NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

MARK SCHEME
2016-2017

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Notice to Markers

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A11. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

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Section A

Question A1

Gradient = $-\frac{3}{5}$ **[M1]**

$y - 15 = \text{their gradient}(x+5)$ **[M1]**

$3x + 5y - 60 = 0$ **[A1]**

or Writes $3x + 5y + k = 0$ **(M1)**

Substitutes $x = -5 \text{ and } y = 15$ into their expression **(M1)**

$3x + 5y - 60 = 0$ **(A1)**

Question A2

$\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}$ Any one fraction seen **[M1]**

Multiplies their probabilities **[M1]**

$= \frac{5}{56}$ or anything rounding to 0.0893 **[A1]**

Question A3

Factorises or uses formula $[(2x-3)(x+7) = 0 \text{ or } x = \frac{-11 \pm \sqrt{[11^2 - 4 \times 2 \times -21]}}{2 \times 2}]$ **[M1]**

Finds two critical values $(\frac{3}{2} \text{ and} -7)$ **[M1]**

$x \geq \frac{3}{2}$ **(A1)** and $x \leq -7$ **(A1)** **[A2]**

Question A4

$\frac{ar^8}{ar^5} = \frac{13122}{3888}$ either way up **[M1]**

Reaches $r^3 = \cdots \text{or } \frac{1}{r^3} = \cdots \ (\frac{27}{8} \text{ or } \frac{8}{27})$ **[M1]**

$r = \frac{3}{2}$ or equivalent **[A1]**

Substitutes their value of r into either equation **[M1]**

$a = 512$ **[A1]**

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Question A5

Uses correct version of cosine formula in any form **[M1]**
$(263^2 = 161^2 + 190^2 - 2 \times 161 \times 190 \times \cos A)$

Carries out correct calculation and reaches $\cos A = \cdots$ $(-\frac{1787}{15295}$ or $-0.1168 \ldots)$ **[M1]**

Angle $A =$ anything rounding to 96.7 (degrees) or 1.69 (radians). **[A1]**

Question A6

$\log_2\left(\frac{3x-1}{2x+3}\right) = \log_2(\frac{1}{8})$ Uses log subtraction law correctly **[M1*]**
Expresses RHS as a log **[M1*]**

Removes logs at the right time and solves **[M1]**

$x = \frac{1}{2}$ **[A1]**

Question A7

$\frac{dy}{dx} = 3x^2 - 14$ Attempts to differentiate (Presence of x^2 or 14 is sufficient) **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$ $(= -2)$ **[M1]**

Inverts and changes sign $(= \frac{1}{2})$ **[M1]**

$y + 12 = \frac{1}{2}(x - 2)$ (in any form) **[A1]**

Question A8

$-10x + 3x^2y^2 + 2x^3y\frac{dy}{dx} - 7\frac{dy}{dx} = 0$ Correct use of Product Rule **[M1*]**

Correct implicit differentiation (sight of $2x^3y\frac{dy}{dx}$ or $7\frac{dy}{dx}$ is sufficient for this mark) **[M1*]**

Factorises and makes $\frac{dy}{dx}$ the subject (this mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{10x - 3x^2y^2}{2x^3y - 7}$ **[A1]**

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Question A9

$\frac{1}{x-3}$ **(M1*)** (Takes reciprocal) $2x-7$ **(M1*)** (Takes inverse) **[M2*]**

Sets equal to each other and forms a quadratic equation $(2x^2-13x+20=0)$ **[M1]**

Factorises or uses formula $[(2x-5)(x-4)=0 \text{ or } x=\frac{13 \pm\sqrt{[(-13)^2-4\times 2\times 20]}}{2\times 2}]$ **[M1]**

$x=\frac{5}{2} \text{ or } 4$ **[A1]**

Question A10

Please note: this is a 'show that' question so all working must be seen.

$\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}$ Forms a single quotient **[M1*]**

$=\frac{1}{\sin\theta\cos\theta}$ Uses $\cos^2\theta+\sin^2\theta \equiv 1$ **[M1*]**

$=\sec\theta\csc\theta$ (in either order) Both M marks scored and no errors seen. **[A1]**

Question A11

Writes $g'(x)=\cdots(3x^2)$ and substitutes into correct formula $(7-\frac{7^3-340}{3\times 7^2})$ **[M1*]**

$=6.97959...$ **[A1+]**

$=6.980$ to four significant figures **[A1ft]**

+This mark can be implied if this line is not seen but the 6.980 emerges.
Allow follow through as long as a more accurate answer is seen earlier.

