NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Mathematics
Part 1 Examination

Mark Scheme

Level of accuracy: If a question specifies how many decimal places or significant figures are required, there is a mark for this. Otherwise accept any reasonable level of accuracy and alternative form.

Error carried forward: Where numerical errors have been made, students lose a mark/marks at that stage but may be awarded marks for using correct methods subsequently if the student demonstrates basic understanding.

Section A

A1 The equation of the line has the form $y = \frac{2}{3}x + c$. **1**

Since it passes through $(4,5)$, $5 = \frac{8}{3} + c$, $c = 5 - \frac{8}{3} = \frac{7}{3}$ **1**

So the equation is $y = \frac{2}{3}x + \frac{7}{3}$ or $2x - 3y + 7 = 0$ **1**

A2 Multiplying the first equation by 3 and the second by 4

$15x + 12y = -9$ **1**

$8x - 12y = 32$ **1**

Adding: $23x = 23,$ $x = 1$ **1**

Then $12y = -9 - 15 = -24$ $y = -2$ **1**

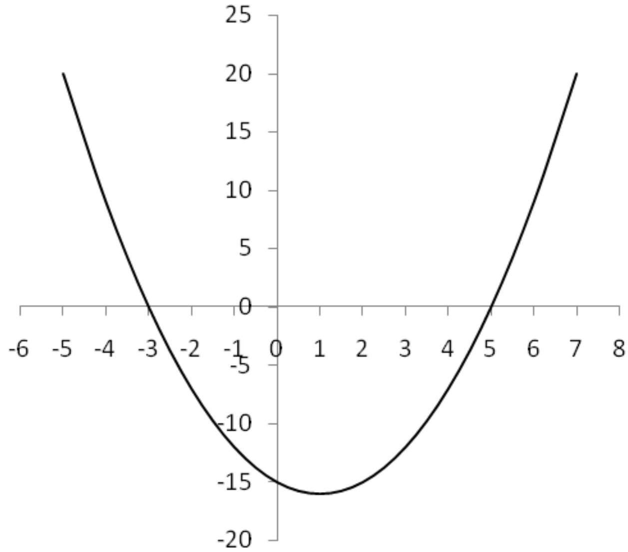
A3 The determinant is $-3 - 10 = -13$ **1**

The inverse is $-\frac{1}{13}\begin{pmatrix} -1 & -5 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & -\frac{3}{13} \end{pmatrix}$ **M2A1**

A4 When $x = 0,\ y = -15$. (Show on axis) **1**

When $y = 0,\ x^2 - 2x - 15 = 0,\ (x-5)(x+3) = 0,\ x = 5, -3$. (Show on axis) **2**

Shape **1**



A5 $5x + 3 < 3x - 7$

$2x < -10$ **2**

$x < -5$ **1**

A6 $(1+5x)^4 = 1 + 4 \times 5x + \frac{4 \times 3}{1 \times 2}(5x)^2 + \ldots$ **M1**

$= 1 + 20x + 150x^2 + \ldots$ **A1**

$(3+x)(1+5x)^4 = 3 + 60x + 450x^2 + \ldots + x + 20x^2 + \ldots$ **M1**

$3 + 61x + 470x^2 + \ldots$ **A1**

A7 $x^{-3/2} = \frac{5}{x}$

$x^{-1/2} = 5$ **1**

$x^{1/2} = \frac{1}{5}$ **1**

$x = \frac{1}{25}$ **1**

A8 $\log_2(x-3) + \log_2(x-5) = 3$

$\log_2(x-3)(x-5) = 3$ **1**

$(x-3)(x-5) = 2^3 = 8$ **1**

$x^2 - 8x + 15 = 8, \qquad x^2 - 8x + 7 = 0$ **1**

$(x-1)(x-7) = 0 \qquad x = 1, 7$ **1**

But $x > 5$, so $x = 7$ **1**

A9	$y = 3\sin x + 5x^3 - 2\ln x$	
	$\dfrac{dy}{dx} = 3\cos x + 15x^2 - \dfrac{2}{x}$	**3**
	$= 32.63$	**1**
	$= 32.6$ correct to 3 significant figures	**1**
A10	$\int_2^3 \left(4\cos x - \dfrac{3}{x}\right)dx$	
	$= \left[4\sin x - 3\ln x\right]_2^3$	**2**
	$= 4\sin 3 - 3\ln 3 - 4\sin 2 + 3\ln 2$	**2**
	$= -4.289 \approx -4.29$	**1**

