NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM001 Further Mathematics Examination

2013-14

Examination Session	**Time Allowed**
Semester Two	3 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet and graph paper.

- **Answers must not be written during the first 10 minutes.**

- Write your NCUK ID Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No electronic devices** (e.g. mobile phones, tablets, iPads) are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Show **ALL** workings in your answer booklet. Marks will be awarded for correct workings.
- Examination materials must not be removed from the examination room.
- Write your name and candidate number on all loose sheets/diagrams.

Section A

Answer ALL questions. This section carries 40 marks.

Question A1

The complex numbers z_1 and z_2 are given by

$z_1 = -9 + 2i, \;\; z_2 = 7 - 6i$

a) Find the values of $z_1 - z_2$ and $|z_1 + z_2|$ **[2]**

b) Show that the real part of the complex number $\frac{z_1 - z_2}{z_1 + z_2}$ is equal to 0. **[3]**

Question A2

Solve the inequality $2x - 2 > \frac{13x-10}{x+3}$ **[5]**

Question A3

A non-uniform ladder *AB* of length $3a$ and mass 5kg has its centre of mass at *G* where *AG* = a. The ladder rests in limiting equilibrium with *B* against a smooth vertical wall and *A* resting on a rough horizontal floor. The coefficient of friction between the floor and the ladder is $\frac{3}{10}$. If *AB* makes an angle of θ with the horizontal, calculate θ. **[5]**

Question A4

Find the particular solution of the differential equation

$(x+3)\frac{\mathrm{dy}}{\mathrm{dx}} + 2y = 2x$, given that $x = 0$ when $y = 1$

Write your answer in the form $y = f(x)$ **[5]**

Question A5

a) Find the value of **[2]**

$$\sum_{r=1}^{r=n}(3r-2)$$

Simplify your answer.

b) Find the smallest value of n such that **[3]**

$$\sum_{r=n+1}^{r=3n}(3r-2) > 1000$$

Question A6

A smooth sphere A of mass 4 kg is travelling in a straight line on a smooth horizontal surface with speed 6 ms^{-1} when it collides with a smooth sphere B of mass 2 kg which is moving in the same direction with speed 3 ms^{-1}. The coefficient of restitution between the spheres is equal to $\frac{1}{2}$.

a) Find the speeds of A and B after the collision. **[4]**

b) Find the magnitude of the impulse on B due to the collision. **[1]**

Question A7

The plane Π has equation

$$\boldsymbol{r} = 2\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}) + \mu(-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$$

The line L has equation

$$\boldsymbol{r} = 3\boldsymbol{i} - \boldsymbol{k} + t(2\boldsymbol{j} + 3\boldsymbol{k})$$

a) Find the coordinates of the point of intersection of Π and L. **[4]**

b) Find the distance of Π from the origin. **[1]**

Section A continues on the following page.

Question A8

Find the value of

$$\int_2^3 \frac{dx}{\sqrt{9x^2+4}}$$

Give your answer to **four** significant figures. **[5]**

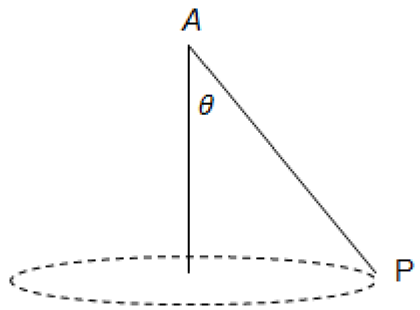
In this question, 1 mark will be awarded for the correct use of significant figures.

Section B

Answer 4 questions. This section carries 60 marks.

Question B1

a)



The diagram above shows a conical pendulum which consists of a light inextensible string with one end attached to a fixed point *A*. A particle P, of mass 2kg, is attached to the other end of the string and moves in a horizontal circular path at a constant speed, completing 2 full revolutions per second. The tension in the string is 8g N and the string makes an angle θ with the vertical.

i. Show that $\cos\theta = \frac{1}{4}$ **[2]**

ii. Find the radius of the circular path of P. **[3]**

iii. Find the speed of P. **[2]**

b) i. A car is travelling in a circle of radius 20m on a rough horizontal surface. The coefficient of friction between the tyres of the car and the surface is $\frac{2}{5}$. Find the maximum speed of the car given that it does not slide. Take g = 9.8 ms^{-2}. **[3]**

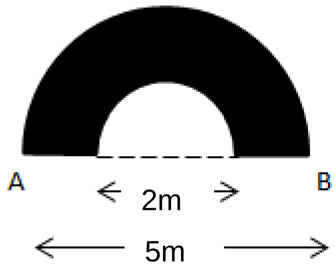
ii. The car now travels in a circle of radius r metres on a flat rough surface which is inclined to the horizontal at an angle of α degrees. The coefficient of friction between the tyres and the surface is μ. Show that the maximum speed v ms^{-1}, that the car can reach without sliding up the plane is given by **[5]**

$$v = \sqrt{\frac{rg(\mu + tan\alpha)}{1 - \mu tan\alpha}}$$

Section B continues on the following page.

