THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM002 Further Mathematics Examination 2017-18

Examination Session
Semester Two

Time Allowed
2 Hours 40 minutes
(including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 45 marks.**
SECTION B	**Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

The complex number z is such that $z = 8 + 6i$. Find $(z^*)^2$ where z^* is the complex conjugate of z.

Give your answer in Cartesian form. **[3]**

Question A2

Solve the inequality

$$x - 3 > \frac{3 - x}{x}.$$

[5]

Show all working.

Question A3

The roots of the quadratic equation $x^2 - 32x + 4 = 0$ are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ $(\alpha,\ \beta > 0)$.

Find the quadratic equation with roots α and β. **[4]**

Question A4

A particle has an initial speed of 7 ms^{-1} and accelerates at 2 ms^{-2}.

How long does it take to cover 250 metres? Give your answer to **3** significant figures.

[4]

In this question, 1 mark will be given for the correct use of significant figures.

Question A5

You do not have to simplify your answers in this question.

A curve has parametric equations $x = 3t - 2$ and $y = 4t^3 + 1$

a) Find, in terms of t, the equation of a tangent to curve C. **[2]**

b) Find a Cartesian equation of curve C. **[2]**

Question A6

By using exponentials, find the value of $\tanh^2 [\ln(\frac{1}{2})]$.

Give your answer in the form $\frac{m}{n}$ where m and n are integers.

Each stage of your working must be shown. Just giving the answer, even the correct one, will receive no marks if this working is not seen. **[4]**

Question A7

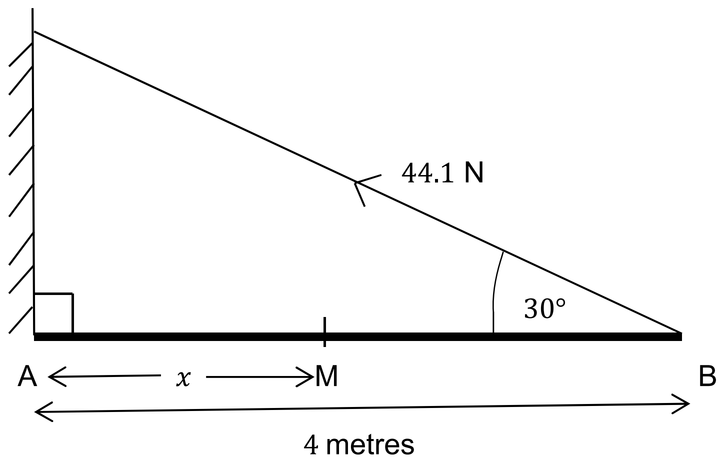

Figure 1

Figure 1 shows a non-uniform rod, AB, of mass 5 kg and length 4 metres. A is attached to a vertical wall by a smooth hinge and AB is perpendicular to the wall. A length of wire is attached to the rod at B and the wire makes an angle of $30°$ to the rod. The other end of the wire is fixed to the wall directly above A. The tension in the wire is 44.1 Newtons.

The centre of mass of rod AB is at point M which is x metres from A.

Find the value of x. **[3]**

Question A8

The complex number z is defined as $z = \sqrt{12} - 2i$.

Express z^4 in the form $a + bi$ where a is an integer and b is a number in surd form. *Show all working*. **[4]**

Question A9

Solve the differential equation

$$x\,\frac{dy}{dx} + 5y = \frac{1}{x}$$

given $y = \frac{1}{8}$ when $x = 2$. Write your answer in the form $y = f(x)$. **[4]**

Question A10

Evaluate

$$\int_0^2 e^{3x}\sinh x\;dx.$$

Each stage of your working must be clearly shown. An answer, even the correct one, will receive no marks if this working is not seen. **[4]**

Question A11

The displacement, s metres, of a particle from the origin O after t seconds is given by

$$s = t^3\boldsymbol{i} - 3t^2\boldsymbol{j} + 48t\boldsymbol{k}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are mutually perpendicular vectors.

