NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

IFYHM001 Mathematics
Part 1 Examination

MARK SCHEME
Version 1 2011–12

~~**Level of accuracy:** If a question specifies how many decimal places or significant figures are required, there is a mark for this. Otherwise accept any reasonable level of accuracy and alternative form.~~

~~**Error carried forward:** Where numerical errors have been made, students lose a mark/marks at that stage but may be awarded marks for using correct methods subsequently if the student demonstrates basic understanding.~~

Section A

A1 The gradient of the given line is $\frac{4}{3}$.

The equation of the line has the form $y=\frac{4}{3}x+c$. **1**

Since it passes through $(6,0)$, $0=\frac{24}{3}+c$, $c=-8$ **1**

So the equation is $y=\frac{4}{3}x-8$ or $4x-3y=24$. **1**

A2 We have to solve the simultaneous equations $5x-4y+1=0$ and $y=4$. **1**

	$5x-16+1=0$	**1**
	$x=3$, $y=4$. The point of intersection is $(3,4)$.	**1**
A3	$\begin{pmatrix}7 & 0\\ -2 & 5\end{pmatrix}\begin{pmatrix}3 & -4\\ 1 & 5\end{pmatrix}=\begin{pmatrix}7\times3+0\times1 & 7\times-4+0\times5\\ -2\times3+5\times1 & -2\times-4+5\times5\end{pmatrix}=\begin{pmatrix}21 & -28\\ -1 & 33\end{pmatrix}$	**4**
A4	$6x^2+x-15=(2x-3)(3x+5)=0$	**2**
	Either $2x-3=0$ so $x=\frac{3}{2}$	**1**
	or $3x+5=0$ so $x=-\frac{5}{3}$	**1**
A5	$4y^2+7y>2y+6$	
	$4y^2+5y-6>0$	**1**
	$(4y-3)(y+2)>0$	**1**
	$y<-2$ or $y>\frac{3}{4}$	**2**
A6	The term in x^4 is ${}^6C_4(2x)^4(-3)^2$ so the coefficient of x^4 is	**1**
	$\frac{6\times5\times4\times3}{1\times2\times3\times4}\times2^4\times3^2$	**1**
	$=15\times16\times9=2160$	**1**
A7	$x^{3/2}x^{-2/3}=x^{3/2-2/3}=x^{9/6-4/6}=x^{5/6}=243=3^5$	**3**
	So $x=3^6=729$.	**1**
A8	$e^{4x}+e^{2x}-12=0$	
	$(e^{2x})^2+e^{2x}-12=0$	
	$(e^{2x}+4)(e^{2x}-3)=0$	**1**
	$e^{2x}+4=0$ or $e^{2x}-3=0$	
	$e^{2x}=-4$ or $e^{2x}=3$	**1**
	But $e^{2x}>0$, so $e^{2x}=3$	**1**
	So $2x=\ln 3$ $x=\frac{1}{2}\ln 3$	**1**
	$x=0.5493$ correct to 4 decimal places	**1**
A9	$y=x^{-2}+4e^x-3\cos x$	
	$\frac{dy}{dx}=-2x^{-3}+4e^x+3\sin x$	**3**

When $x = 2$ this equals $-\frac{2}{8} + 4\mathrm{e}^2 + 3\sin 2 \approx 32$ **2**

A10 $\int_{-1}^{3} \left(3\mathrm{e}^x + 5x\right)\mathrm{d}x = \left[3\mathrm{e}^x + \frac{5}{2}x^2\right]_{-1}^{3}$ **2**

$= 3\mathrm{e}^3 + \frac{5 \times 9}{2} - 3\mathrm{e}^{-1} - \frac{5}{2} = 3\mathrm{e}^3 - 3\mathrm{e}^{-1} + 20$ **2**

≈ 79.2 **1**

B1 (i) The curved surface has area equal to the height times the circumference.

This is $h \times 2\pi r$. **2**

Each end has area πr^2 and there are two ends. **1**

The total area is $A = 2\pi r^2 + 2\pi rh.$

(ii) The volume is the area of the base times the height. **1**

This is $\pi r^2 h,$ and this must equal 200. Therefore $h = \frac{200}{\pi r^2}.$ **2**

Therefore $A = 2\pi r^2 + \frac{400}{r}.$ **1**

(iii) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - \frac{400}{r^2} = 0$ **2**

So $4\pi r^3 = 400$ and $r = \sqrt[3]{\frac{100}{\pi}} \approx 3.169$. So the radius is 3.169 cm. **2**

To show this gives a minimum, $\frac{d^2A}{dr^2} = 4\pi + \frac{800}{r^3} > 0$. **2**

(iv) $h = \frac{200}{\pi r^2} \approx 6.34$. So the height is 6.34 cm. **2**

B2 (i) $u_1 = 5, u_2 = 3.4, u_3 = 4.529, u_4 = 3.649$. This is convergent. **2**

$v_1 = 2, v_2 = 0, v_3 = -2, v_4 = 2$. This is periodic. **2**

$w_1 = 1, w_2 = 2, w_3 = 3, w_4 = 6, w_5 = 27$. This is divergent. **2**

(ii) If the limit is u then $u = \frac{12}{u} + 1$. So $u^2 - u - 12 = 0 = (u-4)(u+3)$.

