NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A4. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

$z^* = 8 - 6i$ (seen or implied) **[M1]**

Multiplies correctly **[M1]**

$28 - 96i$ **[A1]**

Question A2

Multiplies both sides by x^2 **[M1*]**

Reaches a cubic inequality in the form of a product of a linear expression and a quadratic expression $[x(x^2 - 2x - 3) > 0]$ **[M1*]**

Finds 3 critical values $(-1,\ 0,\ 3)$ **[M1]**

$x > -1 \quad x < 0$ (or $-1 < x < 0$) **(A1)** $\quad x > 3$ **(A1)** **[A2]**

Please note: the second and third ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen. The first and second ranges, if not given in the form $-1 < x < 0$, can be separated by a space or a comma; but in this case the word 'and' is also permitted and the word 'or' will lose the final mark [but not if it has already been lost earlier i.e. a maximum of 1 mark is lost overall].

Special case If the candidate multiplies through by x and obtains critical values 3 and -1, the only mark available is the second M mark.

Question A3

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 32, \quad \frac{1}{\alpha^2\beta^2} = 4$ and finds a value for $\alpha\beta$ $\quad [\alpha^2\beta^2 = \frac{1}{4}$ so $\alpha\beta = \frac{1}{2}]$. **[M1]**

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = 32$ **[M1]**

Substitutes their value of $\alpha\beta$ into expression and finds a value for $(\alpha + \beta)$ $[= 3]$ **[M1]**

$x^2 - 3x + \frac{1}{2} = 0$ or any equivalent form. **[A1]**

Question A4

Uses a suitable equation of motion and forms a quadratic equation in t. **[M1]**
$[250 = 7t + \frac{1}{2} \times 2 \times t^2$ giving $t^2 + 7t - 250 = 0]$

Solves **[M1]**

$t = 12.6941 \ldots$ (ignore any negative answers) [this can be implied] **[A1]**

$= 12.7$ (seconds) to 3 significant figures (Allow follow through) **[A1ft]**

Question A5

a) $\frac{dy}{dx} = \frac{12t^2}{3}$ **[M1]**

$y - (4t^3 + 1) = 4t^2[x - (3t - 2)]$ or equivalent **[A1]**

b) $t = \frac{x+2}{3}$ **[M1]**

$y = 4(\frac{x+2}{3})^3 + 1$ or equivalent **[A1]**

Question A6

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ **[M1*]**

Substitutes $x = \ln(\frac{1}{2})$ into their expression **[M1*]**

Obtains a numerical answer $[\frac{\frac{1}{2} - 2}{\frac{1}{2} + 2} = \cdots = -\frac{3}{5}]$ and squares **[M1*]**

$= \frac{9}{25}$ **[A1]**

Question A7

Takes moments about a point and states clockwise [If about A: $5gx$] **[M1]**

States anti-clockwise [if about A: $44.1 \times 4\sin 30°$] and sets equal to clockwise **[M1]**

$x = 1.8$ (metres) **[A1]**

Question A8

Finds the modulus and argument of z $[4 \text{ and } -\frac{\pi}{6}]$ **[M1*]**

Takes their modulus to power 4 and multiplies their argument by 4 $[256, -\frac{4\pi}{6}]$ **[M1*]**

Attempts to convert back to Cartesian form. **[M1*]**

$-128 - 128\sqrt{3}i$ or any equivalent form. **[A1]**

or: keeps z in Cartesian form and squares it **(M1*)**; squares their answer **(M1*)**

Correct real part **(A1)**; correct imaginary part **(A1)** in required form.

Question A9

Divides by x and identifies the correct integrating factor for their expression $[e^{\int \frac{5}{x}dx}]$ **[M1]**

Multiplies through by their integrating factor and simplifies $[\ \frac{d}{dx}(x^5y) = x^3\ ]$ **[M1]**

Integrates both sides $[\ x^5y = \frac{1}{4}x^4 + c$ and attempts to find a value for c $[= 0]$ (if the constant is missed, this mark is lost). **[M1]**

$y = \frac{1}{4x}$ or equivalent. [This mark is lost if the constant has been missed] **[A1]**

