NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Further Mathematics
Second Semester Examination

Examination Session
Summer 2007

Time Allowed
3 hours 10 minutes
(Including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

Answer all the questions in Section A.
Answer 6 questions from the 9 questions in Section B.
You are recommended to spend 1 hour on Section A and 2 hours on Section B.
Write your answers in the Answer Book provided. Graph paper will be provided. Additional sheets will be provided on request.
Write your Candidate Number clearly on the Answer Book in the space provided.

- **The marks for each part of the question are indicated in square brackets [ ].**
- **No answers must be written during the first 10 minutes.**

- Clearly write the section letter, the number and parts of questions attempted at the start of each answer.
- Candidates are reminded of the need to use clear and accurate English.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- State the units where necessary.
- Where appropriate, working should be carried out to 4 significant figures and **answers given to 3 significant figures.**
- Full marks will only be given for full and detailed answers.
- Students will receive a formula book.

Section A [34 marks]

Answer all questions in this section. Questions are <u>not</u> equally weighted.

Question A1

Solve the inequality $\frac{3}{2x+5} > x$ for x. **[5]**

Question A2

Find $\sum_{r=1}^{n}(r+5)(3r+4)$ giving your answer in the factorised form. **[5]**

Question A3

Let $z = 2-3\mathrm{i}$ and $w = 1+4\mathrm{i}$.

(i) Write down the values of $w-z$. **[1]**

(ii) Write down the complex conjugate, w^* of w. **[1]**

(iii) Express $\frac{w-z}{w^*}$ in the form $a+b\mathrm{i}$. **[3]**

Question A4

Let $A = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$.

(i) Evaluate $A-B$. **[1]**

(ii) Evaluate AB. **[4]**

Question A5

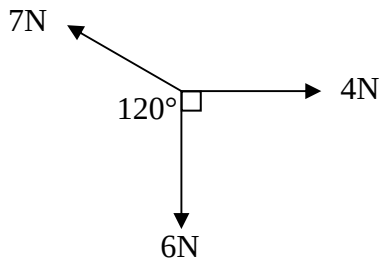

Figure 1

Figure 1 shows three forces acting on a particle.

Find the magnitude and direction of the resultant force.

Show this on a diagram, indicating clearly which angle you have calculated. **[5]**

Question A6

Evaluate $\int_1^6 \frac{1}{\sqrt{9+x^2}}\,\mathrm{d}x$ giving your answer correct to 2 decimal places. **[5]**

Question A7

Write the equation of the plane $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k} + s(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) + t(2\mathbf{j} + 5\mathbf{k})$

in the cartesian form $ax + by + cz = d$. **[4]**

Section B [66 marks]

Answer six questions in this section. Questions in this section have equal weight.

Question B1

(i) The transformation represented by the matrix A is a clockwise rotation about the origin through an angle of $30°$.

Find the matrix A. Use surds in your answer. **[3]**

(ii) The matrix B is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Describe geometrically the transformation represented by B. **[2]**

(iii) Write down the inverse, A^{-1}, of A. **[1]**

(iv) Find $C = A^{-1}BA$. **[3]**

(v) What single transformation is represented by the matrix C? **[2]**

Question B2

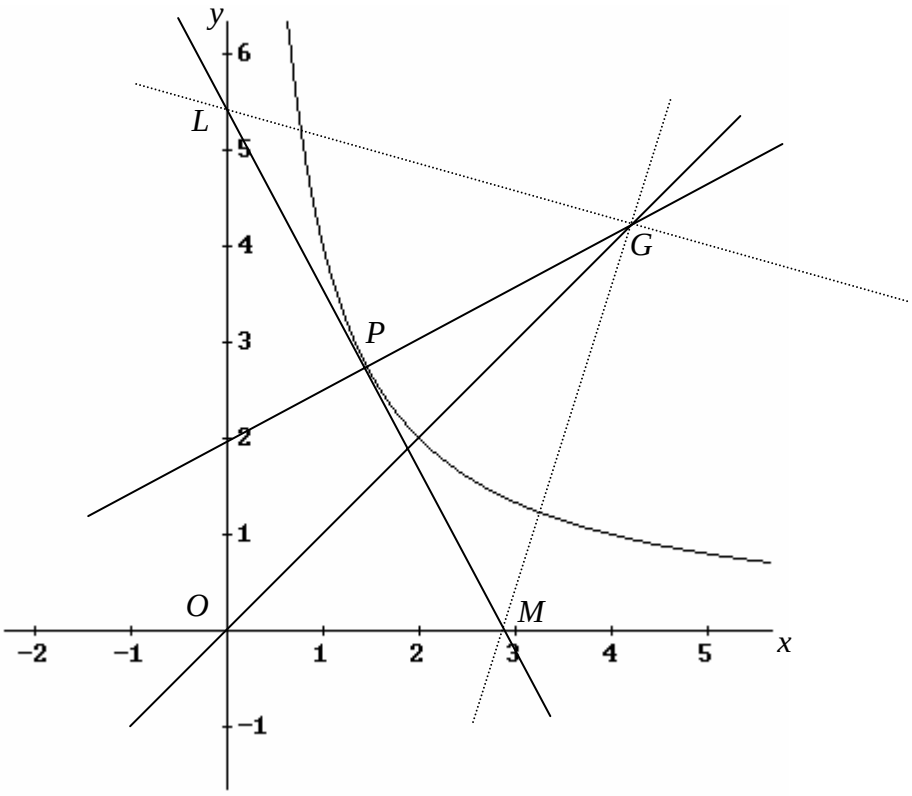

Figure 2

(i) Let P be the point $\left(2t, \frac{2}{t}\right)$ on a regular hyperbola.