Question A12

$5x^2-13x+23=A(x^2+1)+(Bx+C)(x-4)$ **[M1]**

$A=3$ $B=2$ $C=-5$ **(A1)** for each **[A3]**

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Section B

Question B1

a) i. Substitutes $x = -1$ into $f(x)$. **[M1*]**

$= -36$ **[A1]**

ii.

$$
\begin{array}{r|l}
 & x^2 - 2x - 15 \\
\hline
x^2 + 2 & x^4 - 2x^3 - 13x^2 - 4x - 30 \\
 & x^4 \quad\quad + 2x^2 \\
\hline
 & -2x^3 - 15x^2 - 4x \\
 & -2x^3 \quad\quad - 4x \\
\hline
 & -15x^2 \quad - 30 \\
 & -15x^2 \quad - 30 \\
\hline
 & \ldots\ldots\ldots \\
\hline
\end{array}
$$

Correct first division **[M1]**

Correct subsequent division **[M1]**

Correct quotient **[A1]**

$f(x) = (x^2 + 2)(x - 5)(x + 3)$ **[A1]**

b) $k^2 - 4 \times 2 \times -3k = 0$ **[M1]**

Solves **[M1]**

$k = -24$ (ignore 0) **[A1]**

c) i. $\frac{20}{2}[2 \times 28 - (20 - 1) \times d]$ **[M1]**

$\frac{21}{2}[2 \times 20 - (21 - 1) \times d]$ **[M1]**

Sets two expressions equal to each other and solves **[M1]**

$d = -7$ **[A1]**

ii. $r = \frac{1}{3}$ or $-1 < r < 1$ so series is convergent (Allow this mark if their r lies between -1 and +1 exclusive). **[M1]**

Sum to infinity = $\frac{243}{1 - \text{their } r}$ **[M1]**

= 364.5 or equivalent so sum of series never reaches 365 (Answer and conclusion needed). **[A1]**

d) X lies at $(5, 0)$ **[B1]**
Y lies at $(0, 12)$ **[B1]**

$XY^2 = \text{their } 5^2 + \text{their } 12^2$ **[M1]**

$XY = 13$ (units) **[A1ft]**

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Question B2

Please note: this is a 'show that' question so all working must be seen.

a)

i. Substitutes and makes e^{2k} the subject $(\frac{144}{100})$ **[M1*]**

Uses logs correctly and makes $2k$ the subject [$\ln(\frac{144}{100})$] **[M1*]**

$k = \frac{1}{2}\ln\left(\frac{144}{100}\right) = \ln(\frac{144}{100})^{1/2} = \ln(\frac{6}{5})$ (all stages must be seen: allow equivalent fractions or decimals for $\frac{144}{100}$) **[A1]**

ii. Substitutes $t = 3$ into expression **[M1]**

$= 172.8$ (kN m^{-2}) (allow 173) **[A1]**

iii. $\frac{dP}{dt} = 100 \times k \times e^{kt}$ **[M1*]**

Substitutes $t = 4$ into their $\frac{dP}{dt}$ **[M1]**

= anything rounding to 37.8 $(\text{kN m}^{-2}$ per minute) **[A1]**

iv. After 4 minutes, the rate of pressure increase is their 37.8 $(\text{kN m}^{-2}$ per minute) or similar words. The 4 minutes must be mentioned and rate of pressure change stated or implied [their 37.8 per minute is good enough]. Allow follow through on their answer to part iii. **[B1ft]**

b) Recognises the 'hidden' quadratic **[M1*]**

Factorises or uses formula **[M1]**

$[(3\log_8 x - 2)(\log_8 x - 3) = 0$ or $\log_8 x = \frac{11 \pm\sqrt{[(-11)^2 - 4 \times 3 \times 6]}}{2 \times 3}]$

$\log_8 x = \frac{2}{3}$ or 3 (This mark can be implied if the correct solutions follow) **[A1]**

