NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYFM001 Further Mathematics Examination

Examination Session	**Time Allowed**
Semester Two	3 Hours 10 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 40% of the exam marks.**

SECTION B **Answer 4 questions. This section carries 60% of the exam marks.**

The marks for each question are indicated in square brackets [ ].

Your School or College will provide a Formula Booklet.

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the answer books in the space provided.
- Write the answers in the answer books provided. Additional sheets will be provided on request.
- Write the section letter, the question number and numbers to parts of questions attempted clearly at the start of each answer.
- **No** written material is to be brought into the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for **full and detailed answers**.

Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Let $z = 4 + \mathrm{i}$ and $w = 3 - 2\mathrm{i}$ be complex numbers.

a) Find the value of $2z - 5w$. **[1]**

b) Express **[4]**

$$\frac{2\mathrm{i}z}{w^* + z}$$

in the form $a + b\mathrm{i}$, where w^* is the complex conjugate of w.

Question A2

Let the matrix B be defined by $B = \begin{pmatrix} 5 & 8 & 12 \\ 8 & 5 & 12 \\ -8 & -8 & -15 \end{pmatrix}$.

a) Show that $B^2 + 2B$ is a multiple of the identity matrix. **[3]**

b) Hence, find B^{-1}. **[2]**

Question A3

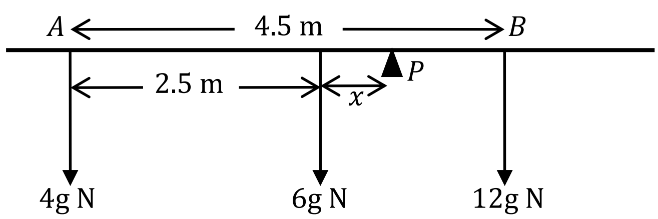

Figure 1

As shown in Figure 1, a uniform rod of weight $6\mathrm{g}$ Newtons rests horizontally on a pivot P, a distance x metres from its centre. Weights A and B are resting on the rod.

a) Find the reaction force acting at P. **[1]**

b) Calculate the distance, $\boldsymbol{x}$, of the pivot from the centre of the rod. **[4]**

Question A4

Solve the differential equation **[5]**

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = x^5$$

completely, given that $y = 9$ when $x = 3$.

Question A5

The lines l_1 and l_2 have equations

$$l_1\text{:}\ \mathbf{r} = (3, -1, -1) + s(1, 4, -2)$$
$$l_2\text{:}\ \mathbf{r} = (4, -9, 7) + t(1, -2, 3).$$

a) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection. **[3]**

b) Find the Cartesian equation of the plane in which l_1 and l_2 lie. **[2]**

Question A6

A car of mass 560 kg takes a bend by travelling along an arc of a circle of radius 30 m at a speed of 45 kilometres per hour. Calculate the frictional force, in Newtons, required to stop the car from sliding on the road. You may assume that the road is level. **[5]**

Question A7

Find the value of the sum **[5]**

$$\sum_{r=n}^{2n}(2r - 1)(3r - 2).$$

Question A8

The curve, C, has equation $y = 29\cosh x + 21\sinh x$. Find the exact value of the coordinates of the turning point on C and determine its nature. **[5]**

Questions continue on the next page.

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

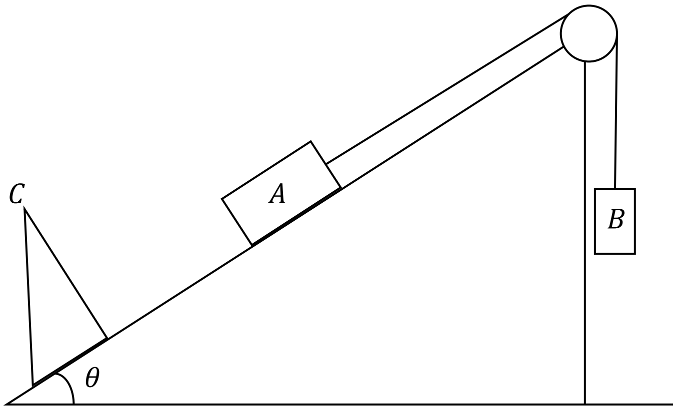

Figure 2

Figure 2 shows a rough slope inclined at an angle θ to the horizontal, where $\cos\theta = 0.96$. A body, A, of mass 11 kg lies on the slope. A string joins A to a second body, B, of mass 0.2 kg. The string is parallel to the line of greatest slope of the plane and passes over a small smooth fixed pulley at the top of the plane. The body B hangs freely from the pulley.
Let the acceleration due to gravity be $g = 9.8$ ms^{-2}.

a) Draw a free-body diagram to show the forces acting on the bodies A and B. **[2]**

b) If the coefficient of friction, μ, is 0.14, calculate the friction force acting on body A. **[1]**

c) Find the acceleration of body A. **[5]**

d) After travelling 4 m from rest down the slope, A collides with a fixed barrier, C, without rebounding. Find the impulse of A upon C. **[3]**

e) Calculate the time taken for the string to become taut again after the collision. **[4]**

Question B2

a) Use integration to find the centre of mass of a uniform semi-circular lamina of radius 20 cm. **[6]**

b)

