THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYHM002 Mathematics
End of Semester 1 Test

MARK SCHEME

Notice to Markers

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate-or otherwise produce-a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.)

When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Gradient = $\frac{-1-5}{6--2}$ or equivalent $\left(= -\frac{3}{4}\right)$ **[M1]**

$y-5 =$ their gradient$(x+2)$ or $y+1 =$ their gradient $(x-6)$ **[M1]**

Multiplies through and rearranges **[M1]**

$3x+4y-14=0$ (Must be in this form) **[A1]**

Question A2

$\frac{3}{8} \times \frac{4}{7} \times \frac{3}{6}$ Any one fraction seen **[M1]**

Multiplies their 3 fractions **[M1]**

$= \frac{3}{28}$ or equivalent or answers rounding to 0.107 **[A1]**

Question A3

$k^2 - 4 \times 9 \times 4 < 0$ **[M1]**

$k > -12$ **(A1)** $k < 12$ **(A1)** or $-12 < k < 12$ **(A1)** for each end **[A2]**

Question A4

${}^6C_2$ or ${}^6C_4$ × 3^4 × p^2 = ${}^6C_3$ × 3^3 × p^3 or equivalent
(Allow ${}^xC_y$ for ${}^yC_x$ and the presence of x for this mark) **[M1*]**

Reaches $p = \cdots$ **[M1]**

$p = \frac{9}{4}$ or equivalent **[A1]**

Question A5

$\log_2\left(\frac{x^2}{x-4}\right) = \log_2 16$ Uses power law and subtraction law **[M1*]**

Write RHS as a log **[M1*]**

Cancels logs at the right time and forms a quadratic equation $(x^2 - 16x + 64 = 0)$ **[M1*]**

Factorises or uses the formula $[(x-8)^2 = 0$ or $x = \frac{16 \pm \sqrt{(16^2 - 4 \times 1 \times 64)}}{2 \times 1}]$ **[M1]**
$x = 8$ **[A1]**

Question A6

Uses cosine formula $(18^2 = 15^2 + 13^2 - 2\times15\times13\cos\theta)$ **[M1]**

Reaches a value for $\cos\theta$ $(= \frac{-70}{-390})$ **[M1]**

$\theta \approx 79.6601 \ldots$ (can be implied if this line is not seen, but 79.7 appears below) **[A1]**

$= 79.7$ to 3 significant figures. (Allow follow through here) **[A1ft]**

<u>Special case</u>: If the answer is given in radians (1.39 to 3 significant figures), then award M1 M1 A0 A1ft.

Question A7

$\frac{dy}{dx} = -\frac{1}{x^2} - \frac{1}{x}$ Differentiates (at least one correct term is sufficient for this mark) **[M1*]**

Substitutes $x = 1$ into their $\frac{dy}{dx}$ and obtains a numerical value for the gradient (-2) **[M1*]**

Inverts their gradient and changes sign ($\frac{1}{2}$) **[M1]**

Finds the $y -$ value $(=1)$ and writes
$y - \text{their } y \text{ value} = \text{their inverted gradient } (x - 1)$ **[M1]**

$y - 1 = \frac{1}{2}(x - 1)$ or equivalent **[A1]**

Question A8

$\frac{dy}{dx} = 6x^2 + \cos x$ Differentiates (at least one correct term is sufficient for this mark) **[M1*]**

$\frac{d^2y}{dx^2} = 12x - \sin x$ Differentiates a second time (at least one correct term is sufficient for this mark) **[M1*]**

Substitutes $x = \frac{\pi}{2}$ into their $\frac{d^2y}{dx^2}$ **[M1]**

$= 6\pi - 1$ (must be exact) **[A1]**

Question A9

$100x + \ln x + e^{-x} + \frac{x^{e+1}}{e+1} + c$ (Allow anything rounding to $0.27x^{3.72}$ for the last term.) **[B4]**

(B1) any one part correct; **(B2)** any two correct; **(B3)** any three correct;
(B4) all correct and $+\ c$

Question A10

$x^3 - 2x^2 + x$ Any one term correct **[M1*]**

Correct answer (which must be seen) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 0$ **[A1]**

Any reasonable explanation e.g. 'areas above and below $x -$ axis are equal' but 'there is no area' scores B0. **[B1ft]**

Please note: if the candidate obtains 0 with no working, then award 1 mark out of 4.

If the candidate obtains a different answer but gives a correct interpretation, then the B mark can be given as follow through (e.g. for a positive answer, give the mark for comments like 'most of the area is above the $x -$ axis' but do not allow 'the area is above the $x -$ axis' or 'the whole area is above the $x -$ axis'. Apply similarly for negative answers.)

