NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Further Mathematics

Examination Session	**Time Allowed**
Summer 2009	3 hours 10 minutes (Including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A
Answer ALL questions. This section carries 40% of the exam marks.

SECTION B
Answer FOUR questions. This section carries 60% of the exam marks.

The marks for each part of the question are indicated in square brackets [ ].

- **No answers must be written during the first 10 minutes.**
- Write your Candidate Number clearly on the Answer Book in the space provided.
- Write your answers in the Answer Book provided. Additional sheets will be provided on request.
- Clearly write the number and parts of questions attempted at the start of each answer.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for full and detailed answers.
- Students will receive a formula book.

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Section A
Answer ALL questions.
This section carries 40 marks.

Question A1

Let $z = 4 - 5\mathrm{i}$ and $w = 3 - \mathrm{i}$.

(i) Find the value of $z - 3w$. **[1]**

(ii) Express $\dfrac{w}{w + z^*}$ in the form $a + b\mathrm{i}$, where z^* is the complex conjugate of z. **[4]**

Question A2

Let the matrix $\mathbf{B} = \begin{pmatrix} -1 & -6 & -6 \\ -3 & -4 & -6 \\ 3 & 6 & 8 \end{pmatrix}$.

(i) Find $\mathbf{B}^2$. **[2]**

(ii) Hence show that $\mathbf{B}^2 - \mathbf{B}$ is a multiple of the identity matrix, $\mathbf{I}$. **[1]**

(iii) Hence find $\mathbf{B}^{-1}$. **[2]**

Question A3

A uniform rod, AB, of length 80 cm and mass 6 kg, is freely hinged at a point A on a vertical wall. The rod is held in equilibrium horizontally by a light inextensible string attached to the wall at C and to a point P on the rod 50 cm from A as shown in Figure 1. The string makes an angle of $45°$ with the horizontal.

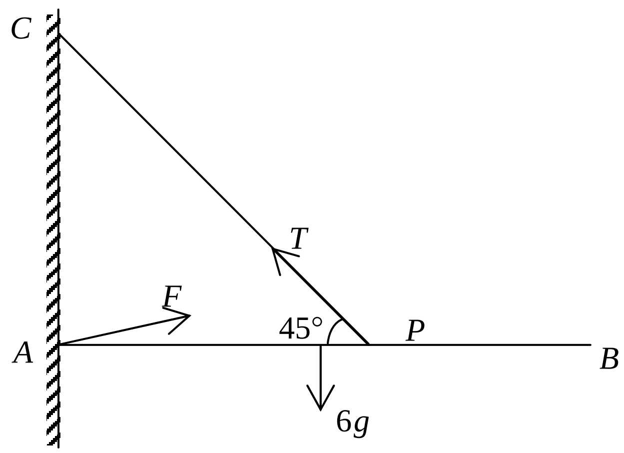

Figure 1

(i) Find the tension in the string. **[2]**

(ii) Show that the magnitude of the reaction force, F, at the hinge A is $\frac{6\sqrt{17}}{5}\text{g}$. **[3]**

Question A4

Solve the differential equation **[5]**

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+7\frac{\mathrm{d}y}{\mathrm{d}t}+10y=50$$

completely, given that $y=9$ and $\frac{\mathrm{d}y}{\mathrm{d}t}=1$ when $t=0$.

Question A5

The lines l_1 and l_2 have equations

$$l_1: \quad \mathbf{r}=(-1,4,6)+s(-2,1,5)$$
$$l_2: \quad \mathbf{r}=(4,--1,0)+t(1,-3,4)$$

(i) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection. **[3]**

(ii) Find the Cartesian equation of the plane in which l_1 and l_2 lie. **[2]**

Question A6

A particle of mass 1.5 kg is attached to the end B of a light inelastic string AB of length 2.5 m, the other end of which is fixed at A. The particle moves in a horizontal circle whose centre is vertically below A with a constant angular speed. The particle takes 1.5 s to complete one revolution.

Let the acceleration due to gravity, g, be 9.8 ms^{-2}, and take π as 3.14.

(i) Calculate the tension in the string. **[2]**

(ii) Find the radius of the circular path of B. **[3]**

Question A7

(i) Express $\frac{1}{(r+1)(r+2)}$ in the form $\frac{A}{r+1}+\frac{B}{r+2}$. **[2]**

(ii) Hence find the exact value of $\sum_{r=5}^{12}\frac{1}{(r+1)(r+2)}$. **[3]**

Question A8

The curve, C, has equation $y = 17\cosh x + 15\sinh x$.

(i) Find the exact value of the x-coordinate of the turning point on C in terms of a natural logarithm. **[3]**

(ii) Hence find the exact value of y. **[1]**

(iii) Determine the nature of the turning point. **[1]**

Section B
Answer 4 questions. This section carries 60 marks.

