THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYHM002 - Mathematics
End of Semester 1 Test
2016-17

Test Session	**Time Allowed**
Semester One	2 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 40 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 60 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the test.
- Show **ALL** workings in your answer booklet.
- Test materials must not be removed from the room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Point A lies at $(-2, 5)$ and point B lies at $(6, -1)$.

Find the equation of the line AB.

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[4]**

Question A2

A box holds 4 red beads, 3 green beads and 1 yellow bead. Three beads are drawn at random from the box, one after another, with no replacement.

Find the probability that the first bead is green and both the second and third beads are red. **[3]**

Question A3

The quadratic equation $9x^2 + kx + 4 = 0$ has no real roots.

Find the range of possible values of k. **[3]**

Question A4

In the expansion of $(3 + px)^6$ where $p \neq 0$, the coefficient of the x^2 term is the same as the coefficient of the x^3 term.

Find the value of p. **[3]**

Question A5

Solve the equation $2\log_2 x - \log_2(x - 4) = 4$.

Each stage of your working must be clearly shown. **[5]**

Question A6

NOT TO SCALE

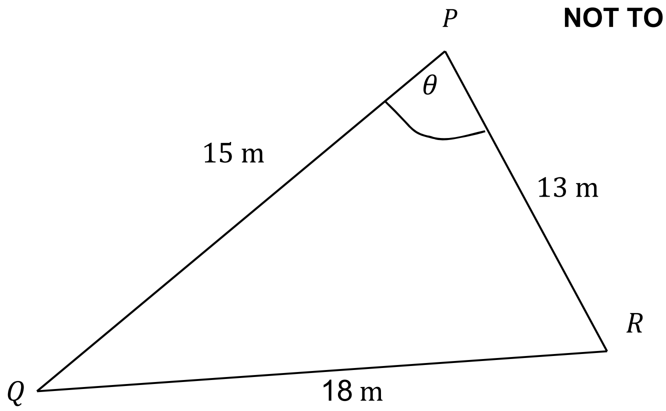


Figure 1

Figure 1 shows triangle PQR with $PQ = 15$ metres, $PR = 13$ metres, $QR = 18$ metres and angle $QPR = \theta$.

Find the size of θ, giving your answer in **degrees** and to **3** significant figures. **[4]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A7

The curve C has equation $y = \frac{1}{x} - \ln x$.

Find the equation of the normal to curve C when $x = 1$. **[5]**

Question A8

A curve has equation $y = 2x^3 + \sin x$. Find the exact value of $\frac{d^2y}{dx^2}$ when $x = \frac{\pi}{2}$. **[4]**

Question A9

Find

$$\int \left(100 + \frac{1}{x} - e^{-x} + x^e\right) dx$$

[4]

Question A10

Showing <u>each</u> stage of your working, find the value of

$$\int_0^1 (3x^2 - 4x + 1)\, dx$$

Explain what your answer means. **[5]**

Section B
Answer <u>4</u> questions ONLY. This section carries 60 marks.

Question B1

a) The function $f(x)$ is defined as $f(x) = x^2 - 10x + 4$.

i. Express $f(x)$ in the form $(x+a)^2 + b$ where a and b are integers. **[2]**

ii. Use your answer in part i to solve the equation $f(x) = 0$ giving your solutions in exact form. **[2]**

b) The function $g(x)$ is defined as $g(x) = 4x^3 + 4x^2 - x - 1$.

i. Use the Remainder Theorem to find the remainder when $g(x)$ is divided by $(x+2)$. *Show your working.* **[2]**

ii. Divide $g(x)$ by $(x+1)$. **[3]**

iii. Factorise $g(x)$ completely. **[1]**

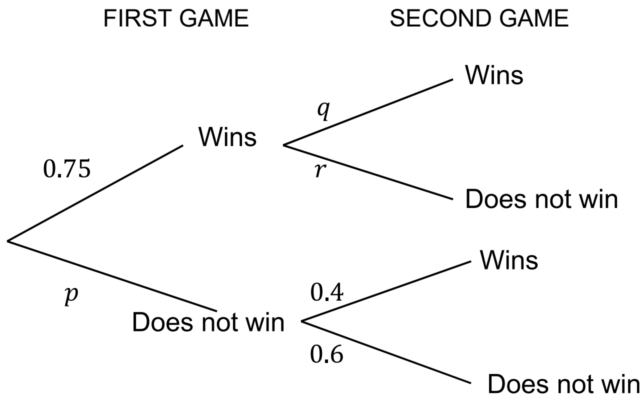
c) Line l_1 has equation $3x - 2y = 27$.

