THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination 2017-18

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

The complex number z is defined as $z = p + 8i$.

a) Find the values of p if $|z| = 17$. **[2]**

The complex number w is defined as $w = 3 + 5i$.

b) Find the argument of w. Give your answer to **2** significant figures. **[3]**

In this question, 1 mark will be awarded for the correct use of significant figures.

Question A2

Matrices **M** and **N** are defined as $\mathbf{M} = \begin{bmatrix} 4 & -1 \\ 3 & 2 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} 5 & -4 \\ -6 & 5 \end{bmatrix}$

Find $\mathbf{M}^{\mathbf{T}}\mathbf{N}^{-1}$. *Each stage of your working must be clearly shown*. **[4]**

Question A3

Find the value of

$$\sum_{r=20}^{35} (3r^3 - 11).$$

*Show all working. Give your answer **in full**, with **no** rounding off.* **[5]**

Question A4

Solve the inequality

$$x + 5 < \frac{6x}{x - 2}.$$

[5]

Question A5

A ball is thrown vertically upwards at 12.25 ms^{-1} and caught at the same level from which it was thrown.

For how long is the ball in the air? **[3]**

Question A6

An ellipse has Cartesian equation

$$\frac{x^2}{144} + \frac{y^2}{81} = 1.$$

Find the coordinates of each focus and the equation of each directrix. **[3]**

Question A7

Solve the equation

$$\coth^2 x - 6\operatorname{cosech} x + 7\operatorname{cosech}^2 x = 0.$$

Give your answers in exact logarithmic form. *All working must be shown*. **[4]**

Question A8

A lorry of mass 4500 kg climbs up a smooth slope which is inclined at $\sin^{-1}\left(\frac{1}{15}\right)$ to the horizontal. The maximum speed of the lorry is 17 ms^{-1}.

Find the power output of the lorry. **[3]**

Question A9

Point A lies at $(0,\ 2, -2)$ and point B lies at $(12, -8, -6)$.

Find $\overrightarrow{OA} \times \overrightarrow{OB}$ where point O is the origin, and hence find the area of triangle OAB. **[3]**

Question A10

By differentiating a suitable number of times, find the Maclaurin expansion of

$f(x) = (1+x)^{½}$ up to the term in x^3. *Show all working*. **[3]**

Question A11

Evaluate

$$\int_{9}^{17} \frac{1}{\sqrt{(x^2 + 6x - 7)}}\ dx.$$

Give your answer as a single logarithm in exact form. *Show all working*. **[4]**

Question A12

The masses in grams of hens' eggs are assumed to follow a Normal distribution. The mean and standard deviation of the masses are unknown.

A sample of 12 eggs is selected. The mean is found to be 49 grams and the standard deviation is found to be 3 grams.

a) Find a 95% confidence interval for the mean mass of the eggs. **[2]**

The person who sells the eggs claims that the eggs have a mean mass of 50 grams.

b) Comment on this claim. **[1]**

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a)

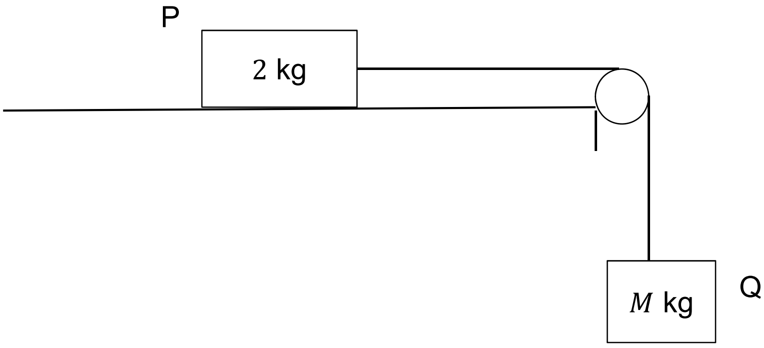

Figure 1

Figure 1 shows two blocks P and Q. P has a mass of 2 kg and rests on a rough horizontal table top. The coefficient of friction between P and the table top is $\frac{1}{2}$. A light inextensible string is attached to P which passes over a smooth pulley. Another block, Q, of mass M kg hangs freely from its end.

The system is released from rest.

i. Make a copy of the diagram and mark in all the forces acting on P and all the forces acting on Q. **[2]**

ii. Show that the acceleration of the blocks, a, is given by

$$a = \frac{g(M-1)}{M+2}$$ **[3]**

The tension in the string is 12.6 Newtons.

iii. Find the value of a. **[2]**

iv. Hence find the value of M. **[2]**

Parts b) and c) are on the next page.

Question B1 – (continued)

b)

Figure 2

Figure 2 shows two spheres A and B approaching each other. A has mass 4 kg and is travelling at u ms^{-1}. B has mass 5 kg and is travelling at v ms^{-1}.

The spheres collide.

After the collision, <u>each</u> sphere moves off in the <u>opposite</u> direction to which it was travelling before colliding. A travels at $\frac{3}{4}$ ms^{-1} and B travels at 1 ms^{-1}.

The coefficient of restitution between the spheres is $\frac{7}{20}$.

i. Find the value of u and the value of v. **[5]**

ii. Find the loss of kinetic energy in the collision. **[3]**

c)

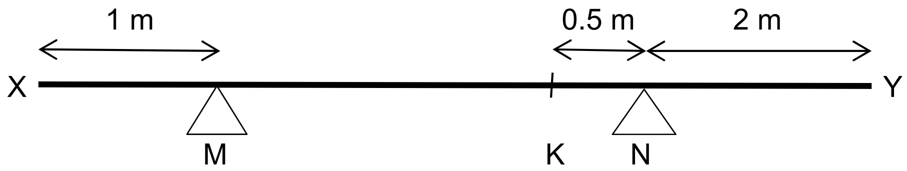

Figure 3

Figure 3 shows a uniform beam XY of mass 28 kg and length 7 metres. It rests on two supports at points M and N. M is 1 metre from X and N is 2 metres from Y. A student of mass x kg stands at point K on the beam which is between points M and N and 0.5 metres from N.