$x = 4$ or 512 (If answers appear with no working, award 1 mark out of 4) **[A1]**

c)

i. Angle $PQR = 59$ (degrees) **[B1]**

Uses correct version of sine formula $\frac{PR}{\sin \text{their } PQR} = \frac{14}{\sin 49}$ **[M1]**

PR = anything rounding to 15.9 (cm) [If $PQ = 17.6$ is seen, give B0 M1 A0; but if just 17.6 is seen, this scores no marks] **[A1]**

ii. $80 = \frac{1}{2} \times \text{their } 15.9 \times 14 \times \sin PRS$ **[M1]**

Angle PRS = anything rounding to 46 (degrees) **[A1]**

iii. $80 \div (\frac{1}{2} \times 14)$ **or** their $PR \times \sin(\text{their } PRS)$ **[M1]**

= anything rounding to 11.4 (cm) **[A1]**

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Question B3

a) i. Attempts to differentiate (sight of x^2 or x^3 is sufficient for this mark) **[M1]**

$\frac{dy}{dx} = 36x^2 - 36x^3$ **[A1]**

Please note: this is a 'show that' question, so all working must be seen.

ii. Substitutes $x = 0$ and $x = 1$ into their $\frac{\mathrm{dy}}{\mathrm{dx}}$ **[M1*]**

Shows $\frac{\mathrm{dy}}{\mathrm{dx}} = 0$ in both cases **[A1]**

or Sets their $\frac{\mathrm{dy}}{\mathrm{dx}} = 0$ and solves $36\mathrm{x}^2(1 - \mathrm{x}) = 0$ **(M1)**

giving $\mathrm{x} = 0$ and 1. **(A1)**

iii. Attempts to differentiate a second time (sight of x or x^2 is sufficient) **[M1*]**

$\frac{d^2y}{dx^2} = 72x - 108x^2$ **[A1]**

$= 0$ when $x = 0$ and changes sign (this must be stated but there is no need to verify) so there is a point of inflexion [reason and conclusion] **[A1]**

Substitutes $x = 1$ into their $\frac{d^2y}{dx^2}$ and shows this is negative **[M1*]**

There is a maximum when $x = 1$. (conclusion needed) **[A1]**

or takes a numerical value of x below 0 and shows $\frac{dy}{dx}$ is positive **(M1*)**

takes a numerical value of x between 0 and 1 and shows $\frac{dy}{dx}$ is positive **(M1*)**

takes a numerical value of x above 1 and shows $\frac{dy}{dx}$ is negative **(M1*)**

There is a point of inflexion when $x = 0$ **(A1)**

There is a maximum when $x = 1$ **(A1)**

Part b) is on the next page.

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Question B3 – (continued)

b) i. **[M1*]**

Sets equations equal to each other and forms a quadratic equation $(2x^2 + 2x - 12 = 0)$

[M1]

Factorises or uses formula

$[2(x+3)(x-2) = 0$ or $x = \frac{-2 \pm \sqrt{2^2 - 4 \times 2 \times -12}}{2 \times 2}]$

[A2]

Coordinates are $(-3, 12)$ **(A1)** and $(2, 7)$ **(A1)**

ii. Attempts to integrate $15 - 2x - x^2$ **[M1*]**
(sight of x, x^2 or x^3 is sufficient for this mark)

$= 15x - x^2 - \frac{x^3}{3}$ **[A1]**

Attempts to integrate $x^2 + 3$ (sight of x^3 or x is sufficient for this mark) **[M1*]**

$= \frac{x^3}{3} + 3x$ **[A1]**

Substitutes -5 and their -3 into their integrated $15 - 2x - x^2$ and subtracts the right way round $(-45 + \frac{175}{3} - = \frac{40}{3})$ **[M1]**

Substitutes their -3 and 0 into their integrated $x^2 + 3$, subtracts the right way round $(0 - (-18) = 18)$ and then adds both areas. **[M1]**

$= \frac{94}{3}$ or equivalent or anything rounding to 31.3 **[A1]**

(If the correct answer appears with no evidence of integration, award 3 marks out of 7.)