Line l_2 has equation $4x + 5y = 13$.

Find the coordinates of the point where lines l_1 and l_2 intersect. **[5]**

Question B2

a) A girl plays two games of squash. The probability that she wins each game is shown in the tree diagram below.



i. Write down the value of p. **[1]**

ii. The probability that she wins both games is 0.6. **[3]**

Find the values of q and r.

iii. Find the probability that she wins exactly one game. **[3]**

b) Figure 2 shows the acute angled triangle ABC with $AB = 18$ metres, angle $A = 77°$ and angle $C = 48°$.

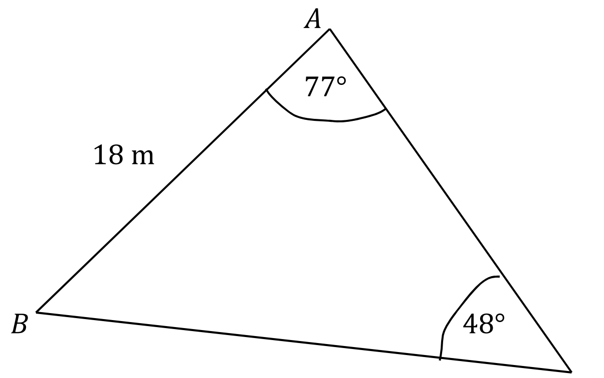


Figure 2

i. Calculate the length of AC.

[2]

ii. Calculate the area of triangle ABC. **[2]**

iii. Find the shortest distance from point A to the line BC. **[1]**

c) Solve the equation $3\tan\theta = -4$ for $0 \leq \theta \leq 360°$. **[3]**

Question B3

a) In an arithmetic series, the 20th term is 150.5 and the 21st term is 158.

i. State the common difference. **[1]**

ii. Find the first term. **[2]**

iii. Find the sum of the first 140 terms. **[3]**

b) The 4^{th} term of a geometric series is 400 and the 7^{th} term is 3.2

i. Find the common ratio, and explain why the series is convergent. **[4]**

ii. Find the first term. **[2]**

iii. Explain why the sum of this series will never reach 70000. **[3]**

Section B continues on the next page

Question B4

a) A large block of ice is melting and its mass, M kg, after t hours from when the ice started to melt is given by the formula

$$M = 540\, e^{kt}$$

where k is a constant.

i. State the mass of the ice when melting started. **[1]**

After 2 hours, the mass is 375 kg.

ii. Find the exact value of k, giving your answer in the form $\ln(\frac{m}{n})$ where m and n are integers. *Show* *__all__* *of your working*. **[3]**

iii. How much ice is left after 3 hours? **[2]**

iv. Explain why this formula is not suitable in the long run. **[1]**

b) ***In part b), each stage of your working __must__ be shown. A correct answer will receive __no__ marks if the working is not present.***

Solve the equations

i. $1.5 \log_x 9 = 3$ **[2]**

ii. $\log_9(1.5x) = 3$ **[2]**

iii. $4^{4x} - 10(4^{2x}) + 16 = 0$ **[4]**

Question B5

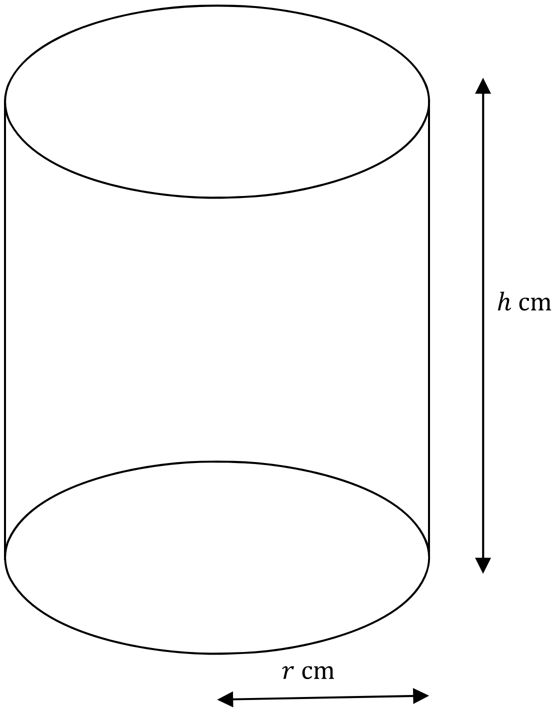


Figure 3

Figure 3 shows a solid cylinder with radius r cm and height h cm. Its **total** surface area is 1734π cm³.

