NCUK
THE NORTHERN CONSORTIUM

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IFYHM001 Mathematics Part 1 Examination

MARK SCHEME
Version 1 2010-2011

Level of accuracy: If a question specifies how many decimal places or significant figures are required, there is a mark for this. Otherwise accept any reasonable level of accuracy and alternative form.

Error carried forward: Where numerical errors have been made, students lose a mark/marks at that stage but may be awarded marks for using correct methods subsequently if the student demonstrates basic understanding.

Section A

A1 The equation of the line has the form $y = -\frac{7}{3}x + c$. **1**

Since it passes through $(-2,0)$, $0 = \frac{14}{3} + c$, $c = -\frac{14}{3}$ **1**

So the equation is $y = -\frac{7}{3}x - \frac{14}{3}$ or $7x + 3y = -14$ **1**

A2 Multiplying the first equation by 3 and the second by -8

$21x + 24y = 12$

$-16x - 24y = 8$ **1**

Adding: $5x = 20$, $x = 4$ **M1A1**

Then $28 + 8y = 4$, $8y = -24$, $y = -3$ **M1**

A3 $\begin{pmatrix} 3 & -2 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} -7 & 21 \\ -4 & 20 \end{pmatrix}$ **4**

A4 $(5x^3 - 13x^2 - 10x + 12) \div (x - 3) = (5x^2 + 2x - 4)$ **3**

A5 $6y^2 + 5y \le 12y + 5$

$6y^2 - 7y - 5 \le 0$ **1**

$(3y - 5)(2y + 1) \le 0$ **1**

$-\frac{1}{2} \le y \le \frac{5}{3}$ **2**

A6 Pascal's triangle looks like:

$$\begin{matrix} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{matrix}$$

1

$(2x - 3y)^3 = (2x)^3 - 3(2x)^2(3y) + 3(2x)(3y)^2 - (3y)^3$ **M2**

$= 8x^3 - 36x^2y + 54xy^2 - 27y^3$ **A1**

A7	$x^{1/6} = 3x^{2/3}$	
	$x^{-1/2} = 3$	**1**
	$x^{1/2} = \dfrac{1}{3}$	**1**
	$x = \dfrac{1}{9}$	**1**
A8	$\log_3(x+3) + \log_3(x-5) = 2$	
	$\log_3(x+3)(x-5) = 2$	**1**
	$(x+3)(x-5) = 3^2 = 9$	**1**
	$x^2 - 2x - 15 = 9, \quad x^2 - 2x - 24 = 0$	**1**
	$(x-6)(x+4) = 0 \quad x = 6, -4$	**1**
	But $x > 5$, so $x = 6$	**1**
A9	$y = 3\cos x + 5x^{-2} - 7e^{3x}$	
	$\dfrac{dy}{dx} = -3\sin x - 10x^{-3} - 21e^{3x}$	**3**
	$= -3\sin(1.2) - \dfrac{10}{(1.2)^3} - 21e^{3.6} = -777.146$	**1**
	$= -780$ correct to 2 significant figures	**1**
A10	$\displaystyle\int_2^3 \left(\frac{2}{x^3} - 3e^x \right) dx$	
	$= \left[-\dfrac{1}{x^2} - 3e^x \right]_2^3$	**2**
	$= -\dfrac{1}{9} - 3e^3 + \dfrac{1}{4} + 3e^2$	**2**
	$= -37.9506 \approx -38.0$	**1**

Section B

B1a(i) The determinant of $\mathbf{A}$ is $3\times(-7)-(-13)\times 2=5$ **1**

$$\begin{pmatrix} 3 & -13 \\ 2 & -7 \end{pmatrix}^{-1} = \frac{1}{5}\begin{pmatrix} -7 & 13 \\ -2 & 3 \end{pmatrix}$$ **M2A1**

(ii) $$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -7 & 13 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 11 \\ 4 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -25 \\ -10 \end{pmatrix} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}$$ **M2**