B1a(i)	The determinant of $\mathbf{A}$ is $-3(3a-5)-2a(2-a)=2a^2-13a+15$.	**1**
	The matrix $\mathbf{A}$ does not have an inverse when $2a^2-13a+15=0$.	**1**
	$(2a-3)(a-5)=0, \quad a=\frac{3}{2}, 5$	**2**
(ii)	For other values of a, $\mathbf{A}^{-1}=\frac{1}{2a^2-13a+15}\begin{pmatrix}3a-5 & -2a \\ a-2 & -3\end{pmatrix}$	**3**
b(i)	By the factor theorem, $f(2)=f(-5)=0$.	**1**
	So $\quad 8+4p+2q-30=0 \quad 4p+2q=22 \quad 2p+q=11$	**1**
	and $\quad -125+25p-5q-30=0 \quad 25p-5q=155 \quad 5p-q=31$	**1**
	Adding $7p=42 \quad p=6 \quad q=-1$	**M1A2**
(ii)	$f(x)=x^3+6x^2-x-30=(x-2)(x+5)(x+3)$	**2**
B2a(i)	The nth term is $12+2(n-1)=2n+10$	**1**
(ii)	The sum of the first n terms is	
	$\frac{1}{2}n(2a+(n-1)d)=\frac{1}{2}n(24+2(n-1))$	**1**
	$=n(n+11)$	**1**
(iii)	When $n(n+11)=200 \quad n^2+11n-200=0$	**1**
	So $n=\frac{-11\pm\sqrt{121+800}}{2}=\frac{-11\pm\sqrt{921}}{2}=9.674$ or -20.67	**M1A1**
	but we need a positive integer.	**1**
	They have completed the 9th floor and so are working on the 10th floor.	**1**
b(i)	If the first term is a and the common ratio is r	
	then $ar^2=125, \quad ar^5=216$	**1**
	So $r^3=\frac{ar^5}{ar^2}=\frac{216}{125} \quad r=\frac{6}{5}$	**M1A1**
	Then $a=\frac{ar^2}{r^2}=\frac{125\times 25}{36}=\frac{3125}{36}(\approx 86.8)$	**1**
(ii)	The sum of the first 5 terms is	
	$\frac{a(1-r^5)}{1-r}=\frac{3125}{36}\frac{1-\frac{6^5}{5^5}}{1-\frac{6}{5}}=\frac{5(6^5-5^5)}{36}=\frac{23255}{36}(\approx 646)$	**2**
(iii)	The common ratio $r=\frac{6}{5}>1$. So the series diverges.	**1**

B3a(i)	$2+2\log_3(x)=\log_3(9)+\log_3(x^2)=\log_3(9x^2)$	**2**
	If this equals $\log_3(y)$, $y=9x^2$.	**1**
(ii)	$2+2\log_3(x)=\log_3(21x-10)$	
	$9x^2=21x-10$	**1**
	$9x^2-21x+10=0$	**1**
	$(3x-2)(3x-5)=0$	**1**
	$x=\frac{2}{3},\frac{5}{3}$	**1**
b(i)	From the given data, $70=Ae^{5k}, 130=Ae^{8k}$	**2**
	Dividing: $e^{3k}=\frac{130}{70}=\frac{13}{7}$	**1**
	Taking logarithms: $3k=\ln\left(\frac{13}{7}\right)$, $k=\frac{1}{3}\ln\left(\frac{13}{7}\right)\approx 0.2063\approx 0.206$	**2**
	Then $A=70e^{-5k}\approx 24.947\approx 24.9$	**1**
(ii)	$V(12)=Ae^{12k}\approx 296.8\approx 297$	**2**
B4a	$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=30°=\frac{\pi}{6}$	**1**
	$\theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$ or $\theta=2\pi-\frac{5\pi}{6}=\frac{7\pi}{6}$	**M1A2**
b(i)	$\angle PRQ=180°-\angle RPQ-\angle RQP=180°-40°-59°=81°$	**M1A1**
(ii)	Using the sine rule $QR=PQ\frac{\sin(RPQ)}{\sin(PRQ)}=9\times\frac{\sin(40°)}{\sin(81°)}\approx 5.857\,\text{m}$	**M2A1**
(iii)	The area is $\frac{1}{2}PQ\times QR\sin(PQR)\approx 22.59\approx 22.6\,\text{m}^2$.	**M2A1**
(iv)	Using the cosine rule $QS^2=QR^2+RS^2-2QR\times RS\cos(QRS)\approx 26.34$	
	So $QS=\sqrt{26.34}\approx 5.13$	**M2A1**

