NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYHM001 Mathematics
Part 1 Examination

Examination Session	**Time Allowed**
Semester One	2 Hours 10 minutes
2010/11	(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for **full and detailed answers**.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Find the equation of the line parallel to the line $7x + 3y = 5$ which cuts the x-axis when $x = -2$. **[3]**

Question A2

Solve the pair of simultaneous equations for x and y: **[4]**

$$7x + 8y = 4$$
$$2x + 3y = -1.$$

Question A3

Multiply the matrices $\begin{pmatrix} 3 & -2 \\ 4 & 0 \end{pmatrix}$ and $\begin{pmatrix} -1 & 5 \\ 2 & -3 \end{pmatrix}$. **[4]**

Question A4

Divide $5x^3 - 13x^2 - 10x + 12$ by $x - 3$. **[3]**

Question A5

Find the values of y for which $6y^2 + 5y \leq 12y + 5$. **[4]**

Question A6

Use Pascal's Triangle to expand $(2x - 3y)^3$. **[4]**

Question A7

Find the value of $x > 0$ for which $x^{1/6} = 3x^{2/3}$. **[3]**

Question A8

Find the value of x when $x > 5$ and $\log_3(x+3) + \log_3(x-5) = 2$. **[5]**

Question A9

If $y = 3\cos x + 5x^{-2} - 7e^{3x}$ find $\frac{dy}{dx}$ when $x = 1.2$ correct to 2 significant figures. **[5]**

Question A10

Find the value of $\int_2^3 \left(\frac{2}{x^3} - 3e^x\right) dx$. **[5]**

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

(a) (i) Find the inverse, $\mathbf{A}^{-1}$, of the matrix **[4]**

$$\mathbf{A} = \begin{pmatrix} 3 & -13 \\ 2 & -7 \end{pmatrix}.$$

(ii) Use $\mathbf{A}^{-1}$ to solve the following simultaneous equations for x and y: **[3]**

$$3x - 13y = 11$$
$$2x - 7y = 4.$$

(b) (i) Find p and q given that $(x - 3)$ and $(x - 4)$ are factors of **[6]**

$$f(x) = x^3 - px^2 - 2x + q.$$

(ii) Hence factorise $f(x)$ completely. **[2]**

Question B2

(a) (i) The ninth term of an arithmetic progression is 31 and the sum of the first nine terms is 171. Find the first term and the common difference. **[5]**

(ii) Find the sum of the first 50 terms of the series in (i) **[2]**

(b) A gardener takes 9 minutes to plant a tray of seeds. With practice they can plant each subsequent tray in 0.97 times the time of the previous one.

(i) Write down the nth term of the geometric progression **[1]**

$$9 + 8.73 + 8.4681 + \cdots.$$

(ii) Show that the time taken to complete their first n trays is **[2]**

$$300(1 - 0.97^n).$$

(iii) How many trays have they completed after 3 hours? **[5]**

Question B3

(a) (i) For which values of x are the logarithms in the equation $\ln(4x-7) = 2\ln(2x-5)$ defined? **[2]**

(ii) Solve the equation $\ln(4x-7) = 2\ln(2x-5)$ for x. **[5]**

(b) A function $V(t)$ is known to have the form $V = A\mathrm{e}^{-kt}$. When $t = 4$, $V = 150$ and when $t = 7$, $V = 80$.

(i) Find the values of k and A. **[6]**

(ii) Find the value of V when $t = 12$. **[2]**

Question B4

a) Sketch the graph of $y = 3\sin 4\theta$ for $0 \leq \theta \leq \pi$. **[4]**

b) In Figure 1, the side PQ has length $15\mathrm{m}$ and side RS has length $12\mathrm{m}$.

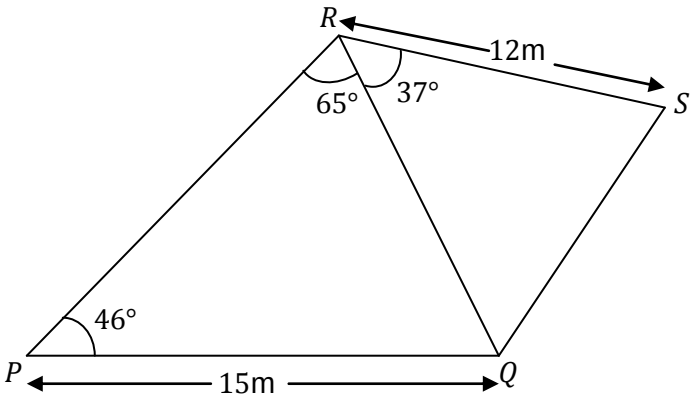

Figure 1

(i) Find the length of QR. **[3]**

(ii) Find the area of triangle PQR. **[5]**

(iii) Find the length of QS. **[3]**

Question B5

A shopkeeper sells DVDs. When the price is £12 he sells 240 per week. When the price is £15 he sells 150 per week. Let p be the price in pounds (£) and d be the number of DVDs sold.

(i) Assuming a linear relationship show that $d = 600 - 30p$. **[3]**

(ii) The shopkeeper buys the DVDs at a cost of £C, where $C = 100 + 5d$. The profit, £N, is given by $N = dp - C$.

Find C and N in terms of p. **[4]**

(iii) Using $\frac{\mathrm{d}N}{\mathrm{d}p}$ and $\frac{\mathrm{d}^2N}{\mathrm{d}p^2}$, find the value of p for which N is a maximum. **[6]**

(iv) Hence find the maximum profit. **[2]**

Question B6

Figure 2 shows a sketch of the graph $y = 2x^2 + \frac{7}{x}$ (not to scale).

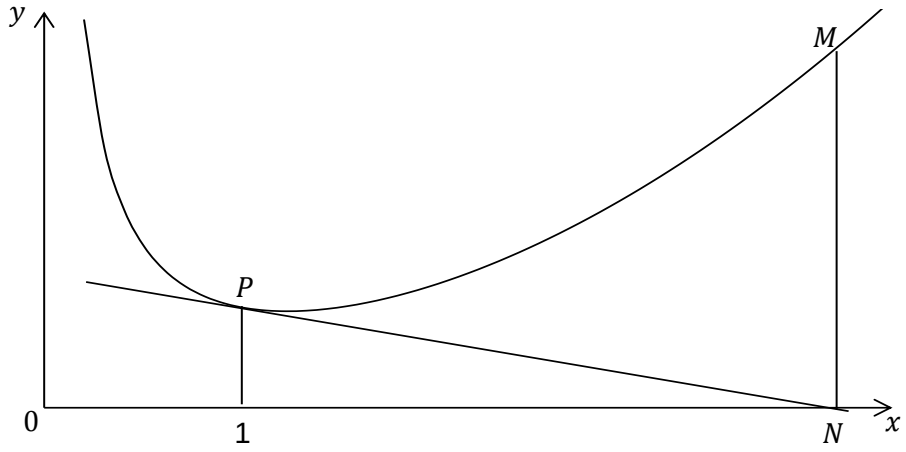

Figure 2

The point P lies on the curve and has x-coordinate equal to 1. The tangent to the curve cuts the x-axis at N.

(i) Find the equation of the line PN. **[5]**

(ii) Show that the x-coordinate of N is 4. **[2]**

(iii) Find the area between the curve and the x-axis for x between 1 and 4. **[4]**

(iv) Show that the line PN divides the area found in (iii) in the ratio $1 : 2.83$ approximately. **[4]**