NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM001 Further Mathematics Examination

Examination Session	**Time Allowed**
Semester Two	3 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**
SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].
Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for **full and detailed answers**.

Section A

Answer ALL questions. This section carries 40 marks.

Question A1

Let z and w be complex numbers with:

$z = 3 + 2i$ and $w = 4 - i$

a) Write the complex number $\frac{1}{z-w}$ in the form $a + ib$ where a and b are real numbers. **[2]**

b) Find real numbers p and q such that: **[3]**

$pz + qw^* = 3i$ (w* is the complex conjugate of w)

Question A2

a) The transformation represented by the matrix A is a rotation of 90° anti-clockwise about the origin. **[1]**

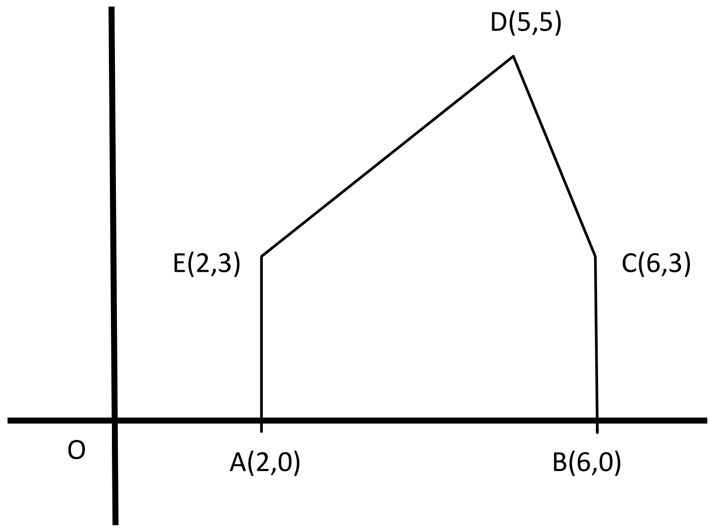
Write down the matrix A.

b) The matrix B is $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$. **[2]**

Describe geometrically the effect of B in the xy plane.

c) Find the matrix $AB^{-1}A$. **[2]**

Question A3



A uniform lamina *ABCDE* is a polygon with vertices at coordinates *A*(2,0), *B*(6,0), *C*(6,3), *D*(5,5) and *E*(2,3) relative to a fixed origin *O* (see diagram above). **[5]**

Find the coordinates of the centre of mass of the lamina relative to *O*.

Question A4

The variables x and y are related by the differential equation: **[5]**

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{1}{x^3}$$

Find the particular solution given that $y = 2$ when $x = 1$, writing your answer in the form $y = f(x)$.

Question A5

Two planes $\prod_1$ and $\prod_2$ have equations:

$\prod_1 : \ 3x - 2y + 2z = 4$

$\prod_2 : \ \boldsymbol{r}.(\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 11$

a) Find an equation for the line of intersection of the planes. **[3]**

b) Find the acute angle between the planes. **[2]**

Question A6

A plane is flying in a horizontal circle of radius 3.3 km with a constant speed of 200 ms^{-1}.

The lifting force on the plane acts at right angles to the wings.

a) Find the angle of the wings to the horizontal. **[3]**

b) If the mass of the plane is 2.5×10^5 kg, find the lifting force acting on the plane, giving your answer to 4 significant figures. **[2]**

Question A7

By using standard summation formulae, find the value of: **[5]**

$$\sum_{r=20}^{r=50} (3n-1)(2n+3)$$

Question A8

If α and β are the roots of the equation $ax^2 + bx + c = 0$, show that: **[5]**

$$(1-3\alpha^3)(1-3\beta^3) = \frac{a^3+3b^3+9c^3-9abc}{a^3}$$

Section B

Answer <u>4</u> questions. This section carries 60 marks.

Question B1

a) A particle *P* of mass 0.1 kg is projected vertically upwards from a point A with a speed of 10 ms^{-1}.

i. Assuming that gravity is the only force acting on the particle, calculate the maximum height reached above the point A. **[2]**

ii. Now assume that *P* is also subject to an air resistance of magnitude $\frac{v^2}{400}$, where *v* is the velocity of *P* in ms^{-1}. **[3]**

Show that:

$$40v\frac{dv}{dx} = -40g - v^2$$

iii. Solve this differential equation and show that the maximum height reached above A is now 4.54m (to 3 significant figures). **[5]**