5(i)	By Pythagoras' theorem $l^2 + 4r^2 = 64$	**1**
	So $r = \sqrt{\dfrac{64 - l^2}{4}}$	**2**
(ii)	$V = l\left(\dfrac{1}{2}\pi r^2\right)$	**1**
	$= l\left(\dfrac{1}{2}\pi \dfrac{64 - l^2}{4}\right)$	**1**
	$= \dfrac{\pi\left(64l - l^3\right)}{8}$	**1**
(iii)	$\dfrac{\mathrm{d}V}{\mathrm{d}l} = \dfrac{\pi\left(64 - 3l^2\right)}{8}$	**2**
	$= 0$ when $64 - 3l^2 = 0,$	**1**
	$l = \sqrt{\dfrac{64}{3}} \approx 4.619\,\mathrm{m}$	**2**
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}l^2} = \dfrac{\pi(-6l)}{8} < 0$	**1**
	so this is a maximum.	**1**
(iv)	$V \approx 77.4\,\mathrm{m}^3$	**1**
	$r \approx 3.27\,\mathrm{m}$	**1**

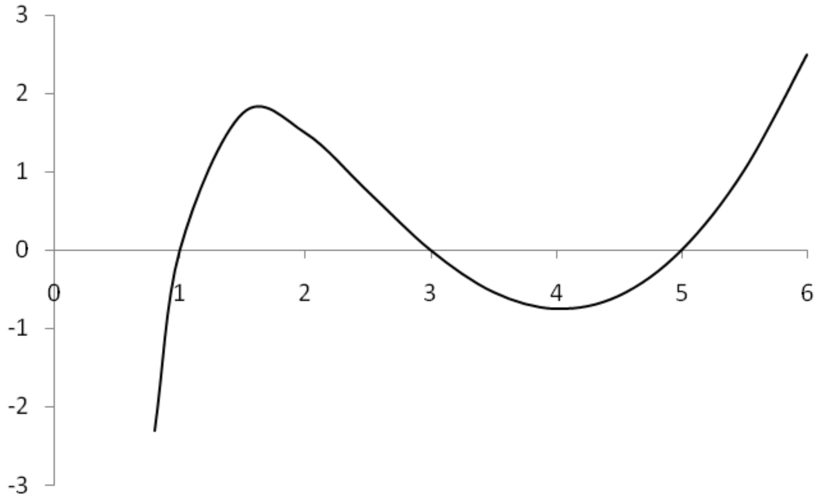
6(i) $x^3 - 9x^2 + 23x - 15 = (x-1)(x^2 - 8x + 15)$ **2**

$= (x-1)(x-3)(x-5)$ **1**

(ii) Shape **1**

Intersections with x-axis **1**

Tends to minus infinity as $x \to 0$ **1**



(iii) $\int_2^4 f(x)\mathrm{d}x = \left[\frac{x^3}{3} - \frac{9x^2}{2} + 23x - 15\ln x\right]_2^4$ **2**

$= \left[\frac{64}{3} - 72 + 92 - 15\ln 4\right] - \left[\frac{8}{3} - 18 + 46 - 15\ln 2\right]$ **2**

$\approx 20.5389 - 20.2695 \approx 0.269$ **1**

(iv) We need to evaluate the integrals from 2 to 3 and 3 to 4 separately and add their absolute values.

$\int_2^3 f(x)\mathrm{d}x = \left[\frac{x^3}{3} - \frac{9x^2}{2} + 23x - 15\ln x\right]_2^3$

$\left[\frac{27}{3} - \frac{81}{2} + 69 - 15\ln 3\right] - \left[\frac{8}{3} - 18 + 46 - 15\ln 2\right]$

$\approx 21.0208 - 20.2695 \approx 0.7514$ **2**

$\int_3^4 f(x)\mathrm{d}x = \left[\frac{x^3}{3} - \frac{9x^2}{2} + 23x - 15\ln x\right]_3^4$

$= \left[\frac{64}{3} - 72 + 92 - 15\ln 4\right] - \left[\frac{27}{3} - \frac{81}{2} + 69 - 15\ln 3\right]$

$\approx 20.5389 - 21.0208 \approx -0.4819$ **1**

So the total area is $0.7514 + 0.4819 \approx 1.23$. **1**