NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM001 Further Mathematics Examination

Examination Session	**Time Allowed**
Semester Two	3 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.

Full marks will only be given for **full and detailed answers**

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Section A

Answer ALL questions. This section carries 40 marks.

Question A1

Let z and w be complex numbers with $z = 1 - 2i$ and $w = 3 + i$.

a) Find the value of $2z – 3w$. **[1]**

b) Write in the form $a + bi$ where a and b are real numbers, **[4]**

$$\frac{i(w-z)}{w^*}$$

(w^* is the complex conjugate of w).

Question A2

The matrix A is given by $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$.

a) Describe geometrically the transformation that A has on the x-y plane. **[1]**

b) Describe geometrically the transformation that A^{-1} has on the x-y plane. **[1]**

c) The transformation represented by the matrix A is then followed by a rotation of 60° anticlockwise about the origin. Find the matrix representing this composite transformation. **[3]**

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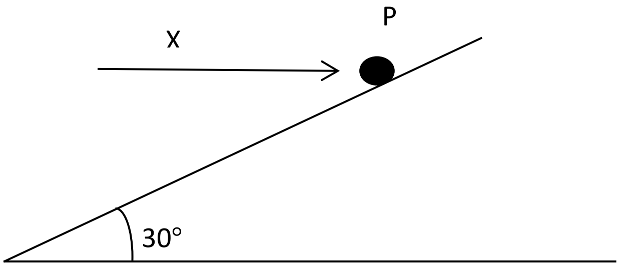
Question A3



A particle P of mass 3kg rests on a rough plane which is inclined to the horizontal at an angle of 30°. P is acted on by a horizontal force X which acts as shown in the diagram above. P is on the point of moving up the plane and the coefficient of friction between P and the plane is $\frac{1}{2}$.

a) Copy the diagram and show all forces acting on the system. **[1]**

b) Find the magnitude of the force X. **[4]**

Question A4

The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} + 4y = 8$$

Find the particular solution given that y = 0 and $\frac{dy}{dx} = 1$ when x = $\frac{\pi}{2}$. **[5]**

Question A5

By differentiating a suitable number of times, find the first three terms of the Maclaurin series for f(x) = $e^{(x^2-4x)}$ **[4]**

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Question A6

A particle P is moving in a straight line with simple harmonic motion with amplitude 4m. When the displacement of P is 3m its speed is $12ms^{-1}$.

a) Find the magnitude of the maximum velocity of P. **[3]**

b) Find the magnitude of the maximum acceleration of P **to 4 significant figures**. **[3]**

In this question one mark will be awarded for the correct use of significant figures.

Question A7

a) By using the exponential definitions of sinhx and coshx, show that **[1]**

$cosh^2x - sinh^2x = 1.$

b) Hence solve the equation **[4]**

$5cosh^2x - 11sinhx - 3 = 0,$

giving your answers in terms of natural logarithms.

Question A8

The curve C is given parametrically by the equations $x = e^{2t} + 3t$, $y = e^{-t}$

a) Find the gradient of C at t = 0. **[1]**

b) Show that the area under C from t = 0 to t = 1 is $2e - 3e^{-1} + 1$ square units. **[4]**

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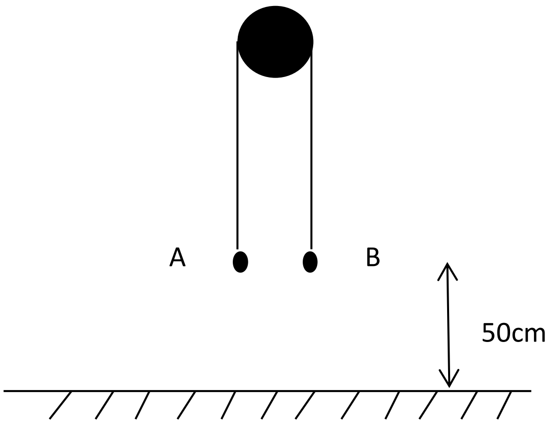
Section B

Answer <u>4</u> questions. This section carries 60 marks.

Question B1

a)



Two particles A, of mass 2kg, and B, of mass 3kg, are connected by a light inextensible string which passes over a smooth fixed pulley. Each particle is initially 50cm above a fixed horizontal plane. The system is released from rest and the effects of air resistance can be ignored. Take $g = 9.8\text{ms}^{-2}$.

i. Find the acceleration of the system. **[3]**

ii. Find the tension in the string while the system is in motion. **[2]**

iii. Particle B hits the plane and does not rebound. Find the maximum height that particle A reaches above the horizontal plane. Assume that A does not reach the pulley. **[3]**

b) A particle P, of mass 2kg, is moving in two dimensions and has velocity vector $\boldsymbol{v}$ ms^{-1} at time t seconds, given by

$$\boldsymbol{v} = (3t^2 - t)\boldsymbol{i} + 4t^3\boldsymbol{j} \qquad t \geq 0$$

where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the direction of the positive x and y axes.

i. Calculate the speed of P at $t = 1$. **[1]**

ii. Given that at $t = 0$, the position vector of P is $3\boldsymbol{i} - 2\boldsymbol{j}$, find the position vector **r**,of P, in terms of t. **[3]**

iii. Find the magnitude of the force acting on P when it crosses the x axis. **[3]**