Find the magnitude of the velocity after 4 seconds. **[3]**

Question A12

The standard deviation of the masses of a certain type of starfish is 20 grams. In a sample of 64 of this type of starfish, the mean mass was found to be 115 grams.

a) Find a 98% confidence interval of the mean masses of all the starfish. **[2]**

b) State what assumption you have made and why you can make it. **[1]**

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a)

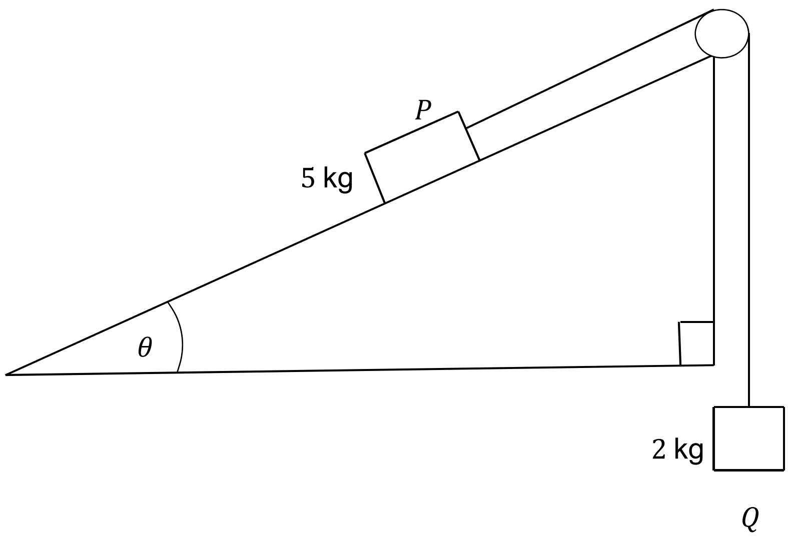

Figure 2

Figure 2 shows a block, P, of mass 5 kg on a rough slope which is inclined at $\theta°$ to the horizontal, where $\tan\theta = \frac{3}{4}$. A light inextensible string is attached to P which passes over a smooth pulley. Another block, Q, of mass 2 kg hangs freely from the end of the string.

The system is released from rest and P moves down the slope.

i. Make a copy of the diagram and mark in all of the forces acting on P and Q. **[3]**

The acceleration of the blocks is 0.5 ms^{-2}.

ii. Find the tension in the string. **[2]**

iii. Find the coefficient of friction between the slope and block P. **[2]**

iv. After 3 seconds the string breaks. Find how far Q will continue to travel upwards before coming to rest. **[3]**

Parts b) and c) are on the next page.

Question B1 – (continued)

b)

Figure 3

Figure 3 shows two spheres, X and Y, approaching each other on a smooth horizontal surface. X has mass M kg and is travelling at 5 ms^{-1}. Y has mass $(M + 2)$ kg and is travelling at 2 ms^{-1}.

The spheres collide. After the collision X is brought to a standstill, and Y goes off at 2 ms^{-1} in the opposite direction to which it was travelling before the collision.

i. Find the value of M. **[3]**

ii. Find the coefficient of restitution between the two spheres. **[1]**

iii. Find the loss of kinetic energy in the collision. **[2]**

c) A van of mass 3000 kg travels up a smooth slope which is inclined at $\phi°$ to the horizontal where $\sin\phi = \frac{1}{15}$.

The van travels at a constant speed of 20 ms^{-1}.

i. Find the power output of the van's engine. **[2]**

ii. Find the gain in potential energy of the van when it travels 80 metres up the slope. **[2]**

Question B2

a) Matrix **A** is defined as $\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$

i. Find the eigenvalues of matrix **A**. **[5]**

ii. For each of the eigenvalues found in part i, find a corresponding eigenvector. **[6]**

b) The second order differential equation is defined as

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$$

i. Find the complementary function. **[2]**

ii. Find a particular integral. **[3]**

iii. Find the particular solution given that when $x = 0,\ y = \frac{dy}{dx} = 1.$ **[4]**

Question B3

a) An ellipse has parametric equations $x = a\cos\theta,\ \ y = a\sin\theta$.

Point A lies on the ellipse where $\theta = \frac{\pi}{3}$ and point B lies on the ellipse where $\theta = \frac{\pi}{4}$.

i. Find the equation of the line l_1 which is the normal to the ellipse at point A. Give your answer in the form $y = f(x)$. **[4]**

ii. Find the equation of the line l_2 which is the tangent to the ellipse at point B. Again, give your answer in the form $y = f(x)$. **[3]**

iii. The lines l_1 and l_2 meet at point C.