Question A10

Writes $\int_0^2 e^{3x}\left(\frac{e^x - e^{-x}}{2}\right)dx.$ **[M1*]**

Integrates their expression $[\frac{1}{2}(\frac{1}{4}e^{4x} - \frac{1}{2}e^{2x})]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{1}{2}(\frac{1}{4}e^8 - \frac{1}{2}e^4 + \frac{1}{4})$ or equivalent, or anything rounding to 359 **[A1]**

or integrates by parts **(M1*)**; integrates by parts a second time **(M1*)**

Makes original integral the subject of the expression: substitutes limits into rest of expression and subtracts the right way round **(M1)**

Correct answer (may be in form $\frac{e^6\cosh 2 - 3e^6\sinh 2 - 1}{-8}$) **(A1)**

Question A11

Differentiates $\boldsymbol{v} = 3t^2\boldsymbol{i} - 6t\boldsymbol{j} + 48\boldsymbol{k}$ **[M1*]**

Substitutes $t = 4$ into their differentiated expression and finds magnitude **[M1]**

$|\boldsymbol{v}| = 72$ (ms^{-1}) **[A1]**

Question A12

a) $115 \pm \frac{20 \times 2.33}{\sqrt{64}}$ (2.3263 may be seen instead of 2.33) **[M1]**

$109 - 121$ (anything rounding to these limits) **[A1]**

b) Large sample used so, (by Central Limit Theorem,) assumption is that mean follows a Normal distribution [or words to this effect]. **[B1]**

Section B

Question B1

a) i. On P: $5g\sin\theta$ down slope and $5g\cos\theta$ into slope [or, instead of these forces, a single vertical force of $5g$]; frictional force up slope, reaction at right-angles to slope, tension in string. On Q: $2g$ downwards; tension in string upwards. [Using $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ is fine].

Any 3 forces correct **(B1)**; 4 correct **(B2)**; all correct **(B3)**. **[B3]**

ii. On Q: $T - 2g = 2 \times 0.5$ **[M1]**

$T = 20.6$ (Newtons) or anything rounding to this. **[A1]**

iii. On P: $5g\sin\theta - \mu \times 5g\cos\theta - T = 5 \times 0.5$ **[M1]**

$\mu = \frac{9}{56}$ or anything rounding to 0.16 **[A1]**

iv. Finds the speed of Q when the string breaks (1.5 ms^{-1}) **[M1]**

Finds the distance travelled by Q before coming to rest **[M1]**

Anything rounding to 0.11 (metres) **[A1]**

b) i. Finds momentum before collision [$5M - 2(M+2)$] **[M1]**

Finds momentum after collision [$2(M+2)$], sets expressions equal to each other and finds a value for M. **[M1]**

$M = 8$ **[A1]**

ii. $e = \frac{2}{7}$ or anything rounding to 0.286 **[B1]**

iii. Finds kinetic energy before and after impact, and subtracts
$[\frac{1}{2} \times \text{their } M \times 5^2 + \frac{1}{2} \times \text{their } (M+2) \times 2^2 - \frac{1}{2} \times \text{their } (M+2) \times 2^2]$ **[M1]**

= 100 (Joules) **[A1]**

c) i. $3000 \times \sin\phi \times 20$ **[M1]**

= 39200 (Watts) **[A1]**

ii. $80 \times \sin\phi \times 3000 \times 9.8$ **[M1]**

156800 (Joules) [Accept anything rounding to 157000] **[A1]**

Question B2

a) i. $\begin{vmatrix} 2-\lambda & -1 & 3 \\ 0 & 2-\lambda & 3 \\ 0 & 2 & 1-\lambda \end{vmatrix} = 0$ **[M1*]**

$(2-\lambda)[(2-\lambda)(1-\lambda)-6]$ **(M1*)** $[-(-1)\times 0-3\times 0]$ **(M1)+** **[M2]**
+ can be implied

Simplifies to form a product of a quadratic expression and a linear expression in λ $[(2-\lambda)(\lambda^2-3\lambda-4)(=0)]$ **[M1]**

$\lambda = -1,\ 2,\ 4.$ **[A1]**

ii. $\lambda = -1$: $\begin{aligned} 2x - y + 3z &= -x \\ 2y + 3z &= -y \\ 2y + z &= -z \end{aligned}$ **[M1]**

Eigenvector is $(4a\boldsymbol{i} + 3a\boldsymbol{j} - 3a\boldsymbol{k})$ **[A1]**