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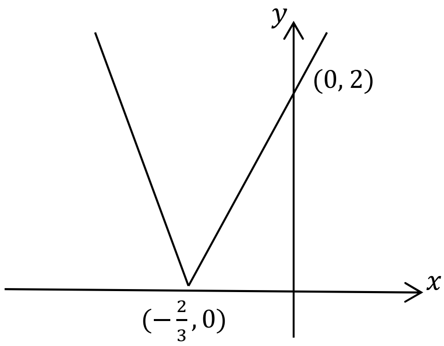
Question B4

a) i.



V-shape on $x-$axis **[B1]**

$(-\frac{2}{3}, 0)$ and $(0, 2)$ **[B1]**

ii. Any correct method *e.g.* solving $3x + 2 = \pm 13$ or squaring both sides and solving a quadratic equation $(9x^2 + 12x - 165 = 0)$ giving $(3x - 11)(3x + 15) = 0$ or $x = \frac{-12 \pm \sqrt{[12^2 - 4 \times 9 \times -16]}}{2 \times 9}$ **[M1]**

$x = \frac{11}{3}$ or -5 or equivalent **(A1)** for each (Accept anything rounding to 3.67) **[A2]**

b) i. Yes **(B1*)** and reason **(B1)** (e.g. 'each element in the domain is mapped to one element in the range' or similar valid statement). **[B2]**
*This mark can only be given if a reason (even a wrong one) follows.

ii. $h(x) > 0$ or $y > 0$ (Note that $x > 0$ is B0) **[B1]**

iii. $\left(h^{-1}(x)\right) = \frac{1}{x}$ **[B1]**

iv. $\frac{1}{4x-3}$ or at least some indication that the functions have been performed in the correct order **[M1]**

Sets equal to 2 and solves **[M1]**

$x = \frac{7}{8}$ or equivalent **[A1]**

Please note: this is a 'show that' question so all working must be seen.

c)

i. Uses $\sin(\theta + \theta)$ **[M1*]**

$= \sin\theta\cos\theta + \cos\theta\sin\theta$ (this must be seen) $= 2\sin\theta\cos\theta$ **[A1]**

ii. Uses the double angle formula in the form $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ **[M1*]**

$\sin\theta = \frac{\sqrt{15}}{8}$. **[A1]**

iii. $2\sin\theta\cos\theta - \sin\theta = 0$ Uses identity **[M1]**

$\sin\theta(2\cos\theta - 1) = 0$ Factorises **[M1]**

$\theta = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

(A1) for any one of $0, \pi$ or 2π and either $\frac{\pi}{3}$ or $\frac{5\pi}{3}$ **(A2)** all correct **[A2]**

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Question B5

a) i. Solves any one of $3 - 5\mu = -7$; $-1 + 2\mu = 3$; $4 + 3\mu = 10$ $(\mu = 2)$ and either solves other two equations or confirms $\mu = 2$ satisfies them. **[M1*]**

Please note: this is a 'show that' question so all working must be seen.

ii. Finds $\overrightarrow{AB}$ $(-10\boldsymbol{i} - 13\boldsymbol{j} - 8\boldsymbol{k})$ **[M1*]**

Their $\overrightarrow{AB} \,.\, (-5\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$ **[M1*]**

$= 50 - 26 - 24$ (must be seen) $= 0$ followed by conclusion **[A1]**

iii. Finds $\overrightarrow{CB}$ and $\overrightarrow{CA}$ $[(-5\boldsymbol{i} - 15\boldsymbol{j} - 11\boldsymbol{k})$ and $(5\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k})]$ **[M1*]**

Finds scalar product of their $\overrightarrow{CB}$ and their $\overrightarrow{CA}$ $(-25 + 30 + 33 = 38)$ **[M1*]**

Their $|\overrightarrow{CB}|(\sqrt{371}) \times$ their $|\overrightarrow{CA}|(\sqrt{38}) \times \cos\theta =$ their scalar product **[M1*]**

$(\cos\theta = \frac{38}{\sqrt{38} \times \sqrt{371}} \approx 0.32)$ $\theta =$ anything rounding to 71.3° or 1.25^{c} **[A1]**

iv. $\frac{1}{2} \times$their $|\overrightarrow{CB}| \times$their $|\overrightarrow{CA}| \times \sin($their $\theta)$ **or** $\frac{1}{2} \times$their $|\overrightarrow{AB}| \times$their $|\overrightarrow{CA}|$ **[M1]**

