NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics
Time-Controlled Assessment

2019-20

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 40 marks.**
SECTION B	**Answer THREE questions ONLY. This section carries 60 marks.**
The marks for each question are indicated in square brackets [ ].	
Guide Time: 2 hours	
The Guide Time is how long you are expected to spend completing this Time-Controlled Assessment. You are allowed 24 hours in total to complete and submit.	

- You **MUST** show **ALL** of your working. This is very important. You will score no marks if there is not enough working shown – even if your answer is correct.
- An approved calculator may be used in the assessment.
- All work must be completed independently. The penalty for collusion is a mark of zero.
- Due to the nature of the questions, there should be no need to use external sources of information to answer them. If you do use external sources of information you must ensure you reference these. Plagiarism is a form of academic misconduct and will be penalised.
- Work must be submitted by the deadline provided. Your Study Centre can be contacted only for guidance on submission of work and cannot comment on the contents of the assessment.
- Your work can be word-processed or handwritten. Once complete, any handwritten work will need to be clearly photographed/scanned and inserted into a single word-processed file for submission
- Work must be submitted in a single word-processed file using the standard NCUK cover page.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

(a) A complex number, z, has modulus 24 and argument $\frac{2\pi}{3}$.

Write z in the form $a + bi$ where a is an integer and b is in surd form. **[2]**

(b) Show that $\frac{5\ +\ 10i}{3\ +\ i}$ can be simplified to $\frac{5}{2} + \frac{5}{2}i$ **[2]**

Question A2

Matrix **M** is defined as **M** $= \begin{bmatrix} 3a & a \\ 21 & 2a \end{bmatrix}$

The determinant of matrix **M** is 12.

Find the values of a. **[3]**

Question A3

Solve the inequality **[5]**

$$\frac{6}{x+1} < \frac{4}{x}$$

Question A4

Find the sum of 2 + 16 + 54 + 128 + ….. + 43904. **[3]**

Give your answer <u>in full</u> with no rounding off.

Question A5

The roots of the equation $2x^2 - 5x - 1 = 0$ are α and β.

Find the equation with roots $(\alpha^2 + \frac{1}{\beta})$ and $\left(\beta^2 + \frac{1}{\alpha}\right)$. **[4]**

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

Question A6

A missile is fired vertically upwards at 500 ms^{-1}.

Find its height after $1\frac{1}{2}$ seconds. Give your answer to **2** significant figures. **[3]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A7

Without using exponentials, solve **[4]**

$$2\sinh^2 x - 9\cosh x + 6 = 0.$$

Give your answers in exact logarithmic form.

Question A8

A parabola has parametric equations $x = 8t^2$ and $y = 16t$.

(a) Find the equation of the tangent to the parabola in terms of t. **[2]**

This tangent meets the directrix of the parabola on the $x-$ axis.

(b) Find the values of t. **[2]**

Question A9

A curve has equation

$$y = \frac{x+3}{x-3}$$

Show that the curve has no stationary values. **[3]**

Question A10

Point A with position vector $(3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ lies in a plane.

The vector $(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$ is perpendicular to the plane.

Find an equation of the plane in the form $\boldsymbol{r}.\boldsymbol{n} = p$. **[3]**

Question A11

Solve the differential equation **[4]**

$$\frac{dy}{dx} - 3y = ae^{5x}$$

(where a is a constant) subject to $y = a$ when $x = 0$. Give your answer in the form $y = f(x)$ where $f(x)$ is in terms of x and a.

Section B begins on the next page.

Section B
Answer <u>THREE</u> questions ONLY. This section carries 60 marks.

Question B1

(a)

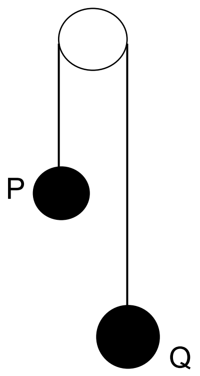

Figure 1

Figure 1 shows two particles P and Q which are connected by a light inextensible string over a smooth pulley.

P has mass M kg and Q has mass 1.8 kg.

The system is released from rest and Q moves downwards with acceleration 0.28 ms^{-2}.

Find the value of M and the tension in the string. **[5]**

(b)

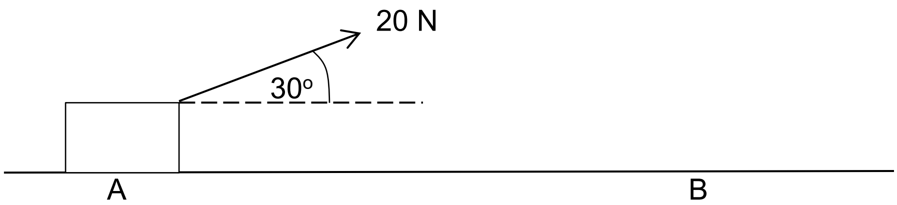

Figure 2

Figure 2 shows a block of mass 16 kg being pulled along a smooth horizontal surface by a light inextensible rope which is inclined at 30° to the horizontal. The tension in the rope is 20 Newtons. The block starts from rest at point A, and it reaches point B after $8\sqrt{3}$ seconds.

i. Find the speed of the block when it reaches point B. **[3]**

The surface beyond point B is rough. The tension in the rope is kept at 20 Newtons but the block now travels at a constant speed.

ii. Find the coefficient of friction between the block and the surface beyond point B. **[3]**

Parts (c) and (d) are on the next page.