$\lambda = 2$: $\begin{aligned} 2x - y + 3z &= 2x \\ 2y + 3z &= 2y \\ 2y + z &= 2z \end{aligned}$ **[M1]**

Eigenvector is $(a\boldsymbol{i})$ **[A1]**

$\lambda = 4$: $\begin{aligned} 2x - y + 3z &= 4x \\ 2y + 3z &= 4y \\ 2y + z &= 4z \end{aligned}$ **[M1]**

Eigenvector is $(3a\boldsymbol{i} + 6a\boldsymbol{j} + 4a\boldsymbol{k})$ **[A1]**

Part b) is on the next sheet.

Question B2 – (continued)

b) i. Forms and solves an auxiliary equation $[m^2 - 5m + 6 = 0]$ **[M1]**

$y = Ae^{2x} + Be^{3x}$ **[A1]**

ii. Takes $y = ke^x$ and differentiates $\frac{dy}{dx} = \frac{d^2y}{dx^2} = ke^x$ **[M1]**

Substitutes into equation and finds a value for k $[= \frac{1}{2}]$ **[M1]**

$y = \frac{1}{2}e^x$ **[A1]**

iii. $y = Ae^{2x} + Be^{3x} + \frac{1}{2}e^x$

Substitutes $x = 0$ and $y = 1$ $\quad [\,1 = A + B + \frac{1}{2}]$ **[M1]**

$\frac{dy}{dx} = 2Ae^{2x} + 3Be^{3x} + \frac{1}{2}e^x$

Substitutes $x = 0$ and $\frac{dy}{dx} = 1$ $\quad [\,1 = 2A + 3B + \frac{1}{2}]$ **[M1]**

Solves equations $[\,A = 1;\ B = -\frac{1}{2}]$ **[M1]**

$y = e^{2x} - \frac{1}{2}e^{3x} + \frac{1}{2}e^x$ **[A1]**

Question B3

a) i. $\frac{dy}{dx} = \frac{a\cos\theta}{-a\sin\theta}$ $(= -\cot\theta)$ **[M1]**

Substitutes $\theta = \frac{\pi}{3}$, inverts and changes sign $(\sqrt{3})$ **[M1]**

Finds coordinates of point A when $\theta = \frac{\pi}{3}$ $[\left(\frac{1}{2}a, \frac{\sqrt{3}}{2}a\right)]$ **[M1]**

$y = \sqrt{3}x.$ **[A1]**

ii. $\frac{dy}{dx} = -\cot\theta$ and substitutes $\theta = \frac{\pi}{4}$ (-1) **[M1]**

Finds coordinates of point B when $\theta = \frac{\pi}{4}$ $[\left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)]$ **[M1]**

$y = -x + \sqrt{2}a$ or equivalent **[A1]**

iii. Sets RHS of equations equal to each other **[M1]**

$x -$ coordinate is $\frac{a\sqrt{2}}{1 + \sqrt{3}}$ **[A1]**

$y -$ coordinate is $\frac{a\sqrt{6}}{1 + \sqrt{3}}$ (Allow ft from their $x -$ coordinate) **[A1ft]**

iv. $x^2 + y^2 = a^2$ or equivalent **[B1]**

v. It is a circle. **[B1]**

b) i. Changes $dx\ to\ \frac{dx}{dt}\ dt$ **[M1*]**

Changes limits and writes integrand in terms of t $[\int_1^2 t(2t+1)\,dt\,]$ **[M1*]**

Integrates and applies limits correctly [evidence of integration only needs to be seen] **[M1*]**

$= \frac{37}{6}$ or anything rounding to 6.17 **[A1]**

ii. Finds coordinates of P $[(2,\ 1)]$ and finds the gradient of the perpendicular to OP (-2) **[M1]**