Find the coordinates of point C. Give each coordinate in the form $\frac{a\sqrt{m}}{1+\sqrt{n}}$ where m and n are integers. **[3]**

iv. Write down the Cartesian equation of the ellipse. **[1]**

v. Write down the name for this type of ellipse. **[1]**

b) A curve has parametric equations $x = t^2 + t, \quad y = \sqrt{t} \quad (t > 0)$.

i. Find

$$\int_2^6 y^2 \ dx.$$

Each stage of your working must be shown. An answer, even the correct one, will receive no marks if this working is not shown. **[4]**

Point P lies on the curve when $t = 1$, and both of the coordinates of point P are positive.

ii. Find the equation of the line which is perpendicular to the line OP (where O is the origin) and passes through point P. **[2]**

The equation of the line found in part ii meets the directrix of the parabola with Cartesian equation $y^2 = 8x$ at point R.

iii. Find the coordinates of point R. **[2]**

Question B4

a) Solve the equation $3\cosh x - e^x = 2$. *All working must be shown*. **[4]**

b) i. Evaluate

$$\int_{-3}^{1} \frac{1}{x^2 + 6x + 25}\ dx.$$ **[4]**

Each stage of your working must be shown.

The section of the line $y = \frac{5}{12}x$ between the points with x −coordinates 6 and 12 is rotated completely about the $x -$ axis.

ii. Use integration to find the area of the surface which is formed.

All working must be shown. **[4]**

c) i. Vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined as $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = 5\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = 2\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$.

Find $(\boldsymbol{a} \times \boldsymbol{b}) \, . \, \boldsymbol{c}$ **[3]**

ii. A force, $\boldsymbol{F}$, when applied to a particle of mass 5 kg produces an acceleration of $(6\boldsymbol{i} - 2\boldsymbol{j} + p\boldsymbol{k})$ ms^{-2} where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are mutually perpendicular vectors.

Find the two possible values of p if the magnitude of force $\boldsymbol{F}$ is 35 N. **[2]**

iii. Find the coordinates where the line with equation $\boldsymbol{r} = (2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) + t(3\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k})$ (where t is a scalar) meets the plane with equation $4x - 3y + 5z + 34 = 0$. **[3]**

Question B5

a) The function $f(x)$ is defined as $f(x) = 12x^3 - 8x^2 + 15x - 10$.

i. Explain why it is impossible for the equation $f(x) = 0$ to have no real roots. **[1]**

ii. Divide $f(x)$ by $(3x - 2)$. **[2]**

iii. Find all the roots of $f(x) = 0$. **[2]**

b) i. For the complex number w, find the Cartesian equation of the locus represented by $|\, w + 4 - 3i \,| = 7$. **[2]**

ii. Describe the region satisfied by $|\, w + 4 - 3i \,| < 7$. **[3]**

c) i. You are given that $z = \cos\theta + i\sin\theta$.

Show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$; and that hence $z - \frac{1}{z} = 2i\sin\theta$. **[2]**

ii. Expand $(z - \frac{1}{z})^4$. Simplify your answer. **[2]**

iii. Use your results to express $\sin^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$ where $a,$ b and c are rational numbers. **[3]**

iv. Hence find the exact value of

$$\int_0^{2\pi} \sin^4\theta \; d\theta.$$

All working must be shown. **[3]**

Question B6

a) i. By differentiating a suitable number of times, obtain the Maclaurin expansion for $(1+x)^{1/3}$ up to the term in x^2. **[3]**

ii. Hence find an approximate value of $\sqrt[3]{0.9}$

Give your answer in the form $\frac{m}{n}$ where m and n are integers. **[2]**

iii. If an accurate estimate of $f(x)$ is needed in a Maclaurin expansion, what condition must apply to the value of x? **[1]**

b) Find the value of n $(n > 0)$ for which

$$\sum_{r=1}^{n} 11r = \sum_{r=1}^{n} r^2$$ **[4]**

Each stage of your working must be clearly shown.

c) A curve C has equation

$$y = \frac{x-3}{x+1}$$

i. Write down the equations of the asymptotes of curve C. **[2]**

ii. Find where curve C crosses the $x-$axis and the $y-$axis. **[2]**

iii. Investigate whether curve C has any stationary values. **[3]**

iv. Sketch curve C (**this must not be done on graph paper**).

Show clearly the asymptotes and the coordinates where the curve crosses the $x-$axis and the $y-$axis. **[3]**

This is the end of the examination.

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