NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2016-17

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

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Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Find the equation of the line which is parallel to $3x + 5y + 7 = 0$ and passes through the point $(-5, 15)$.

Give your answer in the form $ax + by + c = 0$ where $a,\ b$ and c are integers. **[3]**

Question A2

A box holds 5 blue beads and 3 yellow beads. Three beads are drawn, one after the other, with no replacement.

Find the probability the first two beads are yellow and the third bead is blue. **[3]**

Question A3

Find the range of values which satisfy $2x^2 + 11x - 21 \geq 0$. **[4]**

Question A4

The 9th term of a geometric progression is 13122 and the 6th term is 3888. **[5]**

Find the common ratio and the first term.

Question A5

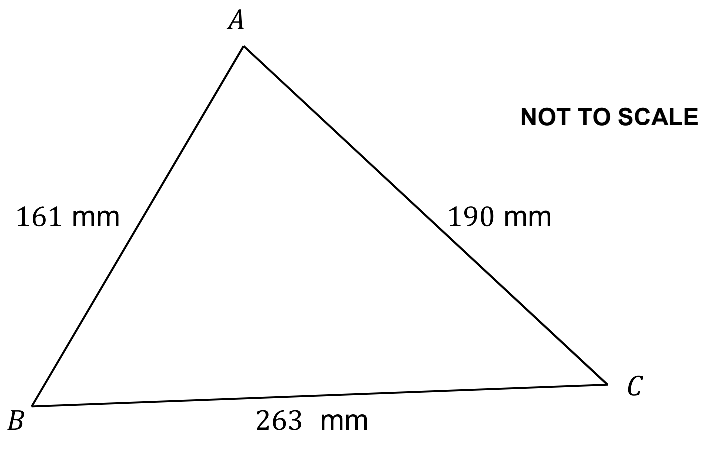

Figure 1

Figure 1 shows triangle ABC with $AB = 161$ mm, $AC = 190$ mm and $BC = 263$ mm.

Find the size of angle A. **[3]**

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Question A6

Solve the equation **[4]**

$$\log_2(3x-1)-\log_2(2x+3) = -3 \qquad (\, x > \frac{1}{3})$$

All working must be shown.

Question A7

Find the equation of the normal to the curve $y = x^3 - 14x + 8$

at the point $(2, -12)$. **[4]**

Question A8

A curve has equation $-5x^2 + x^3y^2 - 7y = 1$.

Find an expression for $\frac{dy}{dx}$ in terms of x and y. **[4]**

Question A9

Function $f(x)$ is defined as $f(x) = x - 3$

Function $g(x)$ is defined as $g(x) = \frac{x+7}{2}$

Solve the equation $(f(x))^{-1} = g^{-1}(x)$ **[5]**

Question A10

Show that $\cot\theta + \tan\theta = \csc\theta\sec\theta$ **[3]**

Each stage of your working must be clearly shown.

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Question A11

The function $g(x)$ is defined as $g(x) = x^3 - 340$.

Starting with $x = 7$, apply the Newton-Raphson method once to obtain a better approximation.

Give your answer to **4** significant figures.

[3]

In this question, 1 mark will be given for the correct use of significant figures.

Question A12

Write $\frac{5x^2 - 13x + 23}{(x-4)(x^2+1)}$ in the form $\frac{A}{x-4} + \frac{Bx + C}{x^2 + 1}$ where A, B and C are constants to be determined.

[4]

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Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) The function $f(x)$ is defined as $f(x) = x^4 - 2x^3 - 13x^2 - 4x - 30$.

i. Use the Remainder Theorem to the find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

ii. Divide $f(x)$ by $(x^2 + 2)$ and hence factorise $f(x)$ completely. **[4]**

b) The quadratic equation $2x^2 + kx - 3k = 0$ (where $k \neq 0$) has one real root.

Find the value of k. **[3]**

c) i. An arithmetic series has first term 28 and common difference d.

A second arithmetic series has first term 20 and the common difference is also d.

The sum of the first 20 terms of the first series is the same as the sum of the first 21 terms of the second series.

Find the value of d. **[4]**

ii. Explain why the geometric series $243 + 81 + 27 + \ldots$ will never reach 365. **[3]**

d) The line with equation $12x + 5y - 60 = 0$ crosses the $x-$ axis at point X and crosses the $y-$ axis at point Y.

Find the length of the line XY. **[4]**

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Question B2

a) A compressor is switched on and operates in such a way that the pressure, P (measured in $\mathrm{kN\ m^{-2}}$) after t minutes from the time the compressor was switched on is given by the formula

$$P = 100e^{kt}$$

where k is a constant.