Section B

Question B1

a) i. $(x-5)^2$ **(B1)** -21 **(B1)** **[B2]**

ii. $x = 5 + \sqrt{21}$ **(B1)** or $x = 5 - \sqrt{21}$ **(B1)** or $x = 5 \pm \sqrt{21}$ **(B2)** **[B2]**

b) i. Substitutes $x = -2$ into expression $(-32 + 16 + 2 - 1)$ **[M1*]**

$= -15$ **[A1]**

ii.

$$\begin{array}{r|l}
 & 4x^2 \quad - 1 \\
\cline{2-2}
x+1 & 4x^3 + 4x^2 - x - 1 \\
 & \underline{4x^3 + 4x^2} \\
 & \qquad\qquad -x - 1 \\
 & \qquad\qquad \underline{-x - 1} \\
 & \qquad\qquad \underline{\ldots\ldots\ldots}
\end{array}$$

Correct first division **[M1]**

Correct second division with no remainder **[M1]**

Correct quotient **[A1]**

iii. $(x+1)(2x-1)(2x+1)$ **[B1]**

c) Realises the need to solve two simultaneous equations. **[M1]**

Any correct method to solve one unknown **[M1]**

Any correct method to solve second unknown **[M1]**

$x = 7$ **[A1]**

$y = -3$ [or quotes $(7, -3)$ - **A1** for each coordinate] **[A1]**

Question B2

a) i. $p = 0.25$ or equivalent **[B1]**

ii. $0.75 \times q = 0.6$ **[M1]**

$q = 0.8$ **[A1]**

$r = 0.2$ (Allow follow through for their q) **[A1ft]**

Accept equivalents

iii. $0.75 \times \text{their } r,$ $\text{their } p \times 0.4$ **[M1]**

Adds **[M1]**

$= 0.25$ or equivalent **[A1]**

b) i. $\frac{AC}{\sin 55} = \frac{18}{\sin 48}$ Uses sine formula correctly **[M1]**

$AC =$ anything rounding to 19.8 (metres) **[A1]**

Please note: if a candidate finds side BC correctly [anything rounding to 23.6 (metres)] then M1A0 can be awarded.

ii. Area = $\frac{1}{2} \times 18 \times \text{their } AC \times \sin 77$ (or alternative valid method using side BC.) **[M1]**

= anything rounding to 174 (m^3) **[A1]**

iii. (18 sin 55) = anything rounding to 14.7 (metres) **[B1]**

c) $\tan\theta = -\frac{4}{3}$ **[M1]**

$\theta =$ anything rounding to 127 (degrees) **[A1]**

or anything rounding to 307 (degrees) **[A1]**

(Any extra solution in the range loses the final A mark. Ignore solutions outside the range)

Question B3

a) i. 7.5 or equivalent **[B1]**

ii. $150.5 = a + 19 \times \text{their } d$ or $158 = a + 20 \times \text{their } d$ **[M1]**

$a = 8$ **[A1]**

iii. $S_{140} = \frac{140}{2}[2 \times \text{their } a + (140 - 1) \times \text{their } d]$ **[M1]**

Correct use of BODMAS **[M1]**

$= 74095$ **[A1]**

b) i. $\frac{ar^6}{ar^3} = \frac{3.2}{400}$ or other way up **[M1]**

Reaches $r^3 = \cdots \left(\frac{1}{125}\right)$ or $r^{-3} = \cdots \ (125)$ **[M1]**

$r = \frac{1}{5}$ or equivalent **[A1]**

As $-1 < r < 1$, the series is convergent (or words to this effect). [Allow follow through if their value of r lies between -1 and $+1$ exclusive] **[B1ft]**

ii. Substitutes their r into either $ar^6 = 3.2$ or $ar^3 = 400$ **[M1]**

$a = 50000$ **[A1]**

iii. $S_\infty = \frac{\text{their } a}{1 - \text{their } r}$ **[M1]**

$= 62500$ **[A1]**

Thus series will never reach 70000. (Allow follow through if their r lies between -1 and $+1$ exclusive and if their sum to infinity is less than 70000). **[B1ft]**

Question B4

a) i. 540 (kg) **[B1]**

ii. Substitutes $M = 375$ and reaches $e^{2k} = \cdots \left(\frac{375}{540}\right)$ **[M1*]**

Uses logs correctly and reaches $2k = \cdots \left[\ln\left(\frac{375}{540}\right)\right]$ **[M1*]**