So $u = 4$ or $u = -3$. But $u > 0$, so $u = 4$. **2**

b(i) If the first term of the arithmetic series is a and the common difference is d

then $\frac{a+3d}{a} = \frac{a+12d}{a+3d}$ **2**

So $a^2 + 6ad + 9d^2 = a^2 + 12ad$ **1**

$6a + 9d = 12a$ $\quad 3d = 2a$ **1**

Also $a + 5d = 78$ $\quad 3d = 2(78 - 5d)$ $\quad 13d = 156$ **1**

So $d = 12$ $\quad a = 18$. **2**

B3a(i)	The first logarithm is defined when $x > \frac{1}{2}$ and the second when $x > \frac{4}{3}$.	**2**
(ii)	$10x - 5 = (3x-4)^2$	**1**
	$= 9x^2 - 24x + 16$	**1**
	$9x^2 - 34x + 21 = 0$	**1**
	$(9x-7)(x-3) = 0$	**1**
	$x = \frac{7}{9}, 3$	**1**
	The first value must be rejected since the second logarithm is not defined, so the only solution is $x = 3$.	**1**
b(i)	From the given data, $75 = Ae^{7k}, 170 = Ae^{12k}$	**2**
	Dividing: $e^{5k} = \frac{170}{75} = \frac{34}{15}$	**1**
	Taking logarithms: $5k = \ln\left(\frac{34}{15}\right)$, $k = \frac{1}{5}\ln\left(\frac{34}{15}\right) \approx 0.1637 \approx 0.164$	**2**
	Then $A = 75e^{-7k} \approx 23.851 \approx 23.9$	**1**
(ii)	$V(15) = Ae^{15k} \approx 277.8 \approx 278$	**2**

B4

a	$5.42 \text{ radians} = \frac{5.42 \times 180^\circ}{\pi} = 310.54^\circ \approx 311^\circ$	**2**
b	Let the distance from the bottom of the cliff to the nearer shore of the lake be x m. **1**	
	Then $\frac{h}{x} = \tan 56^\circ$, $\frac{h}{x+400} = \tan 34^\circ$	**2**
	$x \tan 56^\circ = h = (x+400)\tan 34^\circ$	**1**
	$x(\tan 56^\circ - \tan 34^\circ) = 400 \tan 34^\circ$	**1**
	$x = \frac{400 \tan 34^\circ}{\tan 56^\circ - \tan 34^\circ}$ $(=155.7)$	**1**
	$h = x \tan 56^\circ = \frac{400 \tan 34^\circ \tan 56^\circ}{\tan 56^\circ - \tan 34^\circ}$	**1**
	$= 495.02 \approx 495\text{m}$	**1**
c	$A = 6$	**1**
	$D = 9$	**1**
	$B = \frac{1}{6}$	**1**
	$C = -\frac{4\pi}{3}$ (or $C = \frac{2\pi}{3}$)	**2**

B5(i) $y = 2x - 7\ln x - \dfrac{3}{x}$

$\dfrac{dy}{dx} = 2 - \dfrac{7}{x} + \dfrac{3}{x^2}$ **2**

$\dfrac{d^2y}{dx^2} = \dfrac{7}{x^2} - \dfrac{6}{x^3}$ **1**

(ii) At the stationary points $\dfrac{dy}{dx} = 0$.

$2x^2 - 7x + 3 = 0$ **1**

$(2x-1)(x-3) = 0$

$x = \dfrac{1}{2}, 3$ **1**

When $x = \dfrac{1}{2},\ y = 1 - 7\ln\left(\dfrac{1}{2}\right) - 6 = -0.14797 \approx -0.148$ **1**

When $x = 3,\ y = 6 - 7\ln 3 - 1 = -2.6902 \approx -2.69$ **1**

(iii) When $x = \dfrac{1}{2},\ \dfrac{d^2y}{dx^2} = 28 - 48 = -20$ **1**

< 0 so this is a local maximum. **1**

When $x = 3,\ \dfrac{d^2y}{dx^2} = \dfrac{7}{9} - \dfrac{6}{27} = \dfrac{5}{9}$ **1**

> 0 so this is a local minimum. **1**

(iv) When $x = 1,\ y = 2 - 0 - 3 = -1,\ \dfrac{dy}{dx} = 2 - 7 + 3 = -2$. **2**

The equation of the tangent is $y = -2x + c,\ -1 = -2 + c,\ c = 1$. **1**

That is $y = -2x + 1$ or $2x + y = 1$. **1**

B6(i) The gradient of the tangent is $\dfrac{dy}{dx} = \cos x = \dfrac{\sqrt{3}}{2}$. **1**

So the gradient of PN is $-\dfrac{2}{\sqrt{3}}$ **1**

The coordinates of P are $\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right)$. **1**

The equation of the line is $y - \dfrac{1}{2} = -\dfrac{2}{\sqrt{3}}\left(x - \dfrac{\pi}{6}\right)$. **2**

(ii) When $y = 0, -\frac{1}{2} = -\frac{2}{\sqrt{3}}\left(x - \frac{\pi}{6}\right)$. $x = \frac{\sqrt{3}}{4} + \frac{\pi}{6}$. **2**

(iii) The area is $\int_{\pi/6}^{\pi/6+\sqrt{3}/4} \sin x\,dx = \left[-\cos x\right]_{\pi/6}^{\pi/6+\sqrt{3}/4} = \cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right) \approx 0.2897$. **4**

(iv) The triangle below the line PN has area $\frac{1}{2} \times \frac{\sqrt{3}}{4} \times \frac{1}{2} = \frac{\sqrt{3}}{16} \approx 0.1083$. **2**

So the area between the line and the curve has area 0.1814 approximately. **1**

The ratio is $\frac{0.1814}{0.1083} \approx 1.68$. **1**