THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A1. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

a) Finds modulus $\sqrt{[(-9)^2 + 2^2]}$ **[M1]**

= 9.219544… (can be implied) **[A1]**

= 9.22 to three significant figures. **[A1ft]**

b) $77 - 36i$ **[B1]**

Question A2

$(a-1)(a+6) + 2(-1-10)(+0) = 2a$ **[M1]**

Forms a quadratic equation $(a^2 + 3a - 28 = 0)$ **[M1]**

Solves **[M1]**

$a = -7,\ 4$ **[A1]**

Question A3

$3x(x-1)^2 > 2(x+6)(x-1)$ Multiplies through by $(x-1)^2$ **[M1*]**

Reaches a cubic inequality in the form of a product of a linear expression and a quadratic expression $[(x-1)(3x^2 - 5x - 12) > 0]$ **[M1*]**

Finds 3 critical values ($-\frac{4}{3},\ 1,\ 3)$ **[M1]**

$x > -\frac{4}{3}$ $x < 1$ (or $-\frac{4}{3} < x < 1$) **(A1)** $x > 3$ **(A1)** **[A2]**

Please note: the second and third ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen. The first and second ranges, if not given in the form $-\frac{4}{3} < x < 1$, can be separated by a space or a comma; but in this case the word 'and' is also permitted and the word 'or' will lose the final mark [but not if it has already been lost earlier i.e. a maximum of 1 mark is lost overall].

Special case If the candidate multiplies through by $(x-1)$ and obtains critical values $-\frac{4}{3}$ and 3, the only mark available is the second M mark.

Question A4

$\alpha + \beta = -3;$ $\alpha\beta = -\frac{1}{4}$ **[M1]**

$(\alpha^2 + 3) + (\beta^2 + 3) = \alpha^2 + \beta^2 + 6 = (\alpha + \beta)^2 - 2\alpha\beta + 6 = 9 + \frac{1}{2} + 6 = \frac{31}{2}$ **[M1]**

$(\alpha^2 + 3)(\beta^2 + 3) = \alpha^2\beta^2 + 3(\alpha^2 + \beta^2) + 9 = \frac{1}{16} + 3\left(9 + \frac{1}{2}\right) + 9 = \frac{601}{16}$ **[M1]**

$16x^2 - 248x + 601 = 0.$ **[A1]**

Question A5

a) Uses a suitable equation of motion **[M1]**

22.5 (metres) **[A1]**

b) Finds the deceleration (42 ms^{-2}) and uses a suitable equation of motion **[M1]**

$\frac{1}{2}$ (second) **[A1]**

Question A6

$\frac{dy}{dx} = \frac{1 + \sinh t}{\cosh t}$ and sets $t = 0 \ \ (= 1)$ **[M1]**

Finds coordinates $(1,\ 1)$ **[M1]**

$y = x$ or equivalent **[A1]**

Question A7

Sets $ae = 5$ and $\frac{a}{e} = \frac{16}{5}$, and finds values for e and a. **[M1]**

Uses $b^2 = a^2(e^2 - 1)$ and finds a value for b. **[M1]**

$e = \frac{5}{4}$; $a = 4$; $b = 3$. **[A1]**

Question A8

Breaks integral into $\int \frac{1}{1 + x^2}\ dx + \int \frac{2x}{1 + x^2}\ dx$ **[M1]**

$= \tan^{-1}x + \ln(1 + x^2) + c$ **(A1)** one part correct; **(A2)** all correct and $+\,c$ **[A2]**

Question A9

Finds momentum before winds hits yacht (800×7) **[M1]**

Finds the impulse (240×5) and subtracts from their momentum before the wind hits the yacht. **[M1]**

Finds the new speed **[M1]**

5.5 (ms^{-1}) **[A1]**

Question A10

Finds $\boldsymbol{a} \times \boldsymbol{b}$ $[2\boldsymbol{i} + (4 + 3p)\boldsymbol{j} + p\boldsymbol{k}]$ **[M1]**

Finds magnitude, sets equal to 276 and forms a quadratic equation **[M1]**
$(5p^2 + 12p - 128 = 0)$

