NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination

2017-18

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

The complex number, z, is defined as $z = -9 + 2i$.

a) Find $|z|$ giving your answer to **3** significant figures. **[3]**

In this question, 1 mark will be given for the correct use of significant figures.

b) Find z^2 in Cartesian form. **[1]**

Question A2

Matrix M is defined as **M** = $\begin{bmatrix} a-1 & -2 & 0 \\ -1 & a & 2 \\ 5 & -3 & 1 \end{bmatrix}$

The determinant of matrix **M** is $2a$.

Find the possible values of a. **[4]**

Question A3

Solve the inequality

$$\frac{3x}{2} > \frac{x+6}{x-1}$$

Show all working. **[5]**

Question A4

The quadratic equation $4x^2 + 12x - 1 = 0$ has roots α and β.

Find the equation with roots $\alpha^2 + 3$ and $\beta^2 + 3$.

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers. **[4]**

Question A5

A stone of mass 0.2 kg is dropped down a well. The stone hits the bottom of the well at 21 ms^{-1}.

a) Find the depth of the well. **[2]**

At the bottom of the well, there is a thick layer of mud. Once it hits the mud, the only force acting on the stone is a resistance of 8.4 Newtons.

b) Find the time taken for the stone to come to rest. **[2]**

Question A6

A curve has parametric equations $x = 1 + \sinh t, \quad y = t + \cosh t.$

Find the equation of the tangent when $t = 0$. **[3]**

Question A7

A hyperbola has Cartesian equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

The foci lie at $(\pm 5,\ 0)$ and the directrices have equations $x = \pm\frac{16}{5}$.

Find the eccentricity, and the values of a and b. **[3]**

Question A8

Find

$$\int \frac{1 + 2x}{1 + x^2}\ dx.$$ **[3]**

Question A9

A yacht of mass 800 kg is travelling at 7 ms^{-1}.

A gust of wind of force 240 Newtons meets the yacht head on (i.e. opposite to its direction of travel) and lasts for 5 seconds.

Find the new speed of the yacht. **[4]**

Question A10

Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined as $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{b} = p\boldsymbol{i} - 2\boldsymbol{k}$ where p is an integer.

You are given $|\boldsymbol{a} \times \boldsymbol{b}| = \sqrt{276}$.

Find the value of p. **[4]**

Question A11

Solve the differential equation

$$x^2 \frac{dy}{dx} + xy = 6x^2$$

subject to $y = 1$ when $x = 2$.

Give your answer in the form $y = f(x)$. **[4]**

Question A12

The masses of bags of sugar are assumed to follow a Normal distribution but the standard deviation is unknown.

A sample of 12 bags is selected. The mean is found to be 397 grams and the standard deviation 5 grams.

a) Find a 98% confidence interval for the mean mass of the bags. **[2]**

The person who sells the bags claims that the mean mass is 400 grams.

b) Comment on this claim. **[1]**

Section B

Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a)

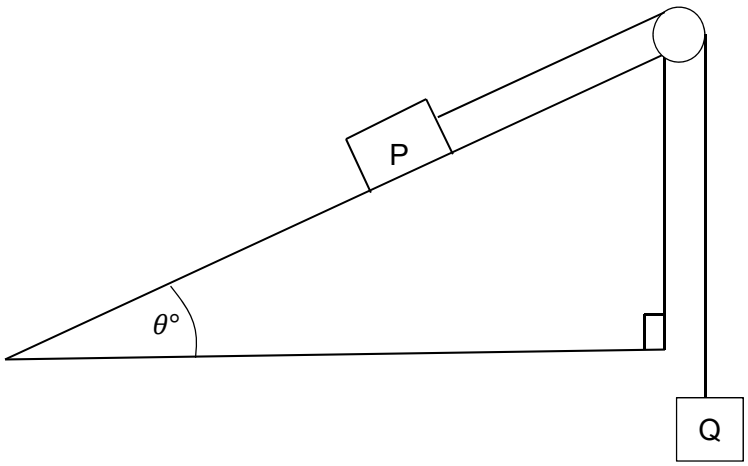

Figure 1

Figure 1 shows a block P of mass 0.51 kg on a rough slope which is inclined at $\theta°$ to the horizontal where $\tan\theta = \frac{8}{15}$. The coefficient of friction between P and the slope is $\frac{1}{3}$.