Write down the cartesian equation of the hyperbola. **[1]**

(ii) Find the equation of the tangent to the hyperbola at the point P. **[3]**

(iii) Find the coordinates of the points L and M

where the tangent intersects the asymptotes (see Figure 2). **[2]**

(iv) Let the normal at P meet the line $y = x$ at G.

Find the coordinates of G. **[3]**

(v) Show that LG is perpendicular to MG. **[2]**

Question B3

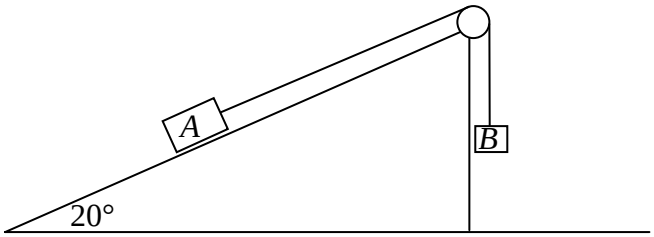

Figure 3

Figure 3 shows a rough slope inclined at an angle of $20°$ to the horizontal.

A body, A, of mass 5 kg is on the slope.

A string joins A to a second body, B, of mass 6 kg.

The string is parallel to the line of greatest slope of the plane and

passes over a small smooth fixed pulley at the top of the plane.

The body B hangs freely from the pulley.

Let the acceleration due to gravity, $g = 9.8\,\text{ms}^{-2}$.

(i) Draw a diagram to show the forces acting on the bodies A and B. **[2]**

(ii) If the coefficient of friction, μ, is 0.2, calculate the friction force acting on body A. **[1]**

(iii) Find the acceleration of body A. **[4]**

(iv) How far has the body travelled from rest after 2 seconds? **[1]**

(v) After 2 seconds body B lands on the floor.

How much further up the slope will body A travel before it stops? **[3]**

Question B4

(a) (i) By using the definitions of $\sinh x$ and $\cosh x$ in terms of exponential functions,

prove $\cosh x \cosh y + \sinh x \sinh y = \cosh(x + y)$. **[2]**

(ii) Hence write $\cosh 3x$ in terms of $\cosh x$. **[3]**

(b) (i) Find the first four terms in the Maclaurin series expansion of

$$f(x) = \tan\left(\frac{\pi}{4} + x\right)$$

in the form $a + bx + cx^2 + dx^3$. **[4]**

(ii) Hence find the value of $\tan 0.8$ correct to 6 decimal places. **[2]**

Question B5

A uniform ladder of length 8 m and mass 20 kg rests with one end on rough horizontal ground and the other end against a smooth vertical wall.

The ladder is inclined at $70°$ to the horizontal.

The coefficient of friction, μ, between the ladder and the ground is 0.3.

A man of mass 65 kg is slowly climbing the ladder.

He wants to know how far up the ladder he can climb safely.

(i) Draw a diagram showing all the forces acting on the ladder when the man is x metres from the foot of the ladder. **[4]**

(ii) Find the reaction of the ground on the ladder. **[1]**

(iii) Show that the reaction of the wall on the ladder is less than $25.5g$ N. **[2]**

(iv) By taking moments about the base of the ladder, find how far up the ladder he can climb safely. **[4]**

Question B6

A plane is approaching Manchester Airport when the pilot is instructed to delay its arrival by 5 minutes.

He therefore flies the plane in a horizontal circle.

The speed of the plane is 500 km per hour.

(i) What is the angular velocity of the plane (in radians per second)? **[2]**

(ii) What is the speed of the plane in metres per second? **[2]**

(iii) Find the radius of the circle. **[2]**

(iv) Given that the reaction between the air and the plane is perpendicular to the wings, draw a diagram to show the forces acting on the plane and calculate the angle to which the plane must be banked to achieve the required flight path. (Use the acceleration due to gravity, $g = 9.8$ ms^{-2}). **[5]**

Question B7

(i) Write $\dfrac{z-(3+2\mathrm{i})}{z-(4+5\mathrm{i})}$ where $z=7+4\mathrm{i}$ in the form $a+b\mathrm{i}$. **[3]**

(ii) Hence show that $\arg\left(\dfrac{z-(3+2\mathrm{i})}{z-(4+5\mathrm{i})}\right)=\dfrac{\pi}{4}$. **[1]**

(iii) Show that for any $z=x+y\mathrm{i}$ for which $\arg\left(\dfrac{z-(3+2\mathrm{i})}{z-(4+5\mathrm{i})}\right)=\dfrac{\pi}{4}$ then

x and y must satisfy the equation $x^2+y^2-10x-6y+29=0$. **[4]**

(iv) Show that the equation in (iii) represents a circle and find its centre and radius. **[3]**

Question B8

The plane Π has equation $3x-2y+5z=6$.

Let A be the point with position vector $7\mathbf{i}-3\mathbf{j}+11\mathbf{k}$.

(i) Find the equation of the line l through A which is perpendicular to the plane Π. **[1]**

(ii) Find the intersection B of the line l and the plane Π. **[2]**

(iii) Find the distance AB. **[2]**

(iv) Let Π intersect the x-axis at C and the y-axis at D.

Find the vectors AC and AD and hence find the volume of the tetrahedron $ABCD$. **[6]**

Question B9

(a) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=2x$$

expressing y in terms of x. **[4]**

(b) (i) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2}+4\frac{\mathrm{d}y}{\mathrm{d}t}+5y=5t+9\,.$$ **[5]**

(ii) Given that $y=1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}t}=3$ when $t=0$, express y in terms of t. **[2]**