Question B1

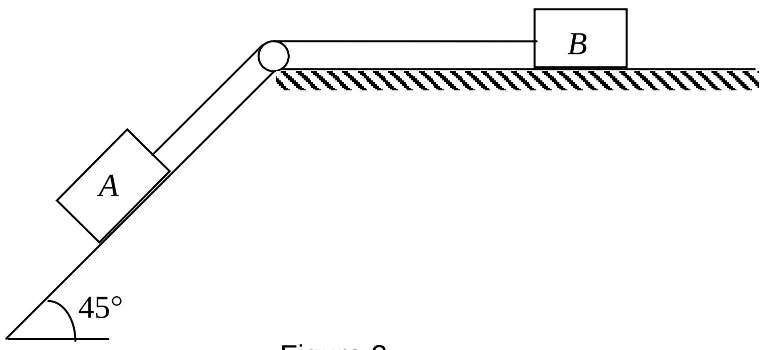

Figure 2

Figure 2 shows two particles A and B, of mass $5\,\text{kg}$ and $4\,\text{kg}$ respectively. They are connected by a light inextensible string which passes over a light smooth pulley P. Particle A rests on a smooth plane inclined at an angle of $45°$ to the horizontal. Particle B rests on a rough horizontal plane. The string is parallel to the line of greatest slope of the inclined plane.
Let the acceleration due to gravity, g be $9.8\,\text{ms}^{-2}$.

(i) Draw a diagram to show the forces acting on the bodies A and B. **[2]**

(ii) If the coefficient of friction, μ, is 0.4, calculate the acceleration of body B. **[5]**

(iii) Find in Newtons, the tension in the string. **[1]**

(iv) After travelling from rest for $1.5\,\text{s}$, the string breaks. Calculate the time taken for B to come to rest given that it does not reach the pulley. **[4]**

(v) What is the total distance that B travels? **[3]**

Question B2

(i) Use integration to find the centre of mass of a uniform semicircular lamina of radius $5\,\text{cm}$. You may use the formula for the area of a circle. **[6]**

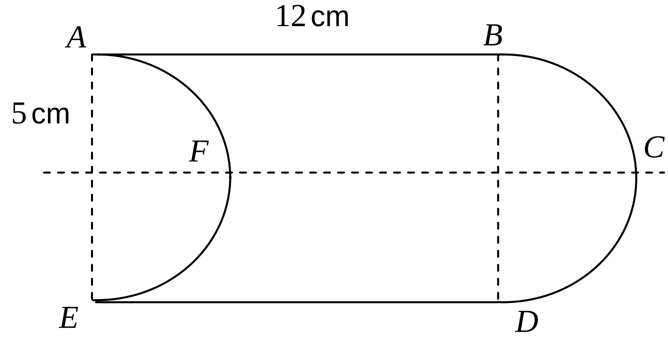

Figure 3

(ii) A semicircular section AFE is removed from a uniform rectangular lamina $ABDE$ and placed at BCD to form the uniform lamina $ABCDEF$ as shown in Figure 3. If the semicircles have radii 5cm find the position of the centre of mass of the lamina. **[3]**

(iii) The lamina $ABCDEF$ is freely suspended from A. Find the angle AB makes with the vertical. **[2]**

(iv) A particle of mass μm, where m is the mass of the lamina, is added at E. Find the value of μ so that AB makes an angle of $45°$ with the vertical. **[4]**

Question B3

(i) Show that the point $P = (4\cos t, 5\sin t)$ lies on the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$. **[1]**

(ii) Find the equation of the tangent at P. **[4]**

(iii) Find the equation of the normal at P. **[3]**

(iv) The normal meets the axes at the points Q and R. Lines are drawn parallel to the axes through the points Q and R; these lines meet at the point V .Find the coordinates of V. **[2]**

(v) Find the equation of the locus of V as t varies in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **[3]**

(vi) Find the eccentricity and foci of the locus of V. **[2]**

Question B4

(a) (i) In an Argand diagram, the point P represents the complex number z, where $z = x + \mathrm{i}y$. Given that $z - 8 = \lambda \mathrm{i}(z - 2)$, where λ is a real parameter, show that as λ varies the locus of P is a circle, and find its centre and radius. **[7]**

(ii) If in (i) above $z = \mu(4 + 3\mathrm{i})$, where μ is real, prove that there is only one possible position for the point P and find its coordinates. **[4]**

(b) Use De Moivre's theorem to solve $(z-1)^3 = 8$ for z. **[4]**

Question B5

(a) The roots of the cubic $x^3 + bx^2 + cx + d$ are α, β and γ.

(i) Show that $\alpha^3 + \beta^3 + \gamma^3 = -b^3 + 3bc - 3d$. **[5]**

(ii) Given the cubic $x^3 + 2x^2 - 4x + 8$ evaluate

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}.$$

[4]

(b) Differentiate $f(x) = \mathrm{e}^{-x}\cos x$ a suitable number of times and hence find the first three non-zero terms of its Maclaurin series. **[6]**

Question B6

(a) (i) Express $4x^2 + 12x + 13$ in the form $(px+q)^2 + r$. **[2]**

(ii) Hence find $\displaystyle\int_{-3/2}^{3/8} \frac{\mathrm{d}x}{\sqrt{4x^2 + 12x + 13}}$ in terms of a natural logarithm. **[8]**

(b) Find the surface area of a parabolic mirror obtained by rotating the parabola $x = 4t^2$, $y = 8t$ from $(0,0)$ to $(4,8)$ about the x-axis, giving your answer correct to one decimal place. **[5]**