THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination 2017-18

Examination Session Semester Two

Time Allowed 2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**
The marks for each question are indicated in square brackets [ ].	

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

The complex numbers z_1 and z_2 are defined as $z_1 = p - 2i$ and $z_2 = 2 + i$.

You are given $z_1 z_2 = 12 + i$.

Find:

a) the value of p. **[2]**

b) the argument of $12 + i$. Give your answer to **2** significant figures. **[3]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A2

Find the value of n if

$$\sum_{r=1}^{n} 3r = 513.$$

[3]

Question A3

Solve the inequality

$$x < \frac{12 - x}{x - 5}$$

Show all working. **[5]**

Question A4

A shell is fired vertically into the air at 245 ms^{-1}.

Find:

a) the time taken to reach its greatest height **[2]**

b) this greatest height. **[2]**

Question A5

A curve has parametric equations $x = t^2 + t, \quad y = t^3 \quad (t \geq 0)$

Evaluate

$$\int_0^6 y \ dx.$$

Each stage of your working must be shown. An answer, even the correct one, will receive no marks if this working is not shown. **[4]**

Question A6

A hyperbola has eccentricity $1\frac{1}{4}$ and foci at $(\pm 15, \ 0)$.

Find its Cartesian equation. **[3]**

Question A7

The power output of a crane is 7350 Watts. How long will it take the crane to lift a mass of 1500 kg through a vertical height of 15 metres? **[3]**

Question A8

By differentiating a suitable number of times, obtain the Maclaurin expansion for $\tan x$ up to the term in x^3. *All working must be shown*. **[3]**

Question A9

The point P with position vector $(3\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k})$ lies in a plane. The vector $(2\boldsymbol{i} + 5\boldsymbol{j} - \boldsymbol{k})$ is perpendicular to the plane.

Find an equation of the plane in the form $\boldsymbol{r}.\boldsymbol{n} = p$. **[4]**

Question A10

Solve the differential equation

$$\frac{dy}{dx} + y \sec^2 x = e^{1 - \tan x}$$

given $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. **[4]**

Question A11

If $y = \sinh^{-1}x$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{(x^2+1)}}$.

<u>Each</u> stage of your working must be clearly shown. **[4]**

Question A12

The mean mass of all apples is assumed to follow a Normal distribution but the standard deviation is unknown. A sample of 9 apples was selected. The mean mass was m grams and the standard deviation s grams.

The 90% confidence interval ranged from 103.8 to 116.2 grams.

Find the values of m and s. **[3]**

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a)

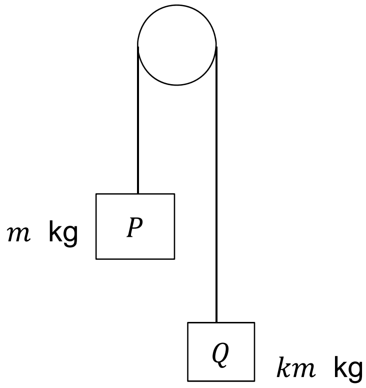

Figure 1

Figure 1 shows two particles, P and Q, which are connected by a light inextensible string over a smooth pulley. P has a mass of m kg and Q has a mass of km kg.

The system is released from rest and P moves downwards.
The acceleration of the particles is 1.4 ms^{-2}.

i. Find the value of k. **[4]**

After 3 seconds the string breaks.

ii. How far does Q continue to rise before coming to rest? (You may assume that Q does not reach the pulley.) **[3]**

iii. Find the speed of P 4 seconds after the string breaks. **[2]**

Parts b) and c) are on the next page.

Question B1 – (continued)

b)

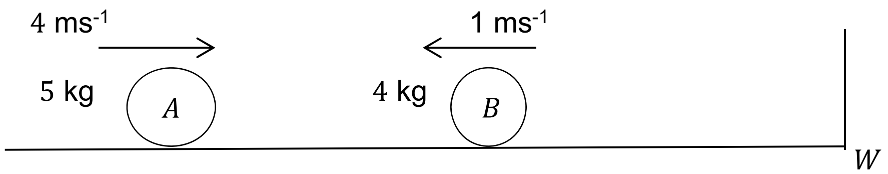

Figure 2

Figure 2 shows two spheres, A and B, approaching each other on a smooth surface. A has mass 5 kg and is travelling at 4 ms^{-1}. B has mass 4 kg and is travelling at 1 ms^{-1}.

The spheres collide. After the collision, sphere A moves at 0.6 ms^{-1} in the same direction in which it was travelling before the collision. Sphere B moves in the opposite direction to which it was travelling before the collision.

i. Find the speed of B after the collision. **[3]**

ii. Find the impulse which each sphere undergoes in the collision. **[1]**

iii. After the collision, B hits a solid wall at point W and rebounds at 1.95 ms^{-1}. Find the coefficient of restitution between B and the wall. **[2]**

c)

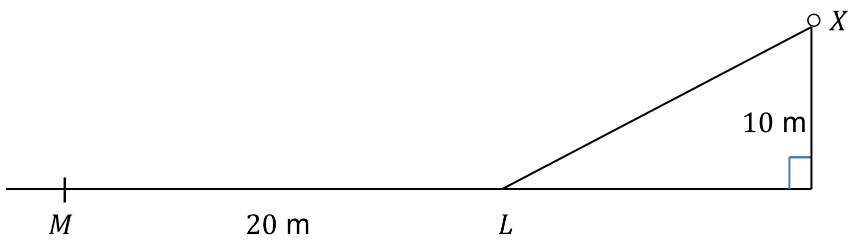

Figure 3

Figure 3 shows a particle, X, of mass 6 kg which is at rest at the top of a smooth slope. X is 10 metres vertically above ground level.