Solves $[(5p + 32)(p - 4) = 0]$ **[M1]**

$p = 4$ [Ignore any reference to the second answer ($-\frac{32}{5}$)] **[A1]**

Question A11

Divides by x^2 and identifies the integrating factor ($e^{\int \frac{1}{x} dx}$) **[M1]**

Multiplies by their integrating factor and reaches $\frac{d}{dx}(\ldots) = \cdots$ $[\frac{d}{dx}(xy) = 6x]$ **[M1]**

Integrates both sides with a constant on one side (if this constant is absent, this mark and the next are both lost) $\left[xy = \frac{6x^2}{2} + c\right]$, and attempts to find c [$= -10$ if c is on the RHS]. **[M1]**

$y = \frac{3x^2 - 10}{x}$ or equivalent **[A1]**

Question A12

a) $397 \pm \frac{5 \times 2.718}{\sqrt{12}}$ **[M1]**

Anything rounding to 393 - 401 (grams) **[A1]**

b) Claim reasonable as 400 lies in the confidence interval. **[A1ft]**

Section B

Question B1

a) i. If P is about to slide up the slope: $mg = 0.51g\sin\theta + \frac{1}{3} \times 0.51g\cos\theta$ **[M1]**

$m \text{ (or } \beta) = 0.39$ (writing $\sin\theta = \frac{8}{17}$ and $\cos\theta$ as $\frac{15}{17}$ is fine) **[A1]**

If P is about to slide down the slope: $mg = 0.51g\sin\theta - \frac{1}{3} \times 0.51g\cos\theta$ **[M1]**

$m \text{ (or } \alpha) = 0.09$ **[A1]**

ii. $0.49g - T = 0.49a$ $\quad T - 0.51g\sin\theta - \frac{1}{3} \times 0.51g\cos\theta = 0.51a$ **[M1]**

Adds and finds a value for a **[M1]**

$a = 0.98 \text{ or } 0.1g$ (ms^{-2}) **[A1]**

iii. Substitutes in either equation **[M1]**

$T =$ anything rounding to 4.32 (Newtons) **[A1]**

b) i. $4 \times 2v + 6(v - 2) = 10 \times 3$ **[M1]**

Solves to find a value for v **[M1]**

$v = 3$ **[A1]**

ii. $e = 2\frac{2}{5} \div \text{their } v$ **[M1]**

$e = \frac{4}{5}$ or equivalent **[A1]**

iii. $28\frac{4}{5}$ (Joules) or equivalent **[B1]**

c) i. Resistance $= 6000g \times \frac{1}{15} + 80 \ (= 4000)$ **[M1]**

Divides 48000 by their resistance. **[M1]**

12 (ms^{-1}) **[A1]**

ii. Their resistance $\times 75$ or P.E. gained $(6000 \times g \times 75 \times \frac{1}{15}) + 80 \times 75$ **[M1]**

300000 (Joules) **[A1]**

Question B2

a) i. $\begin{vmatrix} 3-\lambda & 1 \\ 8 & 1-\lambda \end{vmatrix} = 0$ **[M1*]**

Reaches a quadratic equation $(\lambda^2 - 4\lambda - 5 = 0)$ **[M1]**

$\lambda = -1, \; 5$ **[A1]**

ii. $\lambda = -1$: $3x + y = -x$
$8x + y = -y$ **[M1]**

$(y = -4x)$ Eigenvector is $a\boldsymbol{i} - 4a\boldsymbol{j})$ **[A1]**

$\lambda = 5$: $3x + y = 5x$
$8x + y = 5y$ **[M1]**

$(y = 2x)$ Eigenvector is $(a\boldsymbol{i} + 2a\boldsymbol{j})$ **[A1]**

iii. Multiplies and obtains one correct row or one correct column **[M1]**

$\begin{bmatrix} -17 & -32 \\ 12 & 22 \end{bmatrix}$ <u>Special case</u>: if the candidate has $\mathbf{A}^{\mathbf{T}}$ wrong, then give 1 mark out of 2 if there is one correct row or one correct column. **[A1]**

b) i. Finds an auxiliary equation $(m^2 - 4 = 0)$ and solves **[M1]**

$y = Ae^{2x} + Be^{-2x}$ **[A1]**

ii. ke^{2x} is part of the complementary function (or similar words). **[B1]**

iii. Finds $\frac{dy}{dx}$ $(ke^{2x} + 2kxe^{2x})$ **[M1]**

Finds $\frac{d^2y}{dx^2}$ $(2ke^{2x} + 2ke^{2x} + 4kxe^{2x}$ or $4ke^{2x} + 4kxe^{2x})$ **[M1]**