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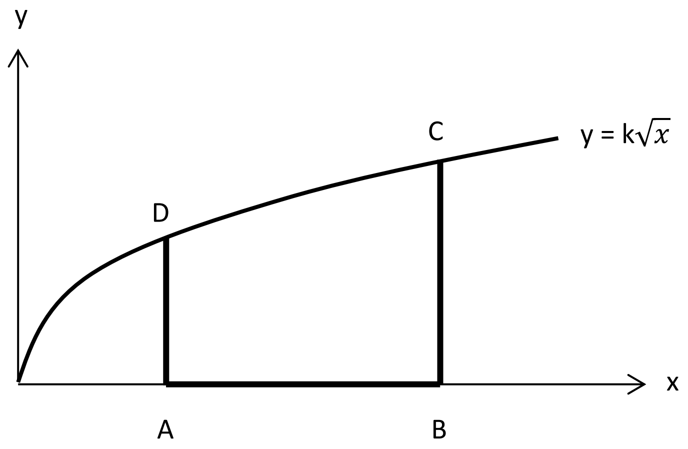
Question B2



A uniform lamina *ABCD* is formed from the curve $y = k\sqrt{x}$ (*k* a constant, *k* > 0), the x axis, and the lines x = 1 and x = 4 (see diagram above). Let the centre of mass of the lamina be at the point *G* (not shown on the diagram).

a) Find the area of the lamina in terms of *k.* **[2]**

b) Show that the distance of *G* from *AD* is $\frac{116}{70}$**.** **[3]**

c) Show that the distance of *G* from *AB* is $\frac{45k}{56}$**.** **[2]**

d) You are given that *k* = 5. The lamina is now suspended freely under gravity from the point *C*. Find the angle that *CB* makes with the vertical. **[4]**

e) The lamina is now placed on a rough inclined plane with the edge *AB* along the line of greatest slope of the plane and such that *B* is higher than *A*. The plane is initially inclined at a small angle θ. If θ is gradually increased and the lamina slides before it topples show that, **[4]**

$\mu < 0.412$ (to 3 significant figures)

where μ is the coefficient of friction between the lamina and the plane.

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Question B3

a) i. Show that, **[5]**

$$\sum_{r=1}^{r=n} r(r^2 + 3r - 3) = \frac{n(n+1)(n^2+5n-4)}{4}.$$

ii. Hence find the value of **[2]**

(11)(151) + (12)(177) + ... + (29)(925) + (30)(987)

b) The curve **[8]**

$y = \sqrt{2x+5} \quad x \geq 0,$

is rotated through 360° about the x axis between the lines x = 1 and

x =15. Show that the area of the surface formed is

$\frac{2\pi}{3}$ (216 - 16$\sqrt{2}$) square units.

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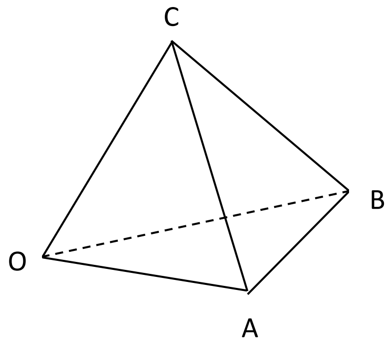
Question B4

a) The diagram shows a tetrahedron *OABC* where *O* is (0, 0, 0), *A* is (2, 1, 3), *B* is (3, -2, 2) and *C* is (4, 2, 3).



i. Determine the area of the face *ABC*. **[2]**

ii. Find the equation of the plane containing *A*, *B* and *C* in the form **[3]**

r.**n** = *d* where **n** and *d* should be found.

iii. Find the perpendicular distance of *O* from the plane containing *A*, *B* and *C*. **[2]**

iv. Hence or otherwise, calculate the volume of the tetrahedron. **[2]**

b) The cubic equation $x^3 - 3x^2 - 5x + 3 = 0$ has roots α, β and γ.

i. Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$, *and* $\alpha\beta\gamma$. **[2]**

ii. Find the value of $\alpha^2 + \beta^2 + \gamma^2 + (\alpha + \beta)(\beta + \gamma)(\gamma + \alpha)$. **[4]**

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Question B5

a) The equation

$4z^4 - 24z^3 + 89z^2 - 124z + 65 = 0$ has a complex root equal to 2 + 3i.

i. Write down another root of the equation. **[1]**

ii. Find the other two roots of the equation. **[4]**

b) On separate Argand diagrams, sketch the following loci for a complex number z and describe the loci geometrically.

i. $|z + 2 - i| = 3$ **[3]**

ii. $\arg(z + 3) = -\frac{\pi}{6}$ **[3]**

c) Let $z = x + iy$ and suppose that z satisfies the equation **[4]**

$\arg\frac{(z-3i)}{(z+3i)} = \frac{\pi}{4}$.

Show that x and y must satisfy

$x^2 + 6x + y^2 = 9$.

Question B6

A parabola has equation $y^2 = 12x$.

a) Write down the coordinates of the focus F, of the parabola. **[1]**

b) Write down the equation of the directrix of the parabola. **[1]**

c) A point P on the parabola has coordinates $(3t^2, 6t)$ where t is a parameter. Find the equation of the tangent to the parabola at P. **[3]**

d) A line is drawn parallel to the x axis through P and intersects the directrix at the point M. The tangent at P intersects the x axis at T. Show that the lines TM and FP are parallel. **[4]**

e) Show that the distances FP and PM are equal. **[3]**

f) Given that at P,$t = 2$, find the area of triangle PFM. **[3]**

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