$y - 1 = -2(x - 2)$ **[A1]**

iii. Realises that directrix is $x = -2$ and substitutes into equation **[M1]**

R lies at $(-2,\ 9)$. **[A1]**

Question B4

a) $3\left(\frac{e^x + e^{-x}}{2}\right) - e^x = 2$ **[M1*]**

Reaches a quadratic equation in e^x [$e^{2x} - 4e^x + 3 = 0$] **[M1*]**

Solves [$e^x = 3 \text{ or } 1$] **[M1]**

$x = 0 \text{ or } \ln 3$ [Do not accept $\ln 1$] **[A1]**

b) i. Writes integral as $\int_{-3}^{1} \frac{1}{(x+3)^2 + 16}\ dx$ **[M1*]**

Integrates [$\frac{1}{4}\tan^{-1}\left(\frac{x+3}{4}\right)$] **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{\pi}{16}$ or anything rounding to 0.196 **[A1]**

ii. Uses $s = 2\pi \int_{6}^{12} y\sqrt{[1 + (\frac{dy}{dx})^2]}\, dx$ with $y = \frac{5}{12}x$ and $\frac{dy}{dx} = \frac{5}{12}$. **[M1*]**

Simplifies and integrates [$2\pi \int_{6}^{12} \frac{65}{144}x\ dx = 2\pi \times \frac{65x^2}{288}$] **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{195\pi}{4}$ or equivalent, or anything rounding to 153. **[A1]**

c) i. Finds $\boldsymbol{a} \times \boldsymbol{b}$ [$-7\boldsymbol{i} + 13\boldsymbol{j} + 17\boldsymbol{k}$] **[M1]**

Finds their $(\boldsymbol{a} \times \boldsymbol{b}).(2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k})$ **[M1]**

= -9 **[A1]**

ii. Acceleration $= 35 \div 5 = 7$ (ms^{-2}) so $6^2 + (-2)^2 + p^2 = 7^2$ **[M1]**

$p = \pm 3$ **[A1]**

iii. $\boldsymbol{r} = \boldsymbol{i}(2 + 3t) + \boldsymbol{j}(-1 - 2t) + \boldsymbol{k}(4 - t)$ **[M1]**

Substitutes into equation and finds a value for t (-5) **[M1]**

Coordinates are $(-13,\ 9,\ 9)$ [Accept if in vector form] **[A1]**

Question B5

a) i. If there are no real roots, there are three complex roots and this is not possible as they must occur in pairs **or** draws sketches showing that the curve has to cross the $x-$ axis at least once **or** any other valid explanation. **[B1]**

ii.

$$\begin{array}{r|l} & 4x^2 + 5 \\ 3x-2 & 12x^3 - 8x^2 + 15x - 10 \\ & 12x^3 - 8x^2 \\ \hline & \qquad\qquad 15x - 10 \\ & \qquad\qquad 15x - 10 \\ \hline & \qquad\qquad \ldots\ldots\ldots \end{array}$$

At least one correct division **[M1]**

Correct quotient **[A1]**

iii. $x = \frac{2}{3}, \quad \pm\frac{\sqrt{5}}{2}i$

Any two correct **[B1]**
All 3 correct **[B1]**

b) i.

$(x+4)^2 + (y-3)^2 = 49$

LHS correct **[M1]**
All correct **[A1]**

ii.

Inside of a circle **(B1)** centre (-4, 3) and radius 7 **(B1)** excluding boundary **(B1)**. **[B3]**

Part c) is on the next sheet.

Question B5 – (continued)

c) i.

Please note: this is a 'show that' question so all working must be seen.

[M1*]

Writes $z^n = \cos n\theta + i\sin n\theta$ and $z^{-n} = \cos n\theta - i\sin n\theta$, then adds and subtracts to obtain results.

Substitutes $n = 1$ into second result and completes proof. **[A1]**

ii. $z^4 + 4z^3\left(-\frac{1}{z}\right) + 6z^2(-\frac{1}{z})^2 + 4z(-\frac{1}{z})^3 + (-\frac{1}{z})^4$ **[M1]**

[A1]

$= z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$

iii. $(z - \frac{1}{z})^4 = \left(z^4 + \frac{1}{z^4}\right) - 4\left(z^2 + \frac{1}{z^2}\right) + 6$ and $(z - \frac{1}{z})^4 = 16\sin^4\theta$ **[M1]**

$16\sin^4\theta = 2\cos 4\theta - 4(2\cos 2\theta) + 6$ **[M1]**

$\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$ **[A1]**

iv. Uses previous result **[M1*]**

Integrates and applies limits $[\frac{1}{32}\sin 4\theta - \frac{1}{4}\sin 2\theta + \frac{3}{8}\theta]$ **[M1*]**