$x=-5,\ y=-2$ **A1**

b(i) By the factor theorem, $f(3)=f(4)=0.$ **1**

So $27-9p-6+q=0$ $\quad 9p-q=21$ **1**

and $64-16p-8+q=0$ $\quad 16p-q=56$ **1**

Subtracting $7p=35$ $\quad p=5 \quad q=45-21=24$ **M1A2**

(ii) $f(x)=x^3-5x^2-2x+24=(x-3)(x-4)(x+2)$ **2**

B2a(i) If the first term is a and the common difference is d

then $a+8d=31$ **1**

and $\frac{1}{2}9(2a+8d)=171 \quad 2a+8d=38$ **2**

so $a=7$ **1**

and $8d=24 \quad d=3$ **1**

(ii) The sum of the first 50 terms is $\frac{1}{2}50(2a+49d)=25(14+147)=4025$. **M1A1**

b(i) The nth term of the GP is $9(0.97)^{n-1}$ **1**

(ii) The sum of the first n terms is $9\frac{1-(0.97)^n}{1-0.97}=300\left(1-(0.97)^n\right)$ **2**

(iii) If $300\left(1-0.97^n\right)=180$ then $0.97^n=1-\frac{180}{300}=\frac{2}{5}$ **1**

Taking logarithms of both sides $n\ln 0.97=\ln(0.4)$ **M1A1**

So $n=\frac{\ln(0.4)}{\ln 0.97}=30.083$ **1**

But we need an integer, so they have completed 30 trays. **1**

B3a(i) The first logarithm is defined when $x > \frac{7}{4}$ and the second when $x > \frac{5}{2}$. **2**

(ii) $4x - 7 = (2x-5)^2$ **1**

$= 4x^2 - 20x + 25$

$4x^2 - 24x + 32 = 0$

$x^2 - 6x + 8 = 0$ **1**

$(x-2)(x-4) = 0$ **1**

$x = 2, 4$ **1**

The first value must be rejected since the second logarithm is not defined, so the only solution is $x = 4$. **1**

b(i) From the given data, $150 = Ae^{-4k}$, $80 = Ae^{-7k}$ **2**

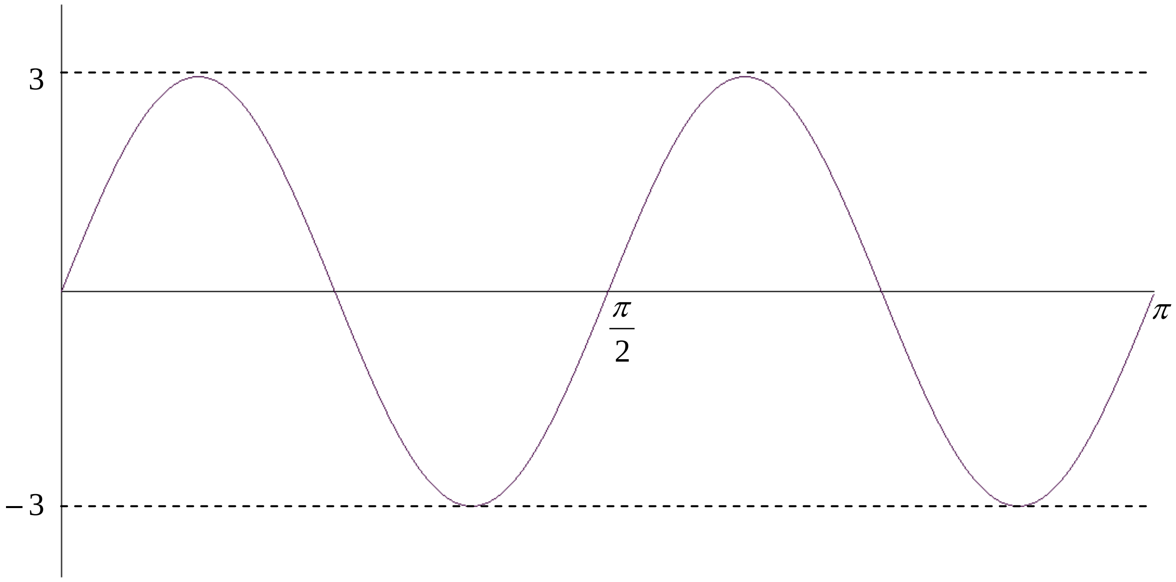
Dividing: $e^{3k} = \frac{150}{80} = \frac{15}{8}$ **1**

