NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

IFYHM001 Mathematics
Part 1 Examination

Examination Session
Semester One

Time Allowed
2 hours 10 minutes
(Including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A
Answer ALL questions. This section carries 40% of the exam marks.

SECTION B
Answer FOUR questions. This section carries 60% of the exam marks.

The marks for each part of the question are indicated in square brackets [ ]

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the Answer Book in the space provided.
- Write your answers in the Answer Book provided. Additional sheets will be provided on request.
- Clearly write the number and parts of questions attempted at the start of each answer.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for full and detailed answers.
- Students will receive a formula book.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Find the equation of the line parallel to the line $4x-3y=5$ which cuts the x-axis when $x=6$. **[3]**

Question A2

Find where the line $5x-4y+1=0$ intersects the horizontal line through the point $(1,4)$. **[3]**

Question A3

Multiply the matrices $\begin{pmatrix} 7 & 0 \\ -2 & 5 \end{pmatrix}$ and $\begin{pmatrix} 3 & -4 \\ 1 & 5 \end{pmatrix}$. **[4]**

Question A4

Solve the quadratic equation $6x^2+x-15=0$ for x. **[4]**

Question A5

Find the values of y for which $4y^2+7y>2y+6$. **[4]**

Question A6

Find the coefficient of x^4 in the expansion of $(2x-3)^6$. **[3]**

Question A7

Solve $\dfrac{\sqrt{x^3}}{\sqrt[3]{x^2}}=243.$ **[4]**

Question A8

Solve the equation $e^{4x} + e^{2x} - 12 = 0$ for x giving your answer to 4 decimal places. **[5]**

Question A9

If $y = x^{-2} + 4e^{x} - 3\cos x$ find $\frac{dy}{dx}$ when $x = 2$ correct to 2 significant figures. **[5]**

Question A10

Find the value of $\int_{-1}^{3} (3e^{x} + 5x)dx$. **[5]**

Section B.
Answer 4 questions. This section carries 60 marks.

Question B1

A manufacturer of canned tomatoes wants to minimise the amount of metal used to make his cans. These are to have a cylindrical shape of height h cm and radius r cm. The metal is of uniform thickness. The volume is to be $200\,\text{cm}^3$.

(a) Show that the area of metal used is $A = 2\pi r^2 + 2\pi rh$. **[3]**

(b) State the volume in terms of r and h and hence find A in terms of r only. **[4]**

(c) Find the value of r for which $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ and show that this gives a minimum value of A. **[6]**

(d) What is the height of the can for this minimum value of A? **[2]**

Question B2

(a) Three sequences are given by the recurrence relations:

$$u_{n+1} = \frac{12}{u_n} + 1, \text{ where } u_1 = 5;$$

$$v_{n+1} = \frac{1}{4}\left(3v_n{}^2 - 2v_n - 8\right), \text{ where } v_1 = 2;$$

$$w_{n+1} = w_n{}^2 - 2w_n + 3, \text{ where } w_1 = 1.$$

(i) Show that one of these sequences is convergent, one is divergent and one is periodic. **[6]**

(ii) Find the limit of the convergent sequence. **[2]**

(b) The first, fourth and thirteenth terms of an arithmetic series are consecutive terms in a (non-constant) geometric series. The sixth term in the arithmetic series is 78.

Find the first term and the common difference of the arithmetic series. **[7]**

Question B3

(a) (i) For which values of x are the logarithms in the equation $\ln(10x-5)=2\ln(3x-4)$ defined? **[2]**

(ii) Solve the equation $\ln(10x-5)=2\ln(3x-4)$ for x. **[5]**

(b) A function $V(t)$ is known to have the form $V=A\mathrm{e}^{kt}$. When $t=7$, $V=75$ and when $t=12$, $V=170$.

(i) Find the values of k and A. **[6]**

(ii) Find the value of V when $t=15$. **[2]**

Question B4

(a) Convert the angle 5.42 radians to degrees. **[2]**

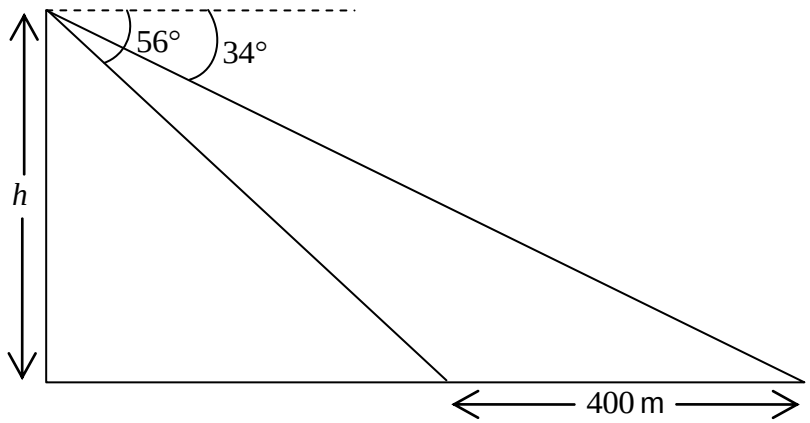

Figure 1

(b) A man standing at the top of a cliff sees a lake in the plain below. The angle of declination of the nearer shore is 56° and that of the further shore is 34°, as shown in Figure 1. The distance across the lake is 400m. Find the height h of the cliff. **[8]**