(Only evidence of integration has to be seen for this mark)

$= \frac{3}{4}\pi$ or equivalent but must be exact. **[A1]**

Question B6

a) i. $f(x) = (1+x)^{1/3}$; $f'(x) = \frac{1}{3}(1+x)^{-2/3}$; $f''(x) = -\frac{2}{9}(1+x)^{-5/3}$ **[M1*]**

$f(0) = 1$; $f'(0) = \frac{1}{3}$; $f''(0) = -\frac{2}{9}$ **[M1]**

$(1+x)^{1/3} \approx 1 + \frac{1}{3}x - (\frac{2}{9} \div 2!)x^2$ (No need to simplify) **[A1]**

ii. Substitutes $x = -\frac{1}{10}$ into their expression **[M1]**

$= \frac{869}{900}$ **[A1]**

iii. It must be small (or words to this effect) **[B1]**

b) $11\left[\frac{n}{2}(n+1)\right] = \frac{n}{6}(n+1)(2n+1)$ **[M1*]**

$\frac{11n}{2}(n+1) - \frac{n(n+1)(2n+1)}{6} = 0$ giving $n(n+1)\left[\frac{11}{2} - \frac{2n+1}{6}\right] = 0$ (at least two lines of intermediate working) **[M1*]**

Finds a value of n **[M1]**

$n = 16$ **[A1]**

c) i. $x = -1$; $y = 1$ **(B1)** for each **[B2]**

ii. $(0, -3)$; $(3, 0)$ **(B1)** for each **[B2]**

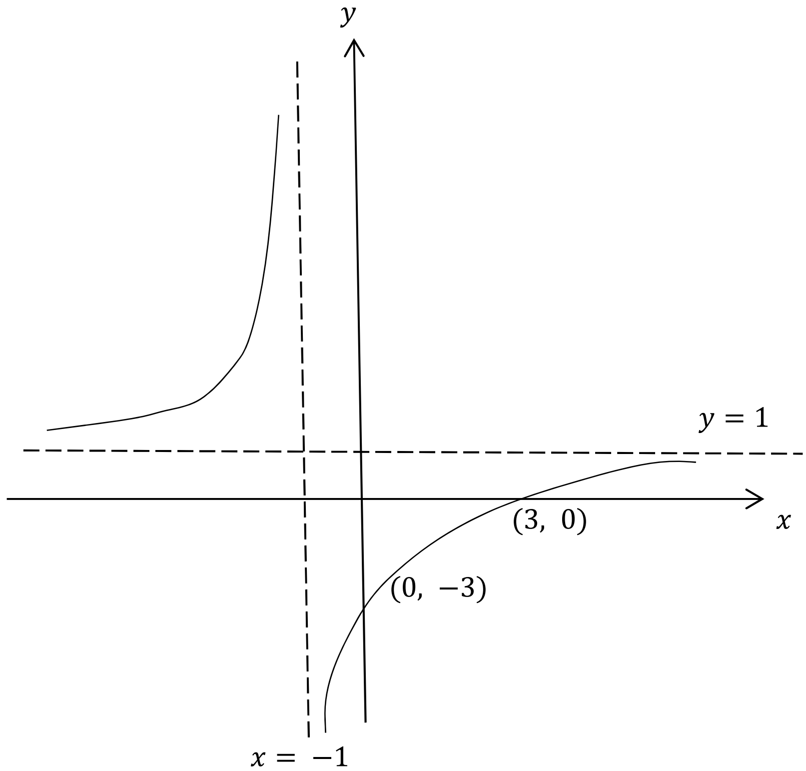
iii. Differentiates using the Quotient Rule or Product Rule **[M1]**

$\frac{dy}{dx} = \frac{4}{(x+1)^2}$ **[A1]**

This is never 0, so there are no stationary values (or similar words) (Allow follow through provided their $\frac{dy}{dx}$ is never 0). **[A1ft]**

Sketch is on next page.

iv.



(B1) for the asymptotes and coordinates

(B1) for one section of the curve. **(B1)** for second correct section. **[B3]**

<u>Please note</u>: if a candidate uses graph paper, apply no penalty if the graph is clearly a sketch. If there is evidence of plotting points, then take off one mark at the end.

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