Particle X is released and it rolls down the slope to ground level at point L.

i. Find the speed of X when it reaches point L. **[2]**

Particle X then travels along a rough horizontal surface before coming to rest at point M which is 20 metres from point L.

ii. Find the coefficient of friction between X and the surface. **[3]**

Question B2

a) The matrix **M** is defined as $\mathbf{M} = \begin{bmatrix} -1 & 4 & 0 \\ 1 & 0 & -1 \\ 0 & -2 & -1 \end{bmatrix}$

i. Find the eigenvalues of matrix **M**. **[5]**

ii. For each eigenvalue found in part i, find a corresponding eigenvector. **[6]**

b) The second order differential equation is defined as

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2x + 6$$

i. Find the complementary function. **[2]**

ii. Find a particular integral. **[3]**

iii. Find the particular solution given that when $x = 0,\ y = 5$ and $\frac{dy}{dx} = 8$. **[4]**

Section B continues on the following page.

Question B3

a) Point $P(ap^2,\ 2ap)$ and point $Q(aq^2,\ 2aq)$ lie on the parabola with Cartesian equation $y^2 = 4ax$.

OP is perpendicular to OQ where point O is the origin.

i. Show that $pq = k$ where k is a constant to be determined.

Show all working. **[3]**

ii. Show that the equation of the tangent to the parabola at P is given by

$$py = x + ap^2\ .$$ **[3]**

iii. Write down the equation of the tangent at Q in terms of a and p. **[2]**

The tangents at P and Q meet at point R.

iv. Find the $x -$ coordinate of point R. **[3]**

b) A curve has parametric equations $x = \tan\theta, \quad y = 2\cos\theta$.

i. Find the equation of the normal to the curve when $\theta = \frac{\pi}{6}$. **[4]**

This normal cuts the $x -$ axis at point X.

ii. Show that the directrix of the parabola with equation $y^2 = \frac{5}{\sqrt{3}}x$ also passes through point X.

<u>Each</u> stage of your working must be clearly shown. **[5]**

Question B4

a) Solve $8\cosh x - 4\sinh x = 7$.

Give your anwers in logarithmic form. **[5]**

b) Evaluate

$$\int_{11}^{23} \frac{1}{\sqrt{(x^2 + 14x + 85)}}\ dx.$$

Give your answer as a single logarithm and in exact form.

All working must be shown. **[4]**

c) Show that

$$\int \operatorname{sech} x\ dx = 2\tan^{-1}(e^x) + constant.$$

[4]

d) A pyramid has a square base $OABC$ and has vertices at $O(0,\ 0,\ 0)$; $A(15, -8,\ 0)$; $B(23,\ 7,\ 0)$; $C(8,\ 15,\ 0)$ and $D\left(11\frac{1}{2},\ 3\frac{1}{2},\ 18\right)$.

The volume of a pyramid is ⅓ × area of base × vertical height.

By using the scalar triple product, find the volume of the pyramid. **[3]**

e) A particle, P, of mass 0.5 kg has position vector $\boldsymbol{r}$ with respect to origin O given by

$$\boldsymbol{r} = \left(t^3 - \frac{3}{2}t^2\right)\boldsymbol{i} + \left(\frac{t^4}{4} - 2t^2 - 6t\right)\boldsymbol{j} + (4t^2 + 4t)\boldsymbol{k}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are mutually perpendicular unit vectors.

Find the magnitude of the momentum of P after 4 seconds. **[4]**

Section B continues on the following page.

Question B5

a) For the complex number z, find the Cartesian equation of the locus which is defined by

$$\frac{|\,z+3-2i\,|}{|\,z-1+i\,|}=1$$

Give your answer in the form $ax+by\ +c=0$ where a, b and c are integers. **[4]**

b) The function $f(x)$ is defined as $f(x)=\ x^4-2x^3+14x^2-18x+45$.

i. Divide $f(x)$ by (x^2+9). **[3]**

ii. Hence solve $f(x)=0$. **[3]**

c) Solve $w^5=243$.

Give your solutions in the form $re^{i\theta}$. **[5]**

d) i. Prove that $\cos x=\frac{e^{ix}+\ e^{-ix}}{2}$ and $\sin x=\frac{e^{ix}-\ e^{-ix}}{2i}$. **[3]**

ii. Hence prove that $\cos^2 x+\ \sin^2 x=1$. **[2]**

Question B6

a) i. Expand and simplify $(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$. **[2]**

The quadratic equation $3x^2 - 12x + 1 = 0$ has roots α and β.

ii. Find the quadratic equation with roots α^3 and β^3.

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers. **[5]**

b) i. Use the Taylor expansion to express $(x + \frac{1}{8})^{1/3}$ in ascending powers of x up to the term in x^2. **[4]**

ii. Hence find an approximate value of $\sqrt[3]{(\frac{9}{40})}$ giving your answer in the form $\frac{m}{n}$ where m and n are integers. **[2]**

c) For the curve $y = \operatorname{sech} x$

i. Write down the equation of its asymptote. **[1]**

ii. Find the coordinates where the curve crosses the $y-$ axis. **[1]**

iii. Show that there is only one stationary value on the curve and state its coordinates. **[3]**

iv. Sketch the curve $y = \operatorname{sech} x$ (**this must not be done on graph paper**).

Show clearly the coordinates where it crosses the $y-$ axis. **[2]**

This is the end of the examination.

Blank Page