NCUK
THE NORTHERN CONSORTIUM

THE NCUK INTERNATIONAL FOUNDATION YEAR (IFY)

Further Mathematics

Examination Session	**Time Allowed**
Summer 2006	3 hours 10 minutes (Including reading time)

INSTRUCTIONS TO STUDENTS

Answer all the questions in Section A.
Answer 6 questions from the 9 questions in Section B.
You are recommended to spend 1 hour on section A and 2 hours on section B.

- **The marks for each question are indicated in square brackets [ ] .**
- **No answers must be written in the first 10 minutes.**

- Write your Candidate Number clearly on the answer books in the space provided. Graph paper will be provided. Additional sheets will be provided on request.
- Clearly write the section letter, the number and parts of questions attempted at the start of each answer.
- **No** written material is allowed in the examination room.
- **No** mobile phones are allowed in the examination room.
- An approved calculator may be used in the examination.
- Unless stated otherwise, round your answers to 3 significant figures.
- Full marks will only be given for full and detailed answers.
- Candidates are reminded of the need to use clear and accurate English.
- Students will receive a formula book.

Section A [34 marks]
Answer all questions in this section. Questions in this section are <u>not</u> equally weighted.

A1 Given that the roots of the equation $x^2 - 5x - 2 = 0$ are α and β;

(i) State the values of $(\alpha + \beta)$ and $\alpha\beta$. **[1]**

(ii) Find the value of $\alpha^2\beta + \alpha\beta^2$. **[2]**

(iii) Find a quadratic equation which has roots $\alpha^2\beta + \alpha\beta^2$ and $\alpha^3\beta^3$. **[2]**

A2 Solve the inequality $\frac{3x}{x-1} > x$ for x. **[5]**

A3 Find the value of the sum $\sum_{r=1}^{n}(r + \frac{1}{2^r})$. **[4]**

A4 A complex number α is given by $\alpha = 1 + i5$. **[4]**

(i) Write down the complex conjugate $\alpha*$.

(ii) Write down the value of the product $\alpha\ \alpha*$.

(iii) Express $\frac{\alpha + \alpha*}{\alpha*}$ in the form $a + ib$.

A5 At time t seconds, a particle P has position vector **r** (in meters), where

$$\mathbf{r} = (t^3 - 3t)\mathbf{i} + 4t^2\mathbf{j}, \quad t \geq 0$$

Find **[2]**

(i) the velocity of P at time t seconds; **[4]**

(ii) the time when P is moving parallel to the vector **i + j**.

A6 Use the substitution $u = \sin x$ to integrate $\int \cos 2x \cos x\, dx$. **[4]**

A7 Starting from the definition of $\cosh x$ in terms of e^x, show that $\cosh 2x = 2\cosh^2 x - 1$. **[3]**

A8 Find the area of the triangle OAB where O is the origin, A has position vector 2**i** – **j** + **k** and B has position vector 3**i** + 4**j** - 2**k** . **[3]**

Section B [66 marks]
Answer six questions in this section. Questions in this section have equal weight.

B1 The matrix **A** is given by **A** = $\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

(i) Draw a diagram showing the unit square, with lower left corner at the origin, and its image under the transformation represented by **A.** **[3]**

(ii) Find the value of the determinant of **A.** Describe how this value relates to your diagram in part (i). **[3]**

(iii) **A** represents a sequence of two geometrical transformations, one of which is a rotation **R**.
Determine the angle of **R**, and describe the other transformation. **[3]**

(iv) State the matrix that represents **R**, giving the elements in an exact form. **[2]**

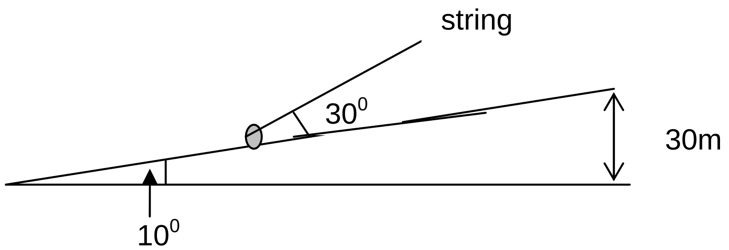
B2 A chord of a parabola from P $(at_1^2, 2at_1)$ to Q $(at_2^2, 2at_2)$ subtends a right angle at the vertex, i.e. angle POQ is a right angle.