Substitutes into equation and compares coefficients **[M1]**

$k = \frac{3}{2}$ or equivalent **[A1]**

iv. Differentiates general solution $\left(\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x} + \frac{3}{2}e^{2x} + 3xe^{2x}\right)$ **[M1]**
Substitutes $x = 0$ into y and $\frac{dy}{dx}$ and obtains 2 equations **[M1]**
$(A + B = 3; \quad 2A - 2B + \frac{3}{2} = \frac{7}{2})$
Solves $(A = 2, \quad B = 1)$ **[M1]**

$y = 2e^{2x} + e^{-2x} + \frac{3}{2}xe^{2x}$ **[A1]**

Question B3

a) i. $16 = 25(1 - e^2)$ and rearranges to find e^2 **[M1*]**

$e = \frac{3}{5}$ **[A1]**

ii. $x = \pm\frac{25}{3}$ **[B1]**

iii. $\frac{dy}{dx} = \frac{4\cos\theta}{-5\sin\theta}$ **[M1*]**

Inverts and changes sign $\left(\frac{5\sin\theta}{4\cos\theta}\right.$ or $\left.\frac{5}{4}\tan\theta\right)$ **[M1*]**

$y - 4\sin\theta = \frac{5}{4}\tan\theta(x - 5\cos\theta)$ or equivalent **[A1]**

iv. Sets $x = \frac{9}{10}$ and $y = 0$ $[-4\sin\theta = \frac{5}{4}\tan\theta(\frac{9}{10} - 5\cos\theta)]$ **[M1]**

Multiplies by $40\cos\theta$ $[-160\sin\theta\cos\theta = 45\sin\theta - 250\sin\theta\cos\theta]$ **[M1+]**

Rearranges and factorises $[45\sin\theta(2\cos\theta - 1) = 0]$ **[M1+]**

+ These marks can be given for any other valid method.

$\theta = \frac{\pi}{3}$ **[A1]**

b) i. Finds coordinates $[(0,\ 1);\ \left(\sqrt{3},\ \frac{1}{2}\right)]$ and uses Pythagoras **[M1]**

KM = $\frac{\sqrt{13}}{2}$ or equivalent but must be exact. **[A1]**

ii. Finds equation of KM ($y - 1 = \frac{-1}{2\sqrt{3}}x$ or equivalent) **[M1]**

Expresses $y^2 = \sqrt{48}\,x$ in the form $y^2 = 4\sqrt{3}x$ and substitutes $x = -\sqrt{3}$ Into their equation of KM. **[M1]**

$y = \frac{3}{2}$ or equivalent **[A1]**

iii. Writes dx as $\frac{dx}{d\phi} \times d\phi$ **[M1]**

Writes integral in terms of ϕ and changes limits $[\int_0^{\pi/4} \cos^4\phi \sec^2\phi \ d\phi]$ **[M1]**

Writes $\cos^2\phi$ as $\frac{1}{2}(\cos 2\phi + 1)$ and integrates $[\frac{1}{2}(\frac{1}{2}\sin 2\phi + \phi)]$ **[M1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$\frac{1}{4} + \frac{\pi}{8}$ or anything rounding to 0.64 **[A1]**

Question B4

a) $\frac{e^{\ln 3} + e^{-\ln 3}}{e^{\ln 3} - e^{-\ln 3}} - \frac{2}{e^{\ln 2} + e^{-\ln 2}}$ **[M1*]**