= anything rounding to 56.2 (units^2) **[A1]**

v. Writes down the negative value of their $\cos\theta$. **[B1ft]**

b) i. $z = \frac{180 - 177}{4}$ or $\frac{177 - 180}{4}$ $(= \pm 0.75)$ **[M1]**

Finds Φ(their z) $(= 0.7734)$ and subtracts from 1. **[M1]**

= anything rounding to 0.227 or equivalent **[A1]**

ii. 0 **[B1]**

iii. (Their part i answer)^3 **[M1]**

= anything rounding to 0.012 **[A1ft]**

iv. $246.5 \pm \frac{1.96 \times 5}{\sqrt{16}}$ **[M1]**

Anything rounding to 244 **(A1)** to 249 **(A1)** **[A2]**

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Question B6

a) i. $(x = \tan y) \quad \frac{dx}{dy} = \sec^2 y$ **[B1]**

ii. *Please note: this is a 'show that' question so all working must be seen.*

Inverts their $\frac{dx}{dy}$ $\left(\frac{dy}{dx} = \frac{1}{\sec^2 y}\right)$ **[M1*]**

Writes in terms of $\tan^2 y$ $\left(= \frac{1}{1 + \tan^2 y}\right)$ **[M1*]**

$\left(\frac{dy}{dx}\right) = \frac{1}{1 + x^2}$ **[A1]**

iii. Separates the variables $\left(y\, dy = \frac{1}{1 + x^2}\, dx \right)$ **[M1*]**

Attempts to integrate both sides (sight of y^2 and $\tan^{-1}x$ is sufficient for this mark) **[M1]**

$\frac{y^2}{2} = \tan^{-1}x + c$ Correct equation and $+\,c$. If the $+\,c$ is missing, this mark is lost and no further credit can be gained. **[A1]**

Substitutes $x = 1$ and $y = 0$ into their equation and finds a value for c $\left(= -\frac{\pi}{4}\right)$ **[M1]**

$y = \sqrt{(2\tan^{-1}x - \frac{\pi}{2})}$ or equivalent **[A1]**

b) i. *Please note: this is a 'show that' question so all working must be seen.*

Correct use of the Quotient Rule **[M1*]**

$\frac{dy}{dx} = \frac{(0) - 1 \times \cos x}{\sin^2 x}$ **[A1]**

$= \frac{-\cos x}{\sin x} \times \frac{1}{\sin x} = -\cot x \csc x$ (either way round) **[A1]**

This stage must be seen (either way round)

or uses the Chain Rule.

Writes $\frac{1}{\sin x}$ as $(\sin x)^{-1}$ **(M1*)**

$\frac{dy}{dx} = -(\sin x)^{-2} \times \cos x$ **(A1)**

$= \frac{-\cos x}{\sin x} \times \frac{1}{\sin x} = -\cot x \csc x$ (either way round) **(A1)**

This stage must be seen (either way round)

Parts ii and iii are on the next page

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ii. $\frac{du}{dx} = 3x^2$ or $du = 3x^2\,dx$ **[M1*]**

Writes integral in terms of u $\left(\frac{1}{3}\int_{-2}^{-1}\frac{1}{u^2}\ du\right)$ The limits do not need to have been changed for this mark. **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. If the original limits are used, the integrated expression must be written in terms of x. **[M1]**

$= \frac{1}{6}$ or equivalent, or anything rounding to 0.167 **[A1]**

(If the correct answer appears with no working, or another method is used, then no marks are awarded.)

iii. Volume = $\pi\int_3^6 (x^2 - 2x + 3)\,dx$ Uses correct formula **[M1*]**

Attempts to integrate (sight of x^3, x^2 or x is sufficient for this mark). This mark and the next one are not lost if the π is dropped. **[M1*]**
$\left(= (\pi)\left[\frac{x^3}{3} - x^2 + 3x\right]\right)$

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 45\pi$ or anything rounding to 141. **[A1]**

(If the correct answer appears with no working, award 1 mark out of 4).

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