NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFY Mathematics Part 1

EXEMPLAR END OF SEMESTER 1 TEST

Examination Session	**Time Allowed**
Semester One	2 Hours 10 minutes
	(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	Answer ALL questions. This section carries 40% of the exam marks.
SECTION B	Answer 4 questions ONLY. This section carries 60% of the exam marks.
The marks for each question are indicated in square brackets [ ].	

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Find the equation of the line which is parallel to $3x + y - 1 = 0$ and passes through the point $(7, 9)$.

Give your answer in the form $y = mx + c$. **[3]**

Question A2

A bag holds 4 red beads and 5 green beads. Two beads are drawn, one after the other, with no replacement.

Find the probability that both beads are of the same colour. **[4]**

Question A3

Solve the equation $3x^2 = 7x - 3$ giving your answers in surd form. **[4]**

Question A4

Write down the first four terms in ascending powers of x (from smallest to largest up to the term in x^3) of

$$(2 - 5x)^8$$

Give your answer in its simplest form. **[4]**

Question A5

On the same axes, sketch $y = 2^x$ and $y = (\frac{1}{2})^x$

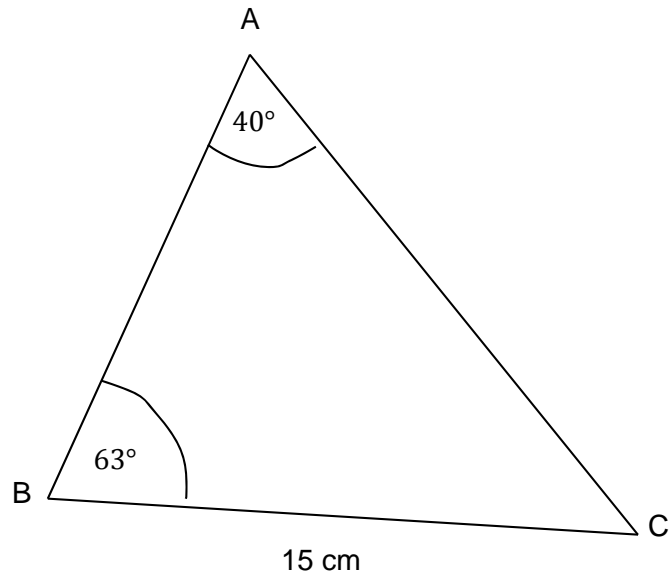Show clearly where your curves cross the $y-$axis. **[3]**

Question A6

Figure 1

Figure 1 shows triangle ABC with angle B = 63º, angle A = 40º and side BC = 15 cm.

Find side AC and the area of triangle ABC.

[4]

Question A7

The function $f(x)$ is defined as $f(x) = 5x^3 + \sin x - \ln x$.

Find $f'(0.2)$ giving your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. [5]

Question A8

A curve has equation $y = x^2 - 10x + 27$. Express the equation in the form

$y = (x + a)^2 + b$ and sketch the curve, showing clearly the coordinates of the turning point and where it crosses the $y-$axis. [4]

Question A9

Find

$$\int \left(3 - \frac{3}{t} + \frac{3}{t^2} - \frac{t^2}{3} \right) \; dt$$

[4]

Question A10

Find the values of a if

$$\int_0^a (2x - 3) \, dx \; = \; 28.$$

[5]

Section B
Answer <u>4</u> questions.
This section carries 60 marks.

Question B1

a) i. Solve the inequalities:

$5 - 2x \leq 4$ **[2]**

ii. $x^2 > 4$ **[2]**

b) Function $f(x)$ is defined as $f(x) = x^3 + 2x^2 - x - 2$.

i. Find the remainder when $f(x)$ is divided by $(x - 2)$. **[2]**

ii. Divide $f(x)$ by $(x + 2)$. **[2]**

iii. Factorise $f(x)$ completely. **[2]**

c) Solve the simultaneous equations $y = x - 4$

$$x^2 + y^2 - 3x - 2y = 9$$ **[5]**

Question B2

a)   i.   The probability that it rains on Monday is 0.4. If it rains on Monday, the probability it rains on Tuesday is 0.3. If it is dry on Monday, then probability that it is dry on Tuesday is 0.8.

