NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Mathematics
Part 1 Examination

Examination Session
January 2009

Time Allowed
2 hours 10 minutes
(Including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A
Answer ALL questions. This section carries 40% of the exam marks.

SECTION B
Answer FOUR questions. This section carries 60% of the exam marks.

The marks for each part of the question are indicated in square brackets [ ]

- **No answers must be written during the first 10 minutes.**
- Write your Candidate Number clearly on the Answer Book in the space provided.
- Write your answers in the Answer Book provided. Additional sheets will be provided on request.
- Clearly write the number and parts of questions attempted at the start of each answer.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- Candidates are reminded of the need to use clear and accurate English.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for full and detailed answers.
- Students will receive a formula book.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Find the equation of the line with gradient $\frac{2}{3}$, passing through the point $(4,5)$. **[3]**

Question A2

Solve the pair of simultaneous equations for x and y: **[4]**

$5x + 4y + 3 = 0$

$2x - 3y = 8$.

Question A3

Find the inverse, $\mathbf{M}^{-1}$, of the matrix $\mathbf{M} = \begin{pmatrix} 3 & 5 \\ 2 & -1 \end{pmatrix}$. **[4]**

Question A4

Sketch the graph of $y = x^2 - 2x - 15$ showing where it crosses the x- and y-axes. **[4]**

Question A5

Find the set of values of x for which $5x + 3 < 3x - 7$. **[3]**

Question A6

Find the first three terms (in ascending powers of x) in the expansion of $(3 + x)(1 - 5x)^4$. **[4]**

Question A7

Solve $x^{-3/2} = \frac{5}{x}$ for x. **[3]**

Question A8

Find the value of x when $x > 5$ and $\log_2(x-3) + \log_2(x-5) = 3$. **[5]**

Question A9

If $y = 3\sin x + 5x^3 - 2\ln x$ find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 1.5$ correct to 3 significant figures. **[5]**

Question A10

Find the value of $\int_2^3 \left(4\cos x - \frac{3}{x}\right)\mathrm{d}x$. **[5]**

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

(a) Let $\mathbf{A} = \begin{pmatrix} -3 & 2a \\ 2-a & 3a-5 \end{pmatrix}$.

(i) Find the values of a for which $\mathbf{A}$ does not have an inverse. **[4]**

(ii) Express $\mathbf{A}^{-1}$ in terms of a for those values of a for which the inverse exists. **[3]**

(b) (i) Find p and q given that $(x-2)$ and $(x+5)$ are factors of $f(x) = x^3 + px^2 + qx - 30$. **[6]**

(ii) Hence factorise $f(x)$ completely. **[2]**

Question B2

(a) A construction company is building a block of apartments. They take 12 days to complete the first floor. Each subsequent floor will take 2 days longer to complete than the one before.

(i) Write down the nth term of the arithmetic progression $12 + 14 + 16 + \ldots$. **[1]**

(ii) Show that the time taken to complete the first n floors is $n(n+11)$ days. **[2]**

(iii) Calculate which floor the company is working on after 200 days. **[5]**

(b) The third term of a geometric series is 125 and the sixth term is 216.

(i) Find the common ratio and the first term. **[4]**

(ii) Find the sum of the first 5 terms. **[2]**

(iii) Show that the series diverges. **[1]**

Question B3

(a) (i) Given that $2+2\log_3(x)=\log_3(y)$, show that $y=9x^2$. **[3]**

(ii) Hence, or otherwise, solve the following equation for x: **[4]**
$2+2\log_3(x)=\log_3(21x-10)$.

(b) A function $V(t)$ is known to have the form $V=A\mathrm{e}^{kt}$.
When $t=5$, $V=70$ and when $t=8$, $V=130$.

(i) Find the values of k and A. **[6]**

(ii) Find the value of V when $t=12$. **[2]**

Question B4

(a) Find all the values of θ, where $0\le\theta\le2\pi$, such that $\cos\theta=-\frac{\sqrt{3}}{2}$. **[4]**
Give your answers as exact multiples of π radians.

(b) In Figure 1, side PQ has length 9 m and side RS has length 7 m.

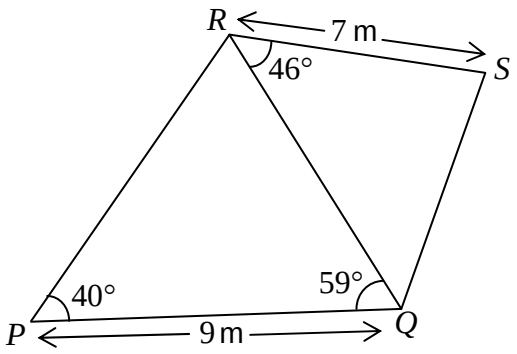

Figure 1

(i) Find the angle PRQ. **[2]**

(ii) Find the length of QR. **[3]**

(iii) Find the area of triangle PQR. **[3]**

(iv) Find the length of QS. **[3]**

(Note: For a triangle ABC with sides a, b, c
the cosine rule is $a^2=b^2+c^2-2bc\cos A$, and
the sine rule is $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.)

Question B5

Bob wants to construct a greenhouse. It is to have a rectangular base and semi-circular cross-section as shown in Figure 2. The diagonal of the floor is to be 8 m.

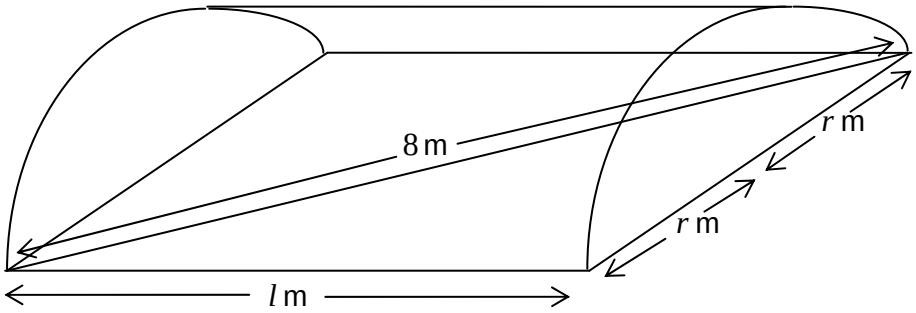

Figure 2

(i) If the length of the greenhouse is l m and the semicircle has radius r m, express r in terms of l. **[3]**

(ii) Express the volume, V, of the greenhouse in terms of l. **[3]**

(iii) Using $\frac{\mathrm{d}V}{\mathrm{d}l}$ and $\frac{\mathrm{d}^2V}{\mathrm{d}l^2}$, find the value of l for which V is a maximum. **[7]**

(iv) Hence find the maximum value of V and the corresponding value of r. **[2]**

Question B6

(i) Factorise $x^3 - 9x^2 + 23x - 15$. **[3]**

(ii) Sketch the graph of $f(x) = x^2 - 9x + 23 - \frac{15}{x}$ for $0 < x \leq 6$, showing clearly the points where the graph crosses the x-axis. **[3]**

(iii) Evaluate $\int_2^4 f(x)\,\mathrm{d}x$. **[5]**

(iv) Find the area between the curve, the x-axis and the lines $x = 2$ and $x = 4$. **[4]**

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