NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Further Mathematics Examination

Examination Session	**Time Allowed**
Semester Two	3 hours 10 minutes (Including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A
Answer ALL questions. This section carries 40% of the exam marks.

SECTION B
Answer FOUR questions. This section carries 60% of the exam marks.

The marks for each part of the question are indicated in square brackets [ ]

- **Answers must not be written during the first 10 minutes.**
- Write your Candidate Number clearly on the Answer Book in the space provided.
- Write your answers in the Answer Book provided. Additional sheets will be provided on request.
- Clearly write the number and parts of questions attempted at the start of each answer.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for full and detailed answers.
- Students will receive a formula book.

Section A
Answer ALL questions.
This section carries 40 marks.

Question A1

(i) Show on an Argand diagram the points A and B representing the complex numbers $3\mathrm{i}$ and $1+2\mathrm{i}$ respectively. **[2]**

(ii) Find the distance AB and the angle AOB where O is the origin. **[3]**

Question A2

The transformation represented by the matrix A is a stretch by a factor of 3 in the x-direction and a reflection in the x-axis.

(i) Write down the matrix A. **[2]**

(ii) The transformation represented by the matrix A is then followed by the rotation represented by the matrix

$$B = \begin{pmatrix} 0.28 & 0.96 \\ -0.96 & 0.28 \end{pmatrix}$$

Find the matrix C representing the composite transformation. **[3]**

Question A3

A particle of mass $4\,\text{kg}$ is suspended in equilibrium by two light inextensible strings which make angles of 25° and 50° to the horizontal.

Find the tension in the strings. **[5]**

Question A4

Solve the differential equation **[5]**

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = \mathrm{e}^{3x}$$

completely, given that $y = 4$ when $x = 0$.

Question A5

The plane Π has equation $3x - 5y + z + 14 = 0$.

Let A be the point with position vector $7\mathbf{i} - 7\mathbf{j}$.

(i) Find the equation of the line l through A which is perpendicular to the plane Π. **[1]**

(ii) Find the point of intersection of the line l and the plane Π. **[2]**

(iii) Find the distance AB. **[2]**

Question A6

A particle is moving in a straight line with simple harmonic motion. Its maximum speed is 7 ms^{-1} and its maximum acceleration is 15 ms^{-2}.

Calculate the amplitude and period of the oscillation. **[5]**

Question A7

Find the first three terms of the Maclaurin series for $y = \mathrm{e}^{2x}\cos x$. **[5]**

Question A8

A curve is defined parametrically by the equations $x = 3\cosh\phi$ and $y = 5\sinh\phi$.

Let P be the point where $\phi = \ln 2$.

Find the equation of the tangent to the curve at the point P. **[5]**

Section B
Answer 4 questions.
This section carries 60 marks.

Question B1

A particle P moves along the x-axis with velocity v and acceleration a.

(i) Show that $a = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2}v^2\right)$. **[4]**

(ii) The particle P has mass 3 kg and the force acting on it is $(4 - \sin 5x)$ N along the positive x-axis. The particle moves through the origin O with speed $3\,\mathrm{ms}^{-1}$.

Calculate the speed of the particle when $x = 3$ m. **[7]**

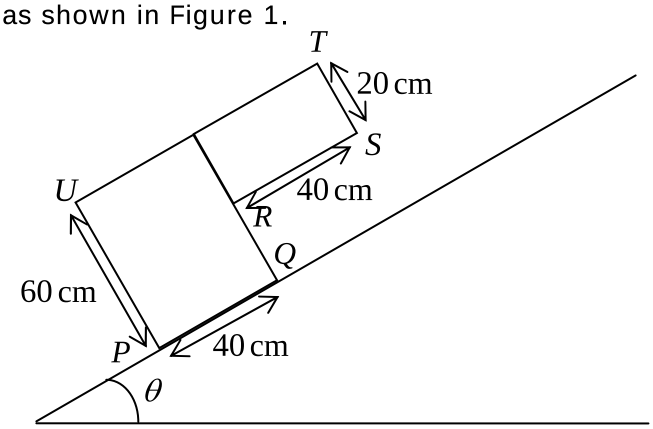
(iii) When $x = 3$ m the particle collides with a stationary particle of mass 7 kg which is free to move along the x-axis. After the collision the particles combine and move as a single body.

Find the speed of the combined particle after the collision. **[4]**

Question B2

A uniform lamina is placed on a rough plane inclined at an angle θ to the horizontal as shown in Figure 1.

Figure 1



(i) Find the distances of the centre of mass of the lamina from PU and PQ. **[6]**

(ii) Determine the maximum value of θ for the lamina to remain in equilibrium in this position. **[3]**

(iii) Let the coefficient of friction between the lamina and the plane be 0.9.

Draw a diagram showing all the forces acting on the lamina and find the value of θ at which the lamina is just about to slip. **[5]**

(iv) From your answers to parts (ii) and (iii), state which occurs first, slipping or toppling. **[1]**

Question B3

Let P be the point $(6,4)$ on the ellipse $\frac{x^2}{10^2}+\frac{y^2}{5^2}=1$.

(i) Find the equation of the tangent to the ellipse at P. **[4]**

(ii) Let the tangent meet the tangents at the ends of the major axis at M_1 and M_2 respectively.

Find the coordinates of M_1 and M_2. **[5]**

(iii) Show that $F=\left(5\sqrt{3},0\right)$ is a focus of the ellipse. **[2]**

(iv) Show that M_1F and M_2F are perpendicular to each other. **[4]**

Question B4

Let $u=9\mathrm{i}$, $v=3+5\mathrm{i}$ and $w=2+3\mathrm{i}$.

(i) Given that $\arg\left(\frac{w-u}{w-v}\right)=k$, find the exact value of k. **[6]**

(ii) If $z=x+\mathrm{i}y$ is a complex number such that $\arg\left(\frac{z-u}{z-v}\right)=k$, show that x and y satisfy the equation $x^2+y^2+x-11y+18=0$. **[5]**

(iii) Hence show that the locus of z lies on a circle and find its centre and radius. **[4]**

Question B5

The curve C has equation $y=\frac{2x^2+11x+5}{x^2-4x+3}$.

(i) Find the equations of the vertical and horizontal asymptotes. **[6]**

(ii) One of the asymptotes has a point in common with C. Determine the coordinates of this point. **[3]**

(iii) Find where the curve crosses the coordinate axes. **[3]**

(iv) Sketch the graph of the curve C. **[3]**

Question B6

(a) Given that α, β and γ are the roots of $x^3+2x^2-3x+6=0$ find the cubic equation whose roots are $\beta\gamma$, $\gamma\alpha$ and $\alpha\beta$. **[7]**

(b) (i) Express $x^2+12x+11$ in the form $(x+p)^2-q$. **[2]**

(ii) Hence show that $\int_0^4 \frac{\mathrm{d}x}{\sqrt{x^2+12x+11}}=\ln\left(\frac{10+5\sqrt{3}}{6+\sqrt{11}}\right)$. **[6]**

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