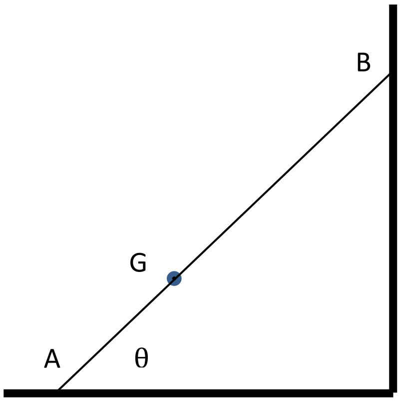
b) Two smooth spheres *A* and *B* are travelling towards each other along a smooth surface. Sphere *A* has mass 3kg and speed 4ms^{-1}, and sphere *B* has mass 6kg and speed 3ms^{-1}. The coefficient of restitution between the spheres is e. All motion takes place in a straight line.

After the spheres collide,

i. Show that the direction of motion of B is reversed if $e > \frac{2}{7}$. **[4]**

ii. Given that e = 0.5, find the speed of *A.* **[1]**

Question B2



A non-uniform ladder *AB* has length 8m and mass 20kg. The end *A* rests on a rough horizontal floor and the end *B* rests against a smooth vertical wall. The angle between the ladder and the horizontal floor is θ. The coefficient of friction between the ladder and the floor is $\frac{\sqrt{3}}{4}$. The centre of mass of the ladder is at G.

a) Draw a diagram showing all forces acting on the ladder. **[4]**

b) Given that θ = 30° and that the ladder is in limiting equilibrium, show that AG = 2m. **[5]**

c) Suppose now that θ is increased so that θ = 60°. **[6]**

A man of mass 60kg begins to climb the ladder.

Calculate the maximum distance from *A* that the man can reach without the ladder slipping.

Question B3

a) The curve C has equation $y = \tanh(3x)$.

i. Find $\frac{dy}{dx}$, giving your answer in terms of a hyperbolic function. **[2]**

ii. Show that $\cosh(ln2) = \frac{5}{4}$ **[1]**

iii. The point P on C has x coordinate equal to $\frac{1}{3}\ln 2$. **[5]**

Find the equation of the tangent to C at P in the form

Ay = Bx + C + D(ln2)

where A, B, C and D are integers to be found.

b) Find the value of: **[7]**

$$\int_{-1}^{0} \frac{dx}{\sqrt{3x^2 + 12x + 11}}$$

leaving your answer in terms of a single natural logarithm.

Question B4

a) Solve the differential equation: **[7]**

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 18x + 21$$

completely, given that $y = 0$ and $\frac{dy}{dx} = 6$ when $x = 0$.

b) The curve *S* has equation $y = \frac{x-4}{(x-1)^2}$

i. Find the equations of the vertical and horizontal asymptotes. **[3]**

ii. Find where the curve crosses the coordinate axes. **[2]**

iii. Sketch the graph of the curve S. **[3]**

Question B5

a) Let $z = cos\theta + isin\theta$

i. Show that $z^{-1} = cos\theta - isin\theta$ **[2]**

ii. You are given that: **[5]**

$z + z^{-1} = 2\cos\theta$ and $z^n + z^{-n} = 2\cos(n\theta)$ $(n \in ℵ)$.

By expanding powers of $z + z^{-1}$, show that:

$16(cos^5\theta + cos^3\theta) \equiv Acos5\theta + Bcos3\theta + Ccos\theta$

where the values of *A*, *B* and *C* should be found.

b) i. Find the modulus and argument of $8 - 8\sqrt{3}i$ **[2]**

ii. Hence solve the equation: **[6]**

$z^3 = 8 - 8\sqrt{3}i$,

giving your answers in the form $a + bi$, with a and b to 2 decimal places.

Question B6

a) Derive the equation of the tangent at the point $P(5t, \frac{5}{t})$ to the rectangular hyperbola $xy = 25$. **[4]**

b) Show that the equation of the normal at P is: **[3]**

$ty - t^3x = 5(1 - t^4)$.

c) The tangent at P intersects the x axis at the point A (a, 0). **[2]**

Show that the coordinates of P are given by $(\frac{a}{2}, \frac{50}{a})$.

d) A line is drawn parallel to the y axis through A and intersects the hyperbola at Q. **[1]**

Find the coordinates of Q in terms of a.

e) Another line is drawn from the origin O to the point Q. **[3]**

Given that OQ and AP are perpendicular, find the value of a, leaving your answer as a surd.

f) Find the distance PQ. **[2]**

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