(c) Question B1 – (continued)

Figure 3

Figure 3 shows two particles K and L which are at the top of two slopes. K has mass 7 kg and is $\frac{72}{g}$ metres vertically above the horizontal. L has mass 4 kg and is $\frac{50}{g}$ metres vertically above the horizontal. All surfaces are smooth.

Both particles are released from rest.

i. Find the speeds of K and L when they reach the bottom of their slopes. **[3]**

The particles then travel along a horizontal surface until they collide. After the collision, K is brought to rest.

ii. Find the speed and direction of L after the collision. **[3]**

iii. Find the coefficient of restitution between the particles. **[1]**

(d) A car is travelling at a constant speed of 20 ms^{-1} up a smooth slope which is inclined at $\theta°$ to the horizontal where $\sin\theta = \frac{1}{15}$.

The power output of the engine is 13720 Watts.

Find the mass of the car. **[2]**

Question B2

(a)

Matrix **A** is defined as $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & a \end{bmatrix}$

i. Find, in terms of a, the characteristic polynomial of Matrix **A.** **[3]**
You do not need to simplify your answer.

ii. Given that one of the eigenvalues of **A** is 3, find the value of a **[3]**

iii. Find the other two eigenvalues of **A.** **[2]**
.

ii. For each eigenvalue found in part iii find a corresponding eigenvector **[4]**

(b) If $y = \tanh^{-1}\left(\frac{1}{3}e^{x}\right)$, prove that $(9 - e^{2x})\frac{dy}{dx} = 3e^{x}$. **[3]**

(c) Evaluate **[5]**

$$\int_{0}^{\ln 2} \sinh^2 x \; dx.$$

Give your answer in the form $\frac{a}{b} - \ln\sqrt{c}$ where a, b and c are integers.

Question B3

(a) The hyperbola with Cartesian equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$ has parametric equations $x = 4\sec t$ and $y = 3\tan t$ where t is a parameter.

i. Find the eccentricity. **[2]**

ii. Explain why your answer to part i is sensible. **[1]**

iii. Find the equation of the tangent to the hyperbola when $t = \frac{5\pi}{6}$.

Give your answer in the form $y = mx + c$. **[3]**

The tangent found in part iii crosses the $y-$ axis at point Y, and the directrix of the hyperbola with positive $x-$ value at point X.

iv. Find the length of XY, giving your answer in the form $\frac{m\sqrt{13}}{n}$ where m and n are integers. **[3]**

(b) A curve has parametric equations $x = \frac{1}{u} + 1$ and $y = \sqrt{(u+1)}$ where u is a parameter.

i. State the range of values of u where the curve is defined. **[1]**

ii. Write a Cartesian equation of the curve. **[2]**

iii. The integral I is defined as

$$I = \int_3^5 y^2 \; dx.$$

Write I in terms of u. **[2]**

iv. Hence evaluate I giving your answer in exact form. **[3]**

v. Find, for $y > 0$, the equation of the normal to the curve when $u = 3$.

Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. **[3]**

Question B4

(a) **i.** Write down the quadratic equation with roots $(4+2i)$ and $(4-2i)$. **[2]**

ii. Explain why it is not possible to write down a quadratic equation with roots $(4+2i)$ and $(-4+2i)$. **[1]**

(b) Solve the equation

$$64x^2 - \frac{9}{x^2} = 20.$$

[3]

(c) Find the values of p and q if $(p+qi)(3+3i) = -33+15i$. **[3]**

(d) The locus of a complex number, w, is defined as

$$|w+6-4i| = |w-2+4i|$$

Find the Cartesian equation of the locus.

Give your answer in the form $ax+by+c=0$ where a, b and c are integers. **[3]**

(e) **i.** Expand $(\cos\theta + i\sin\theta)^3$. **[2]**

ii. Express $\cos^3\theta$ in terms of $\cos 3\theta$ and $\cos\theta$. **[3]**

iii. Hence find the <u>exact</u> value of **[3]**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^3\theta \ d\theta.$$

Question B5

(a) **i.** By differentiating a suitable number of times, obtain a Taylor expansion of $\cos\left(x + \frac{2\pi}{3}\right)$ up to the term in x^2. **[3]**

ii. Hence find an approximate value of $\cos 125°$ in terms of π. **[3]**

(b) **i.** Show that

$$\sum_{r=1}^{n}(3r^2 - r + 1) = n(n^2 + n + 1)$$

[3]

ii. Hence find

$(3 \times 100 - 9) + (3 \times 121 - 10) + (3 \times 144 - 11) + \cdots + (3 \times 900 - 29)$. **[2]**

Give the answer <u>in full</u> with no rounding off.

(c) A second order differential equation is given by

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 18x^2 + 3x + 9p - 14$$

where p is a constant.

i. Find the complementary function. **[2]**

ii. Find a particular integral. **[3]**

iii. Find the particular solution, given when $x = 0,\ y = 2 + p$ and $\frac{dy}{dx} = 10$. **[4]**

- This is the end of the Time-Controlled Assessment. -