a) Write h in terms of r. **[2]**

b) Show that the volume of the cylinder, V, is given by **[3]**

$$V = 867\pi r - \pi r^3$$

c) Use $\frac{dV}{dr}$ to find the value of r which gives the maximum volume. **[5]**

Show <u>all</u> of your working.

d) Confirm that your volume is a maximum. **[3]**

e) Find the maximum volume of the cylinder. **[2]**

Question B6

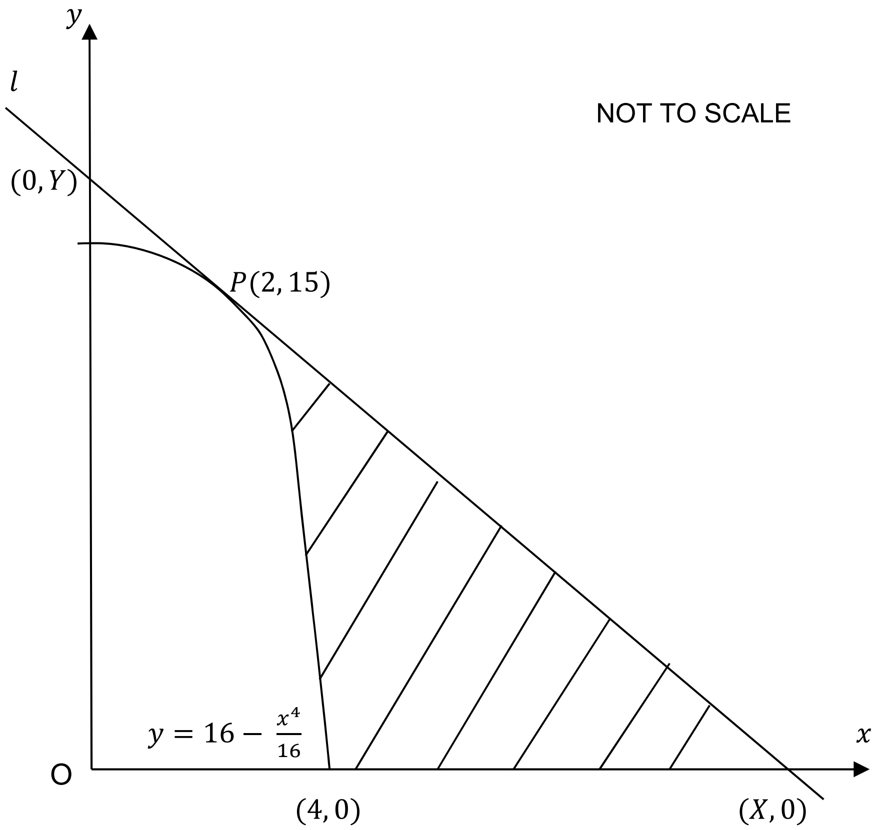


Figure 4

Figure 4 shows the curve $y = 16 - \frac{x^4}{16}$ and the line l which is the tangent to the curve at the point $P(2, 15)$. The line l meets the x −axis at $(X, 0)$ and the $y -$ axis at $(0, Y)$. The curve $y = 16 - \frac{x^4}{16}$ meets the $x -$ axis at $(4,0)$.

a) Find the equation of the line l, giving your answer in the form $y = mx + c$. **[4]**

b) Find the value of x where the gradient of the curve $y = 16 - \frac{x^4}{16}$ is equal to $-\frac{1}{32}$. **[2]**

c) Find the values of X and Y. **[2]**

d) Find the area, which is shaded on the diagram, that is bounded by the line l, the curve $y = 16 - \frac{x^4}{16}$ and the $x -$ axis. *All working must be shown*. **[7]**

This is the end of the test.