When $t = 2,\ P = 144$.

i. Show that $k = \ln\left(\frac{6}{5}\right)$. **[3]**

__Each__ stage of your working must be clearly shown.

ii. Find the pressure after 3 minutes. **[2]**

iii. Find the value of $\frac{dP}{dt}$ when $t = 4$. **[3]**

iv. Explain what your answer to part iii means. **[1]**

b) Solve the equation $3(\log_8 x)^2 - 11(\log_8 x) + 6 = 0$. *Working must be shown*. **[4]**

c)

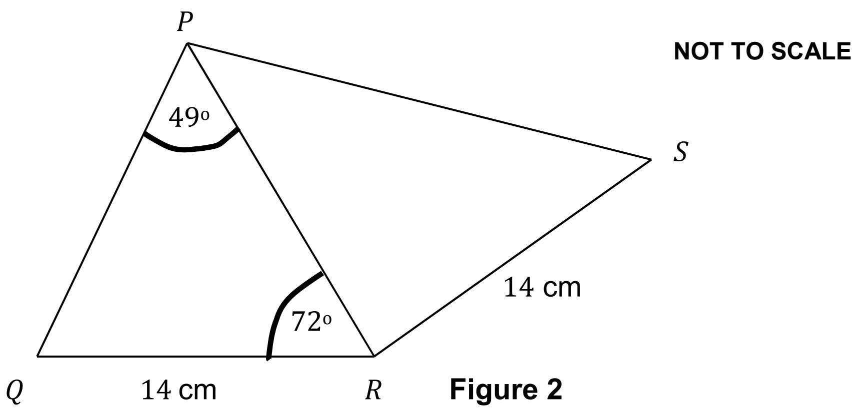

Figure 2

Figure 2 shows the quadrilateral $PQRS$ which is made up of two acute-angled triangles. QR and RS are both 14 cm long. Angle $QPR = 49°$ and angle $PRQ = 72°$.

i. Find the length of PR. **[3]**

ii. The area of triangle PRS is 80 cm².

Find the size of angle PRS. **[2]**

iii. Find the shortest distance from point P to the line RS. **[2]**

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Question B3

a) The equation of a curve is given by $y = 12x^3 - 9x^4 + 5$.

i. Find $\frac{dy}{dx}$. **[2]**

ii. Show that there are stationary values at $x = 0$ and $x = 1$. **[2]**

iii. Confirm that there is a point of inflexion when $x = 0$ and determine whether the stationary value at $x = 1$ is a maximum or a minimum. **[5]**

b)

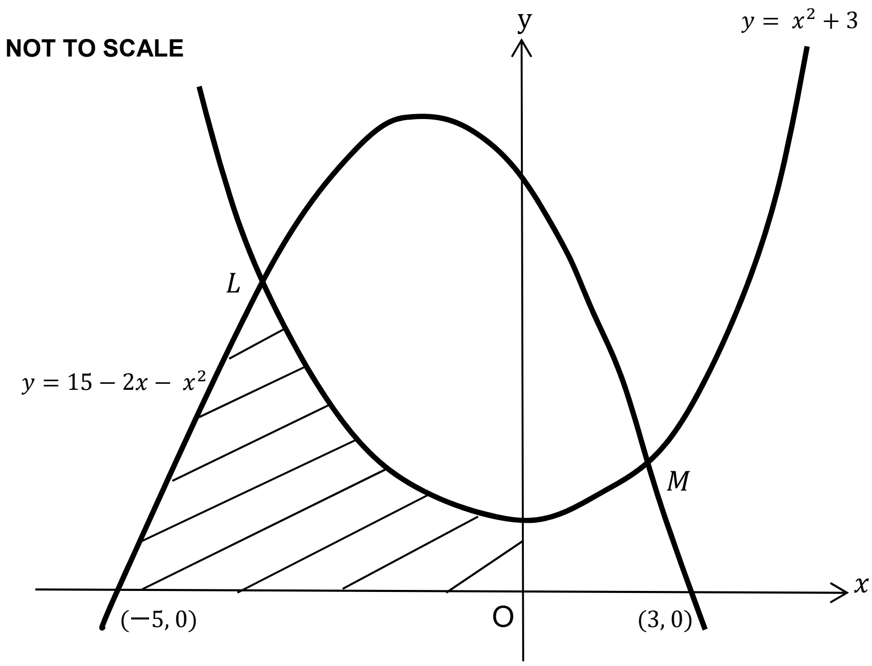

Figure 3

Figure 3 shows the curves $y = 15 - 2x - x^2$ and $y = x^2 + 3$. The curves intersect at points L and M. The curve $y = 15 - 2x - x^2$ crosses the $x -$ axis at $(-5, 0)$ and $(3, 0)$.

i. Find the coordinates of points L and M. **[4]**

ii. Find the area, which is shaded on the diagram, that is bounded by the $x -$ and $y -$ axes; and the curves $y = 15 - 2x - x^2$ and $y = x^2 + 3$. **[7]**

Show all working.