The reaction force at point M is $20g$ Newtons.

Find the value of x. **[3]**

Question B2

a) Matrix A is defined as $\mathbf{A} = \begin{bmatrix} 2 & -4 \\ -7 & 5 \end{bmatrix}$

i. Find the eigenvalues of matrix **A**. **[3]**

ii. For each eigenvalue found in part i, find a corresponding eigenvector. **[4]**

b) Find the value of a if the matrix

$$\begin{bmatrix} 0 & 2 & -3 \\ 6 & 1 & 0 \\ 0 & 3 & a \end{bmatrix}$$ has determinant -6. **[2]**

c) The second order differential equation is defined as

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5\sin x$$

i. Find the complementary function. **[2]**

ii. Find a particular integral. **[5]**

iii. Find the particular solution, given that when $x = 0,\ y = \frac{1}{2}$ and $\frac{dy}{dx} = -\frac{13}{2}$. **[4]**

Question B3

a) Point P lies at $(ap^2,\ 2ap)$ and point Q lies at $(aq^2,\ 2aq)$ on the parabola with parametric equation $y^2 = 4ax$, where p and q are parameters.

i. Find the equation of the chord PQ. **[2]**

Chord PQ and the directrix of the parabola meet on the $x -$ axis.

ii. Show that $pq = k$ where k is a constant to be determined. **[2]**

iii. <u>Derive</u> the equation of the normal at point P in terms of a and p.

All working must be shown. **[3]**

This normal, found in part iii, meets the $x -$ axis at point R.

iv. Find the distance between point R and the focus of the parabola, giving your answer in terms of a and p. **[2]**

b) Curve C has parametric equations $x = \csc\theta,\ y = \cot\theta\ \ (0 < \theta < \frac{\pi}{2})$.

i. Find the equation of the tangent to curve C when $\theta = \frac{\pi}{3}$. **[3]**

ii. This tangent passes through the focus, with positive coordinates, of the rectangular hyperbola with equation $xy = c^2$.

Find the value of c. **[3]**

iii. Find

$$\int_1^{\sqrt{3}} x^{-4}\ dy.$$

[5]

Give your answer in **exact** form.

Each stage of your working must be shown. An answer, even the correct one, will receive no marks if this working is not shown.

Question B4

a) A curve has equation $y = 3\cosh\left(\frac{x}{3}\right)$.

Find the length of arc between $x = \ln 8$ and $x = \ln 27$.

Each stage of your working must be shown. An answer, even the correct one, will receive no marks if this working is not shown. **[5]**

b) By using exponentials, show that if $y = \coth x$, $\frac{dy}{dx} = -\operatorname{cosech}^2 x$. **[4]**

c) Plane Π_1 has equation $\boldsymbol{r}.(8\boldsymbol{i} - 9\boldsymbol{j} + 12\boldsymbol{k}) = 17$.

Plane Π_2 has equation $\boldsymbol{r}.(-6\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 28$.

i. Find the shortest distance from the origin to plane Π_1. **[3]**

ii. Find the **obtuse** angle between plane Π_1 and plane Π_2 . **[3]**

d) A particle starts from point O and its acceleration, $\boldsymbol{a}$, after t seconds is given by

$$\boldsymbol{a} = 4t^3\boldsymbol{i} - 15t^2\boldsymbol{j} + 6t\boldsymbol{k}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are mutually perpendicular vectors.

i. After 1 second the particle has velocity $(2\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$.

Find an expression for the velocity in terms of t. **[3]**

ii. Find the magnitude of the velocity after 2 seconds. **[2]**

Question B5

a) The complex numbers z_1 and z_2 are defined as $z_1 = p + qi$ and $z_2 = 1 + 2i$.

Find the value of p and the value of q if $z_1 \div z_2 = 3 + 5i$. **[3]**

b) Find the Cartesain equation of the locus of points represented by

$$|w + 1| = |w + i|$$

Give your answer in its simplest form. **[2]**

c) Solve the equation

$$9x^4 + 13x^2 + 4 = 0.$$ **[4]**

d) Use De Moivre's Theorem to express $\sin 5\theta$ in terms of $\sin \theta$.

All working must be shown. **[4]**

e) i. Solve the equation

$$z^3 = -27.$$

Give your solutions in Cartesian form. **[5]**

ii. Display your solutions on an Argand diagram. **[2]**

Question B6

a) The quadratic equation $4x^2 + kx + 5 = 0$ has roots α and β.

Your are given $\alpha^2 + \beta^2 = \frac{13}{2}$.

i. Find the value of k, given that k is positive. **[3]**

ii. Find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers. **[2]**

b) i. Use the Taylor expansion to express $\tan(x + \frac{\pi}{4})$ in ascending powers of x up to the term in x^2.

<u>Each</u> stage of your working must be clearly shown. **[4]**

ii. Hence find an approximate value of $\tan 47°$ giving your answer in terms of π. **[3]**

c) The curve C has equation

$$y = \frac{-1}{1+x}.$$

i. Write down the equations of the asymptotes of curve C. **[2]**

ii. State the coordinates where curve C crosses the $y -$ axis. **[1]**

iii. Confirm that curve C has no stationary values. **[2]**

iv. Sketch curve C (**this must not be done on graph paper**).

Show clearly the asymptotes and where the curve crosses the $y -$ axis. **[3]**

This is the end of the examination.

Blank Page