(i) Show that $t_1 t_2 = -4$. **[4]**

(ii) Write down the coordinates of the mid-point of PQ **[2]**

(iii) Find the Cartesian equation of the locus of the mid-point of the chord PQ. **[5]**

B3 A rough slope is inclined at an angle of 10^0 to the horizontal. A particle of mass 6 kg is on the slope. A string is attached to the particle and is at an angle of 30^0 to the slope. The tension in the string is 20 N**.** The diagram shows the slope, the particle and the string.



The particle moves up the slope with a constant acceleration of 0.4 m s^{-2} .

(i) Draw a diagram to show the forces acting on the particle. **[1]**

(ii) Show that the magnitude of the normal reaction is 47.9 N, correct to 3 significant figures. **[3]**

(iii) Find the coefficient of friction between the particle and the slope. **[4]**

(iv) The particle starts from rest at the bottom of the slope. Find the time taken for the particle to reach the top if the length of the slope is 200m . **[3]**

B4 Two smooth spheres A and B, have the same radius. The mass of A is 0.24 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a horizontal table, with a speed of 8 m s^{-1} , when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues along the straight line with a speed of 6 m s^{-1}.

(i) Find the magnitude of the impulse exerted by A on B . **[3]**

(ii) Show that $m \leq 0.08$. **[3]**

(iii) It is given that m = 0.06 **[2]**
Find the coefficient of restitution between A and B

(iv) On another occasion A and B are travelling towards each other, each with a speed of 4 m s^{-1}, when they collide directly. **[3]**
Find the speeds of A and B immediately after the collision, assuming m = 0.06.

B5 (a) A curve has parametric equation **[3]**

$$x = \sin\theta - \cos\theta + 1, \qquad y = \sin 2\theta.$$

Show that the Cartesian equation of the curve is

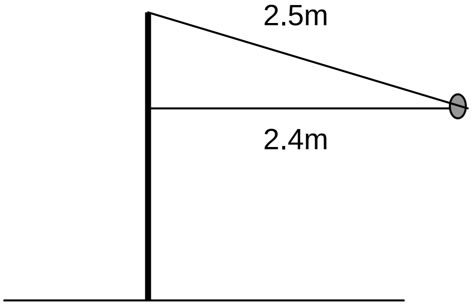
$$y = -x(x-2)$$

(b) $f(x) = 2\sinh^2 x - 5\cosh x$

(i) Solve the equation $f(x) = 10$, giving your answer in an exact logarithmic form. **[4]**

(ii) Find the x coordinates of the stationary points on the curve $y = f(x)$. **[4]**

B6



(a) A ball of mass 0.08 kg is attached by two strings to a vertical post. The strings have lengths 2.5 m and 2.4 m, as shown in the above diagram. The ball moves in a horizontal circle, of radius 2.4 m, with constant tangential speed v m s^{-1}. Each string is taut and the lower string is horizontal. Consider both strings as light and inextensible.

(i) Find the tension in each string when v = 10.5 . **[5]**

(ii) Find the least value of v for which the strings are taut. **[2]**

(b) A particle is moving with simple harmonic motion in a straight line. The period is 0.2 s and the amplitude of the motion is 0.3 m. Find the maximum speed and the maximum acceleration of the particle. **[4]**

B7 (i) Use de Moivre's theorem to show that if $z = \cos\theta + i\sin\theta$, then **[3]**

$$z^n + \frac{1}{z^n} = 2\cos n\theta .$$

(ii) Write down the expansion of $\left(z - \frac{1}{z}\right)^4$ in terms of z. **[2]**

(iii) Hence, or otherwise, show that
$8\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3$ **[4]**

(iv) Solve the equation **[2]**
$$8\sin^4\theta = \cos 4\theta + 1$$
in the interval $-\pi < \theta < \pi$, giving your answers in terms of π.

B8 The points A, B and C have position vectors
$\mathbf{a} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\mathbf{c} = (\mathbf{i} + 2\mathbf{j} - \mathbf{k})$

(a) Find:

(i) the exact value of the area of triangle ABC; **[4]**

(ii) the Cartesian equation of the plane Π containing A, B and C. **[3]**

(b) The line l passes through the point D with position vector $\mathbf{d} = -5\mathbf{j}$ and is perpendicular to the plane Π.
Find:

(i) the equation of l in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$; **[2]**

(ii) the coordinates of the point of intersection of l with Π. **[2]**

B9 (a) Find the general solution of the differential equation **[4]**

$$\frac{dy}{dx} + 3y = 5e^{-4x}$$

expressing y in terms of x.

(b) Solve the differential equation **[7]**

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 5e^{-4x}$$

completely, given that $\frac{dy}{dx} = 0$ and $y = 3$ when $x = 0$.