           Draw a fully labelled tree diagram.   **[2]**

      ii.   Find the probability that it is dry on both days.   **[2]**

b)   i.   In an Arithmetic Progression, the 3rd term is 17 and the 4th term is 13.

           Find the first term and common difference.   **[2]**

      ii.   Find the 48th term.   **[2]**

      iii.   Find the sum of the first 36 terms.   **[3]**

c)   i.   A Geometric Progression is defined as   512,   448,   392, ...

           Find the first term in the progression which falls below 1.   **[3]**

      ii.   Find the sum to infinity.   **[1]**

Question B3

a)   i.   The number of a certain type of fruit fly in a colony (N) after t days is given by the formula $N = Ae^{kt}$ where A and k are constants. Initially, there were 120 fruit flies and 480 fruit flies after 2 days.

           Write down the value of A and show that $k = \ln 2$.   **[4]**

      ii.   Find the number of fruit flies after 5 days.   **[2]**

      iii.   Find $\frac{dN}{dt}$ when $t = 4$.   **[2]**

b)   Solve the equations:

      i. $4n^{\frac{3}{4}} \times 5n^{\frac{1}{2}} = 4860$ **[3]**

      ii. $2\log_8(x - 4) = 2$ **[4]**

Question B4

a) i. Find the value of

$$\int_0^\pi \cos x \; dx$$
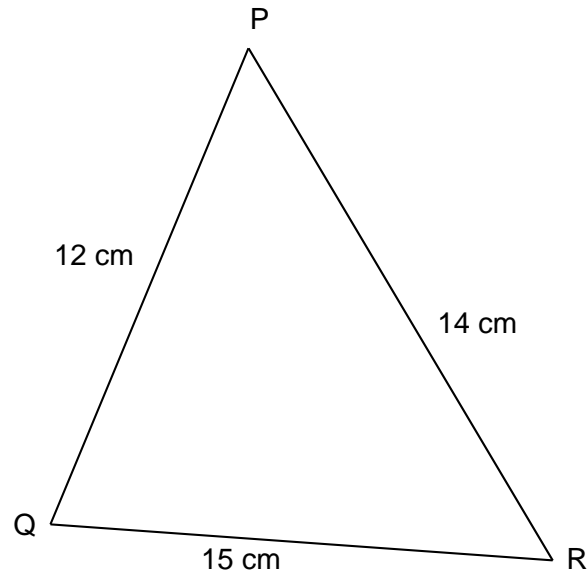
[3]

    ii. What does your answer indicate? **[1]**

b) i.

Figure 2

Figure 2 show triangle PQR where PQ = 12 cm, PR = 14 cm and QR = 15 cm.

Find angle P. **[3]**

    ii. Find the shortest distance from the point Q to the line PR. **[2]**

c) Solve the equation $\tan 2\theta = -\dfrac{1}{\sqrt{3}}$ for $0 \le \theta \le 2\pi$.
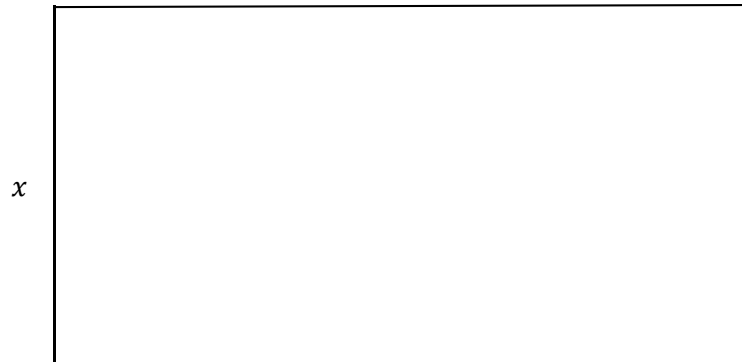
Give your answers as exact multiples of π. **[4]**

d) Angle A is acute and sin A = $\dfrac{9}{41}$.

