NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics
End of Semester 1 Test

2016-17

Test Session	**Time Allowed**
Semester One	2 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 40 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 60 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the test.
- Show **ALL** workings in your answer booklet.
- Test materials must not be removed from the room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

The complex numbers z_1 and z_2 are given by $z_1 = 5 + 3i$ and $z_2 = 3 - i$. **[4]**

Find the values of $|\, z_1 - z_2 \,|$ and $z_1 \div z_2$.

Question A2

Matrix **M** is defined as **M** = $\begin{pmatrix} 8 & 1 \\ 4 & -2 \end{pmatrix}$ and matrix **N** is defined as **N** = $\begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$

Work out $\mathbf{MN}^{-1}$ **[4]**

Question A3

Solve the inequality

$$4x + 3 \le \frac{7x + 12}{x + 1}$$ **[5]**

Question A4

Find the value of

$$\sum_{r=17}^{30} (3r^3 + 7)$$ **[5]**

All working must be shown.

Question A5

The roots of the quadratic equation $3x^2 + 6x - 7 = 0$ are α and β.

Find the quadratic equation with roots α^2 and β^2.

Give your answer in the form $ax^2 + bx + c = 0$ where $a,\ b$ and c are integers. **[4]**

Question A6

A particle is dropped from rest.

Find its speed after it has travelled 40 metres.

Find also the time that elapses between the particle being dropped and reaching this speed, giving your answer to **3** significant figures. **[4]**

In this question, 1 mark will be awarded for the correct use of significant figures.

Question A7

Show that

$$\cosh^2 x \; + \; \sinh^2 x = \cosh 2x$$ **[3]**

Question A8

A curve has parametric equations $x = \tan\theta$ and $y = \cos^2\theta$.

Write down the Cartesian equation of the curve in the form $y = f(x)$. **[3]**

<u>All</u> working must be shown.

Question A9

An ellipse has Cartesian equation

$$\frac{x^2}{100} + \frac{y^2}{64} = 1.$$

Find its eccentricity, and explain why your answer is reasonable. **[3]**

Question A10

By differentiating a suitable number of times, find the first four terms of the

Maclaurin Series for e^{3x}. *<u>All</u> working must be shown*. **[5]**

Section B
Answer <u>4</u> questions ONLY. This section carries 60 marks.

Question B1

a) The complex number z_1 is defined as $z_1 = -\sqrt{8} + \sqrt{8}\,i$.

i. Write down the modulus and argument of z_1. **[2]**

The complex number z_2 has modulus 2 and argument $\frac{\pi}{3}$.

ii. Write z_2 in Cartesian form. **[2]**

iii. Find $z_1 z_2$ giving your answer in *both* modulus argument form *and* in Cartesian form. **[3]**

b) Solve the equation $4w^4 + 15w^2 - 4 = 0$. **[4]**

c) The locus of the complex number z is defined as $|\, z - 5i \,| = 6$.

i. Find the Cartesian equation of the locus of z. **[2]**

ii. Sketch this locus, showing clearly where it crosses the $y-$ axis. **[2]**

Question B2

a) The matrix A is defined as $\mathbf{A} = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

Find the eigenvalues of matrix **A**. *All working must be shown*. **[5]**

b) For each of the eigenvalues found in part a, find a corresponding eigenvector. **[6]**

c) Explain what is meant by an eigenvector. **[1]**

d) Find $\mathbf{A}^2$. **[3]**

Question B3

a) i. The curve C has equation

$$y = \frac{x-2}{x}$$

Write down the equations of the asymptotes of curve C. **[2]**

ii. Find the coordinates of the point where curve C crosses the $x -$ axis. **[1]**

iii. Show that curve C has no stationary points. **[3]**

iv. Sketch curve C (**this must not be done on graph paper**).

Show clearly the asymptotes and the coordinates of the point where the curve crosses the $x -$ axis. **[4]**

b) Solve the equation

$$3\sinh^2 x - 25\cosh x + 31 = 0$$

Give your answers in terms of natural logarithms and in exact form. **[5]**

Show <u>all</u> working.

Question B4

a) A curve has parametric equations $x = \cos\theta$ and $y = \tan\theta$ $\quad (\theta < \frac{\pi}{2})$

i. Find an expression for $\frac{dy}{dx}$ in terms of θ. **[3]**

ii. State the value of θ when the curve is parallel to the $y -$ axis. **[1]**

iii. Find the equation of the normal to the curve when $\theta = \frac{\pi}{6}$. **[3]**

b) Find the Cartesian equation of the curve. Give your answer in the form $y = f(x)$. **[3]**

c) The point P lies on the curve and O represents the origin.

i. Find the coordinates, in exact form, of point P when $\theta = \frac{\pi}{4}$. **[1]**

ii. Find the exact length of OP. **[1]**

iii. Find the angle between OP and the $x -$ axis. **[1]**

Point X lies at $(\sqrt{2}, 0)$.

iv. Find the area of triangle POX. **[2]**

Question B5

a) i. Show that

$$\sum_{r=1}^{n} r(r+7) = \frac{n(n+1)(n+11)}{3}$$ **[4]**

ii. Hence find the value of $(21)(28) + \; 22(29) + \; \ldots + 35(42)$. **[3]**

b) i. Use the Taylor expansion to express $\sin(x - \frac{\pi}{6})$ in ascending powers of x up to the term in x^3. **[5]**

ii. Hence find an approximate value of $\sin 36°$, giving your answer in terms of π. *All working must be shown*. **[3]**

Question B6

a)

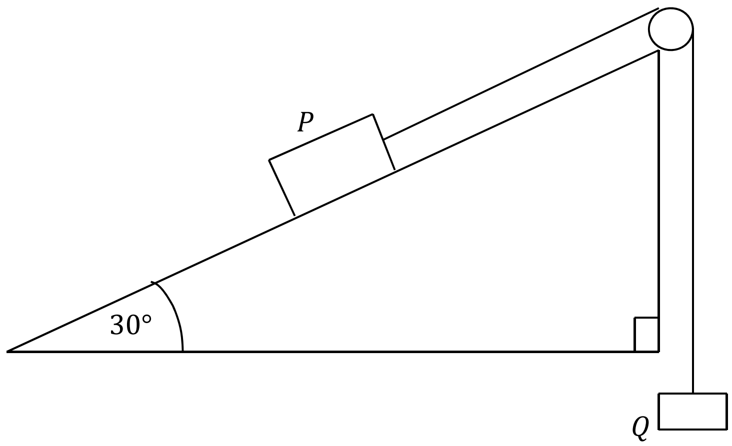

Figure 1

Figure 1 shows two particles P and Q. P has mass 7 kg and is held on a long rough slope which is inclined at $30°$ to the horizontal. Q has mass 2 kg and is suspended freely from a light inextensible string which is connected to P over a smooth pulley. The coefficient of friction between P and the slope is $\frac{1}{10}$.

The system is released from rest and P moves down the slope.

Copy the diagram and show all the forces acting on Q, and all the forces acting on P which are parallel to and perpendicular to the slope. **[3]**

b) Work out the frictional force acting on P. **[2]**

c) Find the acceleration of the particles and the tension in the string. **[5]**

d) After 3 seconds, the string breaks.

Find the speed of P after a <u>further</u> 2 seconds. **[5]**

This is the end of the test.