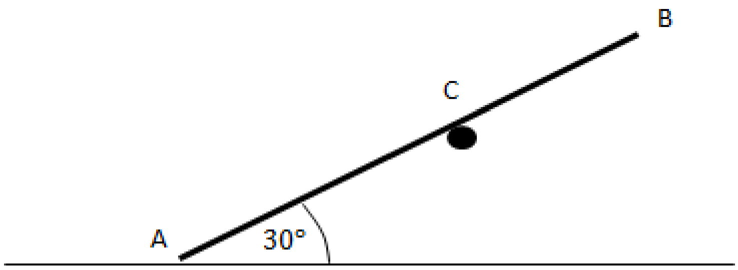
Question B2

a)



The diagram above shows a uniform semi-circular lamina of diameter 5m from which a smaller semi-circular piece of diameter 2m has been removed. The smaller semi-circle has the same centre as the larger semi-circle. Find the distance of the centre of mass of the lamina from the diameter AB. **[5]**

b)



The diagram above shows a uniform rod AB of mass 4kg and length 5m resting in a vertical plane. The end A rests on a rough horizontal plane and a point C of the rod where AC = 3m is in contact with a smooth peg. The rod makes an angle of 30° with the plane. Take g = 9.8 ms^{-2}.

i. Copy the diagram from part b) into your answer booklet and show on it **all** forces acting on the system. **[1]**

ii. Show that the reaction force at C is 28.3 N. **[2]**

iii. Find the magnitude of the reaction force at A. **[2]**

iv. If the rod is in limiting equilibrium, find the coefficient of friction between the rod and the plane. **[5]**

Question B3

a) The curve C has equation

$$y = \frac{(x+3)(2x+1)}{(x+2)(x-3)}$$

i. Write down the equations of all asymptotes to the curve. **[3]**

ii. Find the coordinates of the point where C crosses the horizontal asymptote. **[1]**

iii. Find the coordinates of the points where C crosses the axes. **[3]**

iv. Sketch the curve in your answer booklet. **You do not need to use graph paper for this question.** **[3]**

b) The curve S has equation

$$y = tanh^{-1}\left(\frac{x}{4}\right)$$

i. Find $\frac{dy}{dx}$ **[2]**

ii. Show that the equation of the normal to S at the point x=1 is **[3]**
$4y = -15x + 15 + 2ln\frac{5}{3}$

Question B4

a) You are given the matrix $\boldsymbol{M} = \begin{pmatrix} cos20° & -sin20° \\ sin20° & cos20° \end{pmatrix}$

i. Describe the transformation represented by $\boldsymbol{M}$ **[2]**

ii. Describe the transformation represented by $\boldsymbol{M}^2$ **[2]**

iii. Let $\boldsymbol{A} = \boldsymbol{M}^T$. Describe the transformation represented by $\boldsymbol{A}^3$ **[3]**

iv. Find the **smallest** positive integer n such that $\boldsymbol{M}^n = \boldsymbol{I}$. Explain your answer. **[2]**

b) A curve is given parametrically by

$$x = \frac{t^3}{3} - 25t, \qquad y = 5t^2$$

Find the arc length of the curve between $t = 1$ and $t = 3$ **[6]**

Section B continues on the following page.

Question B5

a) Let w be the complex number $2 - 3i$.

i. Calculate w^2 **[1]**

ii. Calculate w^3 **[1]**

iii. Let $f(z) = z^3 - 6z^2 + 21z - 26$

Show that $f(w) = 0$ **[2]**

iv. Find the other **two** roots of the equation $f(z) = 0$ **[2]**

v. Express all the roots of the equation $f(z) = 0$ in modulus-argument form, giving the arguments in radians. **[3]**

b) Use De Moivre's theorem to find all roots of the equation **[6]**

$$z^4 + 9i = 0$$

giving your answers in the form $a + bi$ where a and b are real numbers.

Question B6

A parabola has equation $y^2 = 4ax$

a) Find the equation of the normal to the parabola at the point $P(ap^2, 2ap)$ **[4]**

b) Let $Q(aq^2, 2aq)$ be a different point on the parabola. Show that the equation of the chord PQ is **[3]**

$$y = \frac{2x}{q+p} + \frac{2apq}{q+p}$$

c) Given that $p = 2$ and that the chord PQ is also the normal to the parabola at P, find the value of q. **[3]**

d) The tangent to the parabola at P intersects the x axis at the point T. Find the coordinates of T. **[2]**

e) Show that the ratio of the distances $PT : PQ = 4 : 5$ **[3]**