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Question B4

a) The function $f(x)$ is defined as $f(x) = |3x + 2|$.

i. Draw a sketch of $y = f(x)$ showing clearly where your graph meets or touches the $x -$ axis and the $y -$ axis. **[2]**

ii. Solve the equation $|3x + 2| = 13$. **[3]**

b) Function $g(x)$ is defined as $g(x) = 4x - 3 \quad (-\infty < x < +\infty)$.

Function $h(x)$ is defined as $h(x) = \frac{1}{x} \quad (x > 0)$.

i. Is $g(x)$ a one-one function? Give a reason. **[2]**

ii. State the range of $h(x)$. **[1]**

iii. Write down $h^{-1}(x)$. **[1]**

iv. Solve the equation $h\big(g(x)\big) = 2$. **[3]**

c) i. Use one of the addition formulae to prove that $\sin 2\theta = 2 \sin \theta \cos \theta$. **[2]**

You are given $\sin\left(\frac{\theta}{2}\right) = \frac{1}{4}$ and $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{15}}{4}$.

ii. Find the value of $\sin \theta$, giving your answer in the form $\frac{\sqrt{m}}{n}$ where m and n are integers. *You must show your working*. **[2]**

iii. Solve the equation $\sin 2\theta - \sin \theta = 0$ for $0 \le \theta \le 2\pi$. **[4]**

Give your answers as exact multiples of π.

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Question B5

a) Line l has vector equation $\boldsymbol{r} = (3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) + \mu(-5\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$ where μ is a scalar.

i. Show that point $A(-7,\ 3,\ 10)$ lies on line l. **[1]**

ii. Point B lies at $(-17, -10,\ 2)$.

Show that $\overrightarrow{AB}$ is perpendicular to line l. **[3]**

iii. You are given that point $C(-12,\ 5,\ 13)$ also lies on line l.

Find the acute angle between $\overrightarrow{CB}$ and $\overrightarrow{CA}$. *Show all working.* **[4]**

iv. Hence, or otherwise, find the area of triangle ABC. **[2]**

v. α is the obtuse angle between $\overrightarrow{CB}$ and $\overrightarrow{CA}$. <u>Without</u> doing any further calculation, write down the **exact** value of $\cos\alpha$. **[1]**

b) A machine manufactures bolts which have a mean mass of 180 grams and standard deviation 4 grams. The masses of the bolts can be assumed to follow a Normal distribution.

i. What proportion of bolts are below 177 grams? **[3]**

ii. A bolt is selected at random.

Find the probability that its mass is exactly 182 grams. **[1]**

iii. Three bolts are selected at random.

Find the probability that all three bolts are below 177 grams. **[2]**

iv. A second machine produces larger bolts with a standard deviation of 5 grams. A random sample of 16 of these larger bolts is selected and the mean mass was found to be 246.5 grams.

Find a 95% confidence interval of the mean mass of bolts produced by this second machine. **[3]**

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Question B6

a) You are given $y = \tan^{-1}x$.

i. Express x in terms of y and write down $\frac{dx}{dy}$. **[1]**

ii. Hence show that $\frac{dy}{dx} = \frac{1}{1+x^2}$. **[3]**

iii. Hence solve the differential equation

$$\frac{dy}{dx} = \frac{1}{y(1+x^2)} \quad (0 < x < \frac{\pi}{2})$$

given $y = 0$ when $x = 1$.

Give your answer in the form $y = f(x)$. *Show all of your working.* **[5]**

b) i. By writing $\csc x$ as $\frac{1}{\sin x}$, show that the differentiation of $\csc x$ is $-\csc x \cot x$. **[3]**

ii. **Use the substitution** $u = x^3 - 1$ to evaluate

$$\int_{-1}^{0} \frac{x^2}{(x^3-1)^2}\ dx$$ **[4]**

Show all working.

iii. The curve $y = \sqrt{(x^2 - 2x + 3)}$ is rotated about the $x -$ axis between

$x = 3$ and $x = 6$.

Find the volume formed. *Show all working.* **[4]**

This is the end of the examination.

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