A light inextensible string connects P over a smooth pulley to another block Q of mass m kg which hangs freely.

The system is released from rest. P remains on the slope without moving if $\alpha < m < \beta$.

i. Find the values of α and β. **[4]**

You are now given that the mass of block Q is 0.49 kg and, when the system is released from rest, block P moves up the slope.

ii. Find the acceleration of the blocks. **[3]**

iii. Find the tension in the string. **[2]**

Parts b) and c) are on the next page.

Question B1 – (continued)

b)

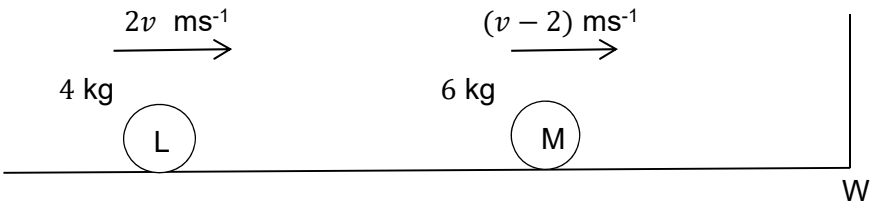

Figure 2

Figure 2 shows two spheres L and M on a smooth horizontal surface. Sphere L has mass 4 kg and is travelling at $2v$ ms^{-1}. Sphere M has mass 6 kg and is travelling at $(v-2)$ ms^{-1} in the same direction as L.

The spheres collide and after the collision they coalesce (*i.e.* join together) to form a single combined mass of 10 kg which travels at 3 ms^{-1} in the same direction as the spheres were travelling before the collision.

i. Find the value of v. **[3]**

The combined mass then hits a solid wall at point W and rebounds at $2\frac{2}{5}$ ms^{-1}.

ii. Find the coefficient of restitution between the combined mass and the wall. **[2]**

iii. Find the kinetic energy of the combined mass after it rebounds from the wall. **[1]**

c) A lorry of mass 6000 kg and with a power output of 48000 Watts goes up a smooth slope inclined at $\sin^{-1}(\frac{1}{15})$ to the horizontal. There is a wind resistance of 80 Newtons acting down the slope.

i. Find the maximum speed that the lorry can achieve. **[3]**

ii. Find the work done when the lorry travels 75 metres up the slope. **[2]**

Question B2

a) Matrix **A** is defined as $\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 8 & 1 \end{bmatrix}$

i. Find the eigenvalues of matrix **A**. **[3]**

ii. For each eigenvalue found in part i, find a corresponding eigenvector. **[4]**

Matrix **B** is defined as $\mathbf{B} = \begin{bmatrix} -3 & -8 \\ 2 & 6 \end{bmatrix}$

iii. Find $\mathbf{BA^T}$. **[2]**

b) A second order differential equation is defined as

$$\frac{d^2y}{dx^2} - 4y = 6e^{2x}.$$

i. Find the complementary function. **[2]**

ii. Explain why a particular integral must take the form $y = kxe^{2x}$ rather than $y = ke^{2x}$. **[1]**

iii. Find a particular integral. **[4]**

iv. Find the particular solution given $y = 3$ and $\frac{dy}{dx} = \frac{7}{2}$ when $x = 0$. **[4]**

Question B3 is on the next page.

Question B3

a) The Cartesian equation of an ellipse is given by

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

i. Show that the eccentricity is $\frac{3}{5}$. **[2]**

ii. Write down the equations of the directrices. **[1]**

Point P lies on the ellipse and has coordinates $(5\cos\theta,\ 4\sin\theta)$ where θ is a parameter.

iii. **Derive** the equation of the normal at point P in terms of θ. *All working must be shown*. **[3]**

The normal at point P passes through the point $\left(\frac{9}{10},\ 0\right)$.

iv. Find the value of θ, given $0 < \theta < \frac{\pi}{2}$. **[4]**

b) A curve has parametric equations $x = \tan\phi, \quad y = \cos\phi$
where $0 \leq \phi < \frac{\pi}{2}$.