$k = \ln\frac{5}{6}$ Both M marks scored and no errors seen. **[A1]**

iii. $M = 540\, e^{3 \times \text{their } k}$ **[M1]**

$= 312\frac{1}{2}$ (kg) or equivalent (Follow through for their k). **[A1ft]**

iv. Using this formula will mean $M > 0$ (or M will never reach 0) and the ice will eventually all melt (or words to this effect) **[B1]**

b) i. Divides by 1.5 and then takes an inverse log $(x^2 = 9)$ **[M1*]**

$x = 3$ [if – 3 is also given, this is A0) **[A1]**

ii. Takes inverse log and then divides by 1.5 $(x = 9^3 \div 1.5)$ **[M1*]**

$x = 486$ **[A1]**

iii. Recognises the 'hidden' quadratic equation **[M1*]**

Factorises or uses formula **[M1]**

$[\,(4^{2x} - 8)(4^{2x} - 2)$ or $4^{2x} = \frac{10 \pm \sqrt{[(-10)^2 - 4 \times 1 \times 16]}}{2 \times 1}\,]$

$4^{2x} = 8$ and $4^{2x} = 2$ **[M1]**

$x = \frac{3}{4}$ or $\frac{1}{4}$ or equivalent **[A1]**

Question B5

a) $1734\pi = 2\pi rh + 2\pi r^2$ **[M1]**

$h = \frac{1734\pi - 2\pi r^2}{2\pi r}$ or equivalent (e.g. $\frac{867}{r} - r$) **[A1]**

b) *Please note: this is a 'show that' question so all working must be seen*.

Uses $V = \pi r^2 h$ **[M1*]**

Replaces h by their answer to part a). **[M1*]**

$V = 867\pi r - \pi r^3$ Both M marks scored and no errors seen. **[A1]**

c) Differentiates ($\frac{dV}{dr} = 867\pi - 3\pi r^2$) (At least one term correct) **[M1*]**

Correct expression **[A1]**

Sets their $\frac{dV}{dr}$ equal to 0. (This can be implied) **[M1]**

Reaches $r^2 = \cdots$ (289) **[M1]**

$r = 17$ (cm) **[A1]**

d) $\frac{d^2V}{dr^2} = -6\pi r$ **[M1*]**

(Differentiates a second time: presence of r term is sufficient for this mark)

Correct expression **[A1]**

This is negative, so there is a maximum (Reason and conclusion) [Allow follow through if their $\frac{d^2V}{dr^2}$ is negative for their value of r.] **[A1ft]**

or Takes a numerical value below 17 and shows $\frac{dV}{dr} > 0$ **(M1*)**

Takes a numerical value above 17 and shows $\frac{dV}{dr} < 0$ **(M1*)**

Maximum at $r = 17$ (Conclusion) **(A1ft)**

[Allow follow through if their r and their $\frac{dV}{dr}$ give the same conclusion.]

e) Substitutes their r into V. **[M1]**

$= 9826\pi$ or any answer rounding to 30900 (cm^3) **[A1]**

Question B6

a) $\frac{dy}{dx} = -\frac{1}{4}x^3$ (Differentiates: presence of x^3 is sufficient for this mark) **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$ and obtains a numerical value **[M1]**

Writes $y - 15 = \text{their gradient } (x - 2)$ **[M1]**

$y = -2x + 19$ **[A1]**

b) Sets their $\frac{dy}{dx} = -\frac{1}{32}$ **[M1]**

$x = \frac{1}{2}$ or equivalent **[A1]**

c) $X = \frac{19}{2}$ or equivalent **[B1]**

$Y = 19$ **[B1]**

d) Works out area under line l: $\frac{1}{2} \times (\text{their } X - 2) \times 15$ **[M1*]**

[The M1* can be obtained if the candidate integrates their equation of the line and reaches as far as substituting the limits (2 and their X) into their integrated expression and subtracting the right way round]

Obtains numerical answer $(= 56\frac{1}{4})$ **[M1]**

Integrates $16 - \frac{x^4}{16}$ giving $16x - \frac{x^5}{80}$ (At least one term correct) **[M1*]**

Correct expression **[A1]**

Substitutes *correct* limits (2 and 4) into their integrated expression and subtracts the right way round $(51\frac{1}{5} - 31\frac{3}{5} = 19\frac{3}{5})$ **[M1]**

Subtracts their area under the curve from their area under the line. **[M1]**

$= 36\frac{13}{20}$ or equivalent **[A1]**