(c) Find a formula in the form $y = A\sin(B\theta + C) + D$ which gives the graph as shown in Figure 2. **[5]**

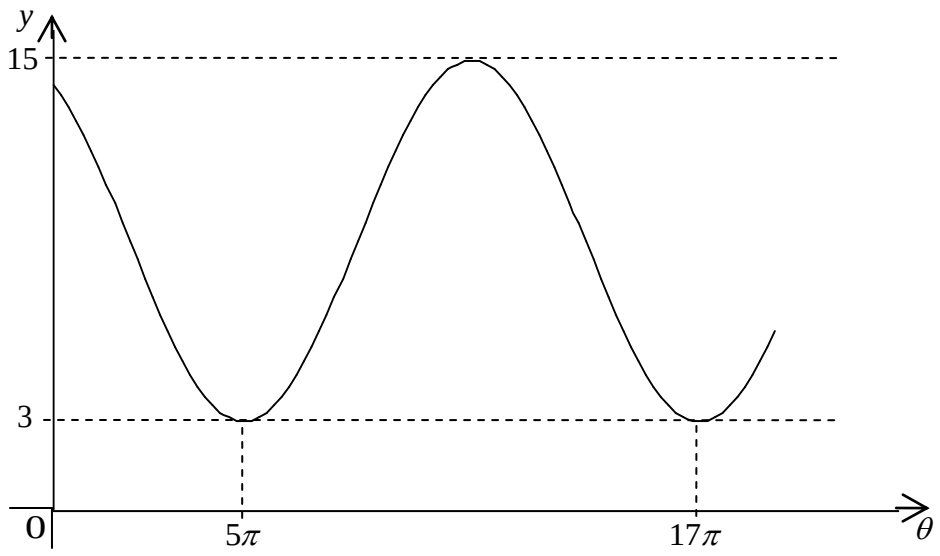

Figure 2

Question B5

A curve has the equation $y = 2x - 7\ln x - \frac{3}{x}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **[3]**

(b) Find the coordinates of the stationary points on the graph $y = 2x - 7\ln x - \frac{3}{x}$. **[4]**

(c) Find $\frac{d^2y}{dx^2}$ at the stationary points and hence determine whether each stationary point is a maximum or a minimum. **[4]**

(d) Find the equation of the tangent at $x = 1$. **[4]**

Section B continues on the next page.

Question B6

Figure 3 shows a sketch of the graph of $y = \sin x$.

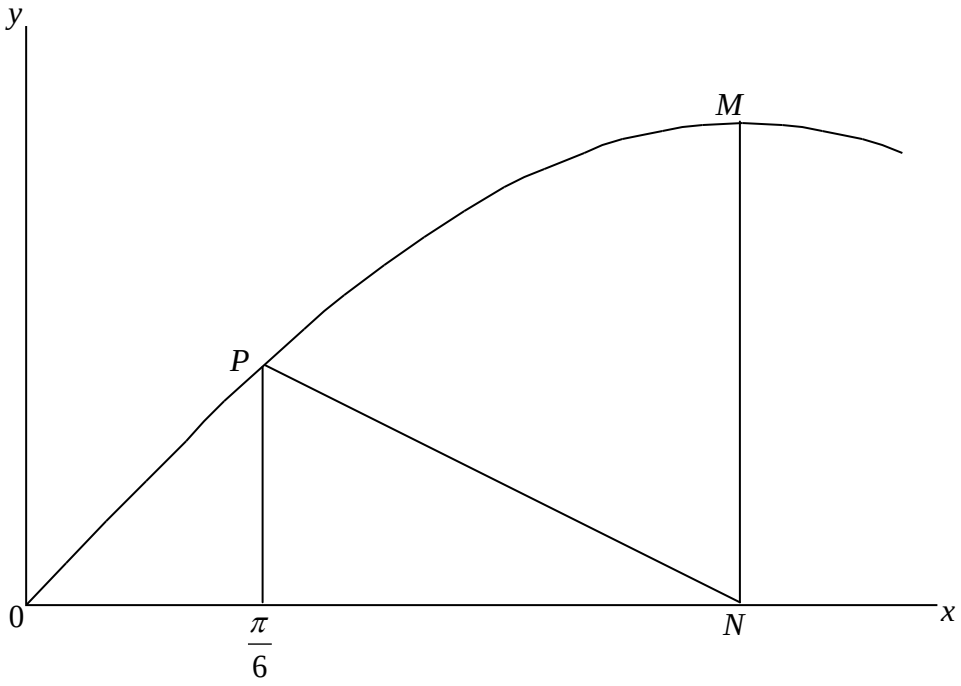

Figure 3

The point P lies on the curve and has x-coordinate equal to $\frac{\pi}{6}$. The normal to the curve at P cuts the x-axis at N.

(a) Find the equation of the line PN. **[5]**

(b) Show that the x-coordinate of N is $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$. **[2]**

(c) Find the area between the curve and the x-axis for x between $\frac{\pi}{6}$ and $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$. **[4]**

(d) Show that the line PN divides the area found in (c) in the approximate ratio $1:1.68$. **[4]**