Point K lies on the curve when $\phi = 0$ and point M lies on the curve when $\phi = \frac{\pi}{3}$.

i. Find the **exact** length of KM. **[2]**

ii. Find the $y-$ coordinate where the line KM meets the directrix of the parabola with equation $y^2 = \sqrt{48}x$. **[3]**

iii. Evaluate

$$\int_0^1 y^4 \ dx.$$

[5]

Show all working.

Question B4

a) **By using exponentials,** find the value of $\coth(\ln 3) - \operatorname{sech}(\ln 2)$.

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

Each stage of your working must be shown. An answer, even a correct one, will receive no marks if this working is not seen.

[4]

b) i. Use the substitution $u = x^2$ to evaluate

$$\int_4^6 \frac{2x}{\sqrt{(x^4 - 1)}}\ dx.$$

Give your answer as a single logarithm in exact form.

Each stage of your working must be shown. An answer, even a correct one, will receive no marks if this working is not seen. **[4]**

ii. If $y = \cosh^2 x$, show that

$$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 - 4y^2 + 2 = 0.$$

All working must be shown. **[5]**

c) i. The line with equation $\boldsymbol{r} = 2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k} + \mu(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k})$ intersects the plane with equation $\boldsymbol{r}.(\boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}) = 77$ at point Q.

Find the coordinates of point Q. **[3]**

ii. A particle, P, has mass 2.5 kg and position vector $\boldsymbol{r}$ at time t where $\boldsymbol{r} = (t^4 + 3t^2 + 1)\boldsymbol{i} + (t^3 - t^2)\boldsymbol{j} + (6t^2 + 5)\boldsymbol{k}$

Find the magnitude of the force acting on P when $t = 1$. **[4]**

Question B5 is on the next page.

Question B5

a) i. Find $(23 - 2i) \div (2 - 3i)$.

Give your answer in the form $p + qi$ where p and q are integers. **[3]**

ii. Solve the equation

$$9x^4 + 35x^2 - 4 = 0.$$ **[3]**

iii. Find the Cartesian equation of the locus represented by

$$|z - 2 + 5i| = 3.$$

Give your answer in the form $x^2 + y^2 + ax + by + c = 0$ where a, b and c are integers. **[3]**

iv. Describe the locus of points defined by $|z - 2 + 5i| \geq 3$. **[2]**

b) i. If $z = \cos\theta + i\sin\theta$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and that hence $z + \frac{1}{z} = 2\cos\theta$. **[2]**

ii. Expand $(z + \frac{1}{z})^3$. Give your answer in its simplest form. **[2]**

iii. Use your answers to parts i and ii to express $\cos^3\theta$ in the form $a\cos 3\theta + b\cos\theta$ where a and b are rational numbers. **[2]**

iv. Hence find the **exact** value of

$$\int_0^{\frac{\pi}{3}} \cos^3\theta \ d\theta.$$

Working must be shown. **[3]**

Question B6

a) i. By differentiating a suitable number of times, obtain the Maclaurin expansion for $(1 + 2x)^{-½}$ up to the term in x^3. **[3]**

ii. Hence find an approximate value of $\frac{1}{\sqrt{(1.2)}}$.

Give your answer in the form $\frac{m}{n}$ where m and n are integers. **[2]**

iii. Use your calculator to find $\frac{1}{\sqrt{(1.2)}}$ and find the difference between the value on your calculator and your answer to part ii. **[1]**

iv. When $\frac{1}{\sqrt{(1.4)}}$ was evaluated using the Maclaurin expansion and a calculator, the difference between the values was approximately 0.00515.

Explain why the difference is larger here than the one found in part iii. **[2]**

b) Find the value of k if

$$\sum_{r=1}^{10} kr^2 = \sum_{r=1}^{12} (3r - 2)$$

[4]

c) A curve C has equation $y = \tanh x$.

i. Find where curve C crosses the $x-$ axis and the $y-$ axis. **[1]**

ii. Write down the equations of the asymptotes of curve C. **[2]**

iii. Investigate whether curve C has any stationary values. **[2]**

iv. Sketch curve C (**this must not be done on graph paper.**)

Show clearly the aspmptotes and where the curve crosses the $x-$ axis and the $y-$ axis. **[3]**

This is the end of the examination.

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