$\frac{3 + \frac{1}{3}}{3 - \frac{1}{3}} - \frac{2}{2 + \frac{1}{2}}$ **[M1*]**

Simplifies **[M1]**

$= \frac{9}{20}$ **[A1]**

b) i. $du = 2x\,dx$ or equivalent **[M1*]**

Writes integral in terms of u and changes limits $[\int_{16}^{36} \frac{du}{\sqrt{u^2 - 1}}]$ **[M1*]**

Integrates $[\cosh^{-1} u]$ and substitutes in correct limits. **[M1*]**

$\ln[\frac{36 + \sqrt{1295}}{16 + \sqrt{255}}]$ **[A1]**

ii. $\frac{dy}{dx} = 2\cosh x \sinh x \qquad \frac{d^2y}{dx^2} = 2\cosh x \cosh x + 2\sinh x \sinh x$ **[M1*]**

LHS = $2(\cosh^2 x + \sinh^2 x) + 4\cosh^2 x \sinh^2 x - 4\cosh^4 x + 2$ **[M1*]**

Replaces $\sinh^2 x$ by $\cosh^2 x - 1$ **[M1*]**
$[2\cosh^2 x + 2(\cosh^2 x - 1) + 4\cosh^2 x(\cosh^2 x - 1) - 4\cosh^4 x + 2]$

Simplifies (at least one line of working must be seen) **[M1*]**

= 0 (all M marks scored and no errors seen) **[A1]**

c) i. $\boldsymbol{r}.(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}) = [(2 + \mu)\boldsymbol{i} + (-3 + 2\mu)\boldsymbol{j} + (1 - 3\mu)\boldsymbol{k}].(\boldsymbol{i} + \boldsymbol{2j} - \boldsymbol{3k})$

Finds scalar product $[(2 + \mu) + 2(-3 + 2\mu) - 3(1 - 3\mu) = 77]$ **[M1]**

Finds a value for μ (6) **[M1]**

Q lies at $(8, 9, -17)$. [Accept in vector form] **[A1]**

ii. Finds $\frac{dr}{dt}$ $[(4t^3 + 6t)\boldsymbol{i} + (3t^2 - 2t)\boldsymbol{j} + 12t\boldsymbol{k}]$ **[M1]**

Finds $\frac{d^2\boldsymbol{r}}{dt^2}$ $[(12t^2 + 6)\boldsymbol{i} + (6t - 2)\boldsymbol{j} + 12\boldsymbol{k}]$ **[M1]**

Substitutes $t = 1$, finds magnitude (22) and multiplies by 2.5 **[M1]**

Force $= 55$ (Newtons) **[A1]**

Question B5

a) i. $\frac{23-2i}{2-3i} \times \frac{2+3i}{2+3i}$ **[M1]**

Multiplies out $\left[\frac{46+65i+6}{4+9}\right]$ **[M1]**

$= 4 + 5i$ **[A1]**

ii. Recognises the quadratic equation in x^2 **[M1]**

Solves $[\,(9x^2-1)(x^2+4)=0\,]$ **[M1]**

$x = \pm\frac{1}{3}, \quad \pm 2i$ **[A1]**

iii. $(x-2)^2 + (y+5)^2 \ldots$ **[M1]**

$\ldots = 9$ **[M1]**

$x^2 + y^2 - 4x + 10y + 20 = 0.$ **[A1]**

iv. Outside circle centre (2, -5) and radius 3 **[B1]**

including boundary **[B1]**

b) i. $z^n = \cos n\theta + i\sin n\theta$; $z^{-n} = \cos n\theta - i\sin n\theta$, adds and obtains result. **[M1*]**

Sets $n = 1$ and obtains second result. **[M1*]**

ii. $z^3 + 3z^2\left(\frac{1}{z}\right) + 3z\left(\frac{1}{z^2}\right) + \frac{1}{z^3}$ **[M1]**

$= z^3 + 3z + 3\left(\frac{1}{z}\right) + \frac{1}{z^3}$ or equivalent in simplest form. **[A1]**

iii. $8\cos^3\theta = \left(z^3 + \frac{1}{z^3}\right) + 3\left(z + \frac{1}{z}\right) = 2\cos 3\theta + 6\cos\theta$ **[M1*]**

