 25 + (48 – 1) × - 4 **(M1)** = - 163 **(A1)** **[2]**

iii. S_{36} = $\frac{36}{2}[2 \times 25 + (36-1) \times -4]$ **(M1)**

Correct use of BODMAS **(M1)**

= - 1620 **(A1)** **[3]**

c) i. $a = 512;\ r = \frac{7}{8}$

$512 \times (\frac{7}{8})^{n-1} \leq 1$ **(M1)** Correct use of formula **[3]**

Correct use of logs and reaches $n - 1 = \text{or} \leq \cdots$ **(M1)**

48th term is the first to fall below 1. **(A1)**

ii. S_∞ = 4096 **(B1)** **[1]**

Question B3

a) i. $A = 120$ **(B1)**

$480 = 120\, e^{2k}$

Rearranges, uses logs and reaches an expression in $2k$ **(M1)**

$k = \frac{1}{2}\ln 4$ or $k = \ln 4^{\frac{1}{2}}$ **(M1)** This, or some other intermediate stage, must be seen

$k = \ln 2$ **(A1)** Correct answer and no errors seen. **[4]**

ii. Substitutes $t = 5$ into the formula **(M1)**

= 3840 **(A1)** **[2]**

iii. $\frac{dN}{dt} = 120 \times \ln 2 \times e^{4\ln 2}$ **(M1)**

= any answer rounding to 1330. **(A1)** **[2]**

b) Multiplies the 4 by the 5 and adds the indices **(M1)**

i. Rearranges to make n the subject **(M1)** [$n = (\frac{4860}{20})^{\frac{4}{5}}$]

$n = 81.$ **(A1)** **[3]**

ii. **(M1)** for either $\log_8 (x-4)^2$ or $\log_8 64$

Removes logs correctly and reaches a three-term quadratic equation

set equal to 0 **(M1)** [$x^2 - 8x - 48 = 0$ or $(x-4)^2 = 64$]

Factorises **(M1)** [$(x-12)(x+4) = 0$ or $x - 4 = \pm 8$]

$x = 12$ **(A1)** (ignore any reference to – 4) **[4]**

Question B4

a) i. $\left[\ \sin x\ \right]_0^{\pi}$ **(M1)**

Substitutes limits into their integrated expression and subtracts the right way round. **(M1)**

= 0 **(A1)** **[3]**

ii. Equal areas are above and below the $x-$ axis. **(B1)**

('No area' scores B0) **[1]**

b) i. Uses $15^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \times \cos P$ or equivalent (M1)

Rearranges correctly (M1)

P = any answer rounding to 70°. (A1) **[3]**

ii. $12 \sin \text{their angle}\ P$ **(M1)**

= any answer rounding to 11.3 (cm) (A1) **[2]**

c) Realises the search has to be made for $0 \leq 2\theta \leq 4\pi$ **(M1)** This can be scored at any time in the question.

$2\theta = \frac{5\pi}{6}$ **(A1)**

Any second angle (any one of $\frac{11\pi}{6}$ $\frac{17\pi}{6}$ $\frac{23\pi}{6}$) and divides by 2 **(M1)**

$\theta = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}$ **(A1)** **[4]**

d) Any correct method seen **(M1)**

Cos A = $\frac{40}{41}$. **(A1)** (Answer only with no working scores 1 out of 2) **[2]**

Question B5

a) i. Length = $40 - x$ **(M1)** Area = width × length so $A = x(40 - x)$ **(A1)** **[2]**

ii. $A = 40x - x^2$

$\frac{dA}{dx} = 40 - 2x = 0$ **(M1)**

$x = 20$ **(A1)** **[2]**

iii. $\frac{d^2A}{dx^2} = -2$ **(A1)** **[2]**

This is negative, therefore a maximum **(B1)***

***Allow follow through from their answer.**

iv. Substitutes their $x -$ value into A **(M1)** **[2]**

= 400 (m^2) **(A1)**

b) i. $\frac{dy}{dx} = 3x^2 - 18x + 27 = 0$ **(M1)** (Attempts to differentiate and sets equal to 0) **[3]**

Factorises **(M1)** [$(x-3)^2 = 0$]

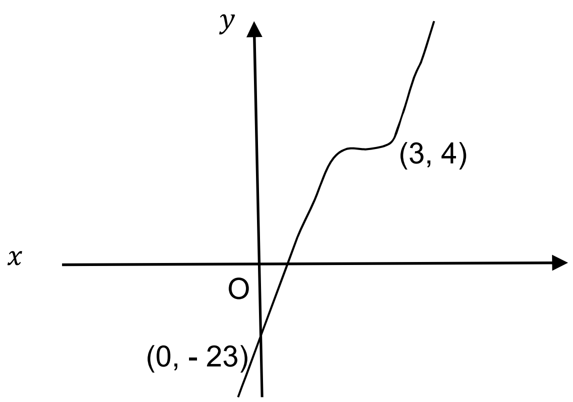
Coordinates are (3, 4) **(A1)**

ii. Either shows gradient is positive when $x < 3$ **(B1)** and positive when $x > 3$ **(B1)**

Or differentiates a second time [$= 2x - 6$] **(B1)**, shows it equals 0 and changes sign. **(B1)** **[2]**

iii. **[2]**

s



(B1) correct shape

(B1) (3, 4) and (0, - 23)

Question B6

i.

Gradient is – 2 **(M1)**

Substitutes in $x = 6 \text{ and } y = 4$ **(M1)**

$y = -2x + 16$ **(A1)** **[3]**

ii.

$\frac{dy}{dx} = 2x - 10$ **(M1)**

Substitutes $x = 6$ into their $\frac{dy}{dx}$ **(M1)** (= 2)

$y = 2x - 8$ **(A1)**

[3]

iii. (4, 0) **(B1)** **[1]**

iv. Area under the line = $\frac{1}{2} \times (12 + 4) \times 4$ **(M1)**

= 32 **(A1)**

(If integration is used, the M mark is scored when the limits are substituted in and subtracted the right way round.)

Area under the curve =

$$\int_2^6 (x^2 - 10x + 28)\ dx$$

(M1) for attempting to integrate

= $\left[\frac{x^3}{3} - 5x^2 + 28x \right]_2^6$ **(A1)** correct answer

Substitutes limits into their integrated expression and subtracts the right way round. **(M1)**

= $\frac{64}{3}$ or equivalent or answers rounding to 21.3 **(A1)**

Subtracts their area under the curve from their area under the line **(M1)**

= $\frac{32}{3}$ or equivalent or answers rounding to 10.7 **(A1)** **[8]**