Find the exact value of cos A. *You must show your working.* **[2]**

Question B5

a)

Figure 3

A farmer has 80 m of fencing and wishes to make a rectangular pen of width x metres as shown in figure 3. She wishes to make the area of the pen as large as possible.

i. Write down the length of the pen in terms of x and show that the area of

the pen, $A,$ is given by
                                                                    [2]
$$A = x(40 - x)$$

ii. Find the value of x which gives the maximum value of A.   **[2]**

iii. Confirm that your value of x gives a maximum.   **[2]**

iv. Find the maximum area.   **[2]**

b) i. A curve has equation $y = x^3 - 9x^2 + 27x - 23$.

Show that there is only one stationary value and find its coordinates.   **[3]**

ii. Show that this stationary value is a point of inflexion.   **[2]**

iii. Draw a sketch of the curve.   **[2]**

Question B6

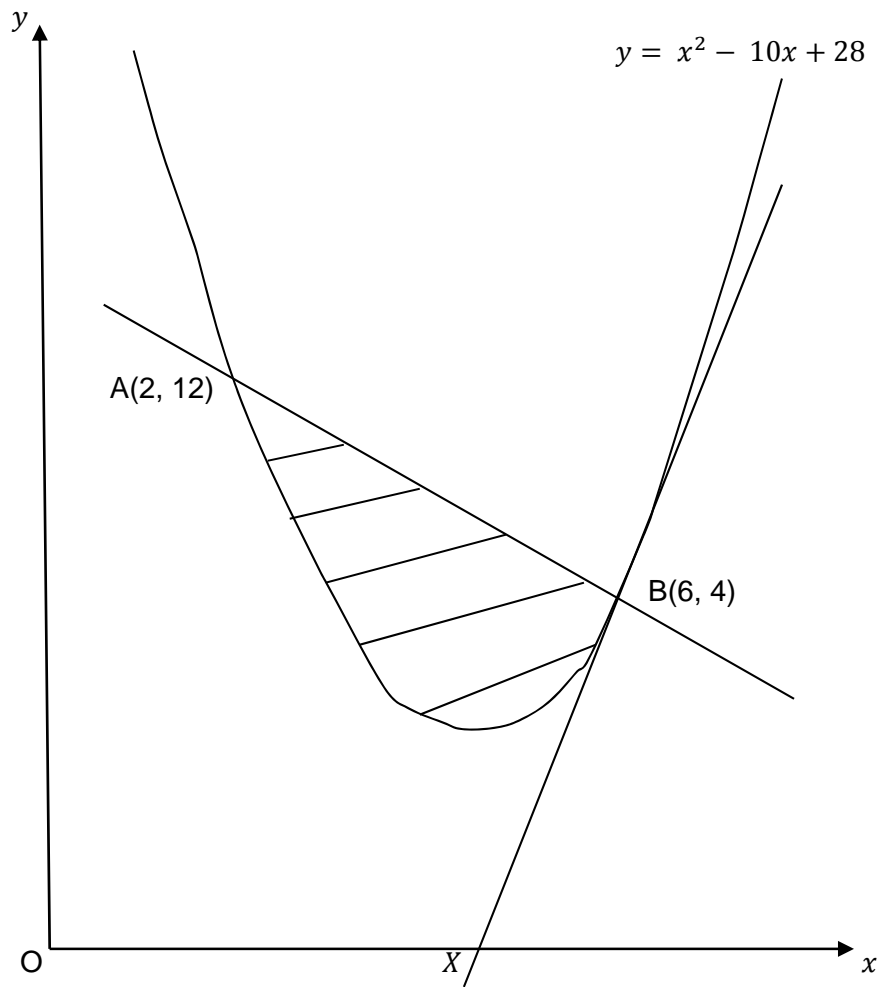$$y = x^2 - 10x + 28$$

A(2, 12)

B(6, 4)

O

X

x

y

Figure 4

Figure 4 shows the curve $y = x^2 - 10x + 28$. It is crossed by the straight line AB at A(2, 12) and B(6, 4). The tangent at point B is also shown.

i.   Find the equation of the line AB.   **[3]**

ii.   Find the equation of the tangent at point B.   **[3]**

iii. The tangent at point B meets the x −axis at point X.

      Find the coordinates of point X.   **[1]**

iv.   Find the area (which is shaded) which is bounded by the line AB and the

      curve $y = x^2 - 10x + 28$.   **[8]**