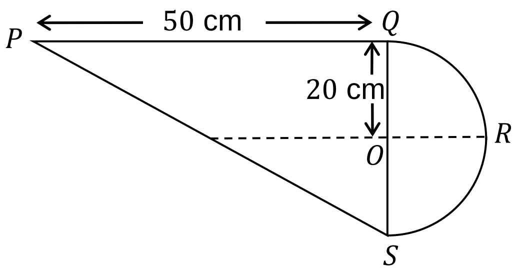

Figure 3

Figure 3 shows a uniform lamina, $PQRS$, where PQS is a right angled triangle and QRS is a semicircle of radius 20 cm.
Taking the origin at O, the mid-point of QS, and the x-axis along OR, find the coordinates of the centre of mass of the lamina. **[6]**

c) The lamina is freely suspended from the vertex P. Find the angle that PQ makes with the vertical. **[3]**

Question B3

a) Show that the point $P = (12, 6)$ lies on the ellipse **[1]**

$$\frac{x^2}{225} + \frac{y^2}{100} = 1.$$

b) Find the equation of the tangent at P. **[4]**

c) Find the coordinates of the point N where the tangent in (b) meets the y-axis. **[2]**

d) Let A and C be the ends of the major axis of the ellipse. Let AP meet the y-axis at L and CP meet the y-axis at M. Find the coordinates of L and M. **[6]**

e) Hence, show that $LN = NM$. **[2]**

Questions continue on the next page.

Question B4

a) Let $u = 1 - 3\mathrm{i}$, $v = 3 + 3\mathrm{i}$ and $w = 4 + \mathrm{i}$ be complex numbers.

i Given that **[3]**

$$\frac{|w-u|}{|w-v|} = k,$$

find the exact value of k.

ii If z is a complex number such that **[7]**

$$\frac{|z-u|}{|z-v|} = k,$$

show that the locus of x is a circle and find its centre and radius.

b) Use De Moivre's theorem to find the possible values of z, where **[5]**

$$z^2 = -5 - 12\mathrm{i},$$

giving your answer in the form $a + \mathrm{i}b$.

Question B5

a) Figure 4 shows a parallelepiped $ABCDPQRS$ where A is $(-2, 1, -3)$, B is $(1, 1, -2)$, D is $(-1, 4, -2)$, and P is $(-1, 2, 1)$.

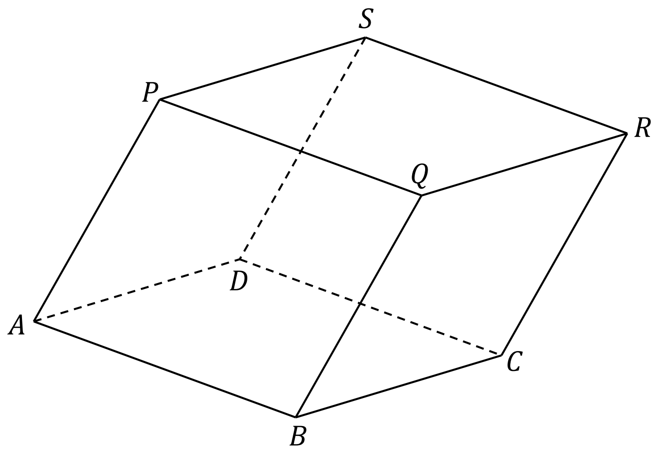

Figure 4

i Find the area of the face $ABCD$. **[4]**

ii Find the volume of the parallelepiped. **[3]**

b) i Show that **[2]**

$$\frac{\mathrm{d}(uvw)}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}vw + u\frac{\mathrm{d}v}{\mathrm{d}x}w + uv\frac{\mathrm{d}w}{\mathrm{d}x}.$$

ii Find the first three terms in the Maclaurin series expansion of **[6]**

$$f(x) = (x+1)\mathrm{e}^x \cos x.$$

Questions continue on the next page.

Question B6

Figure 5 shows a wheel of radius 30 cm which rolls along the line Ox. The centre of the wheel is at C. The fixed point P is on the rim of the wheel. Initially P is at O.

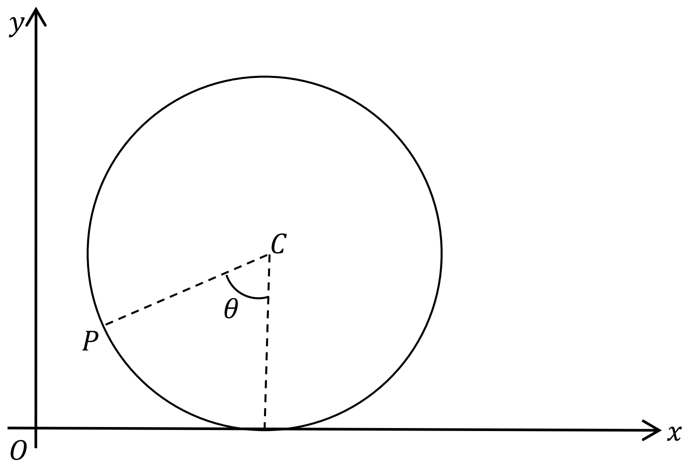

Figure 5

a) Show that when CP has rotated through an angle θ the coordinates of P are **[3]**

$$x = 30\theta - 30\sin\theta\,, \qquad y = 30 - 30\cos\theta.$$

b) Hence, find the length of the path of P when the wheel rolls through a quarter of a revolution. **[8]**

c) Find the exact value of the gradient of the tangent to the path of P where $\theta = \frac{\pi}{2}$. **[4]**

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