Taking logarithms: $3k = \ln\left(\frac{15}{8}\right)$, $k = \frac{1}{3}\ln\left(\frac{15}{8}\right) \approx 0.20954 \approx 0.210$ **M1A1**

Then $A = 150e^{4k} \approx 346.8 \approx 347$ **1**

(ii) $V(12) = Ae^{-12k} \approx 28.06 \approx 28.1$ **2**

B4a	Shape	**1**
	Amplitude	**1**
	Period	**1**
	Phase	**1**



b(i) Using the sine rule

$$QR = PQ\frac{\sin(RPQ)}{\sin(PRQ)} = 15\times\frac{\sin(46°)}{\sin(65°)} \approx 11.906 \approx 11.9\,\text{m}$$ **M2A1**

(ii) $\angle PQR = 180° - \angle RPQ - \angle PRQ = 180° - 46° - 65° = 69°$ **M1A1**

The area is

$$\frac{1}{2}PQ\times QR\sin(PQR) = \frac{1}{2}\times 15\times 11.91\sin(65°) \approx 83.36 \approx 83.4\,\text{m}^2.$$ **M2A1**

(iii) Using the cosine rule $QS^2 = QR^2 + RS^2 - 2QR\times RS\cos(QRS)$

$$= 11.91^2 + 12^2 - 2\times 11.91\times 12\cos(37°) \approx 57.55$$

So $QS = \sqrt{57.55} \approx 7.586 \approx 7.59\,\text{m}$ **M2A1**

B5(i) Since the relationship between price and demand is assumed linear the relation is of the form $d = ap + b$. From the given data:

$240 = 12a + b$

$150 = 15a + b$ **1**

Subtracting: $90 = -3a \quad a = -30 \quad b = 600$ **1**

So $d = 600 - 30p$ **1**

(ii) $C = 100 + 5d = 100 + 5(600 - 30p) = 3100 - 150p$ **1**

$N = dp - C = (600 - 30p)p - (3100 - 150p)$ **1**

$= -30p^2 + 750p - 3100$ **M1A1**

(iii) $\frac{\mathrm{d}N}{\mathrm{d}p} = -60p + 750$ **2**

$= 0$ when $p = \frac{750}{60} = 12.50$ **M1A1**

$\frac{\mathrm{d}^2N}{\mathrm{d}p^2} = -60 < 0$ so this point is a maximum. **M1A1**

(iv) At this point the maximum profit is

$N = -30 \times 12.50^2 + 750 \times 12.50 - 3100 = £1587.50$ **M1A1**

B6(i) The gradient of the line is $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - \frac{7}{x^2} = 4 - 7 = -3.$ **2**

The coordinates of P are $(1, 9)$. **1**

The equation of the line is $y - 9 = -3(x - 1)$ or $3x + y = 12$. **M1A1**

(ii) When $y = 0, 3x = 12, x = 4$. **M1A1**

(iii) The area is

$$\int_1^4 2x^2 + \frac{7}{x}\mathrm{d}x = \left[\frac{2x^3}{3} + 7\ln x\right]_1^4 = \frac{128}{3} + 7\ln 4 - \frac{2}{3} - 0 \approx 51.70 \approx 51.7\,.$$ **4**

(iv) The triangle below the line PN has area $\frac{1}{2} \times 3 \times 9 = 13.5$. **2**

So the area between the line and the curve has area 38.20 approximately. **1**

The ratio is $\frac{38.20}{13.5} \approx 2.8299 \approx 2.83.$ **1**