$\cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$ **[A1]**

iv. Uses previous result and integrates $\left[\frac{1}{12}\sin 3\theta + \frac{3}{4}\sin\theta\right]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{3\sqrt{3}}{8}$ or any equivalent form but it must be exact. **[A1]**

Question B6

a) i. $f(x) = (1+2x)^{-½}$; $f'(x) = -1(1+2x)^{-3/2}$;

$f''(x) = 3(1+2x)^{-5/2}$; $f'''(x) = -15(1+2x)^{-7/2}$ **[M1*]**

$f(0) = 1$; $f'(0) = -1$; $f''(0) = 3$; $f'''(0) = -15$ **[M1]**

$(1+2x)^{-½} \approx 1 - x + \frac{3x^2}{2!} - \frac{15x^3}{3!}$ or $1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3$ **[A1]**

(No need to simplify)

ii. Substitutes $x = \frac{1}{10}$ into their expansion **[M1]**

$= \frac{73}{80}$ **[A1]**

iii. Anything rounding to 0.00037 **[B1]**

iv. The value given to x is larger in this case… **[B1]**

…and the smaller the value of x, the greater the accuracy in the (Maclaurin) expansion. (Allow any similar argument as long as the candidate shows a clear understanding.) **[B1]**

b) $k\sum_{r=1}^{10} r^2 = 3\sum_{r=1}^{12} r - \sum_{r=1}^{12} 2$ **[M1]**

$k\left[\frac{10}{6}(11)21\Big)\right] = 3\left[\frac{12}{2}(13)\right] - 24$ **[M1]**

Solves **[M1]**

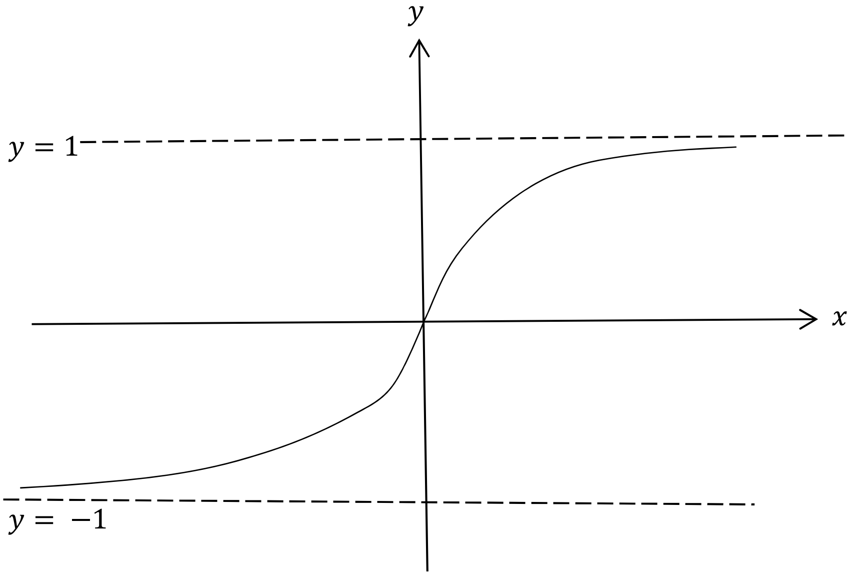
$k = \frac{210}{385}$ or $\frac{6}{11}$ or equivalent, or anything rounding to 0.545 **[A1]**

c) i. $(0,\ 0)$ **[B1]**

ii. $y = 1$; $y = -1$ **(B1)** for each **[B2]**

iii. $\frac{dy}{dx} = \text{sech}^2 x$ **[M1*]**

This is never 0, so there are no stationary values. **[A1]** iv.



(B1) for the correct shape (do not worry if the gradient appears vertical at the origin)

(B1) for asymptotes

(B1) for passing through $(0, 0)$

<u>Please note</u>: if a candidate uses graph paper, apply no penalty if the graph is clearly a sketch. If there is evidence of plotting points, then take off one mark at the end. **[B3]**