THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

MARK SCHEME 2016-2017

Notice to Markers

Significant Figures:
All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A9. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:
Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

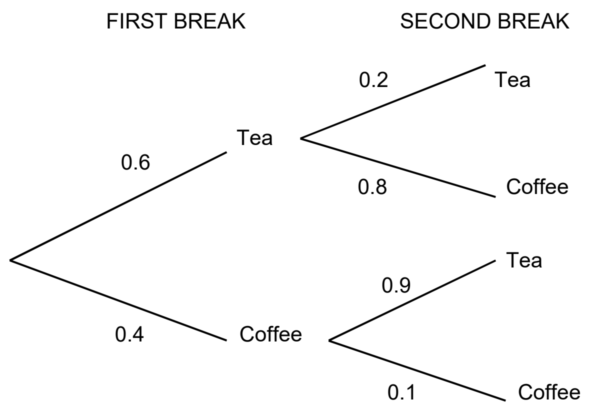
Uses a correct method to find one unknown **[M1]**

Uses a correct method to find the second unknown. **[M1]**

$m = \frac{3}{2}$ or equivalent **[A1]**

$n = -\frac{2}{3}$ or equivalent, or anything rounding to -0.667 **[A1]**

Question A2



First branches correct **[B1]**

Second branches correct **[B1]**

their $0.6 \times$their 0.8,   their $0.4 \times$their 0.9 **[M1]**

Adds their expressions **[M1]**

$= 0.84$ or equivalent **[A1]**

Question A3

$k^2 - 4 \times 2 \times 8 \ < 0$ **[M1]**

$k \ > \ -8$ and $k \ < 8$ **[A1]**

Largest possible value is 7. **[A1]**

Question A4

$S_{65} = \frac{65}{2}[2 \times -100 + (65-1) \times 6]$ **[M1]**

Calculates in the correct order **[M1]**

$= 5980$ **[A1]**

Question A5

$\log_3 12^3 - \log_3 49^{1/2} + \log_3 21 - \log_3 4^3$ Uses log power law **[M1*]**

$\log_3(\frac{1728 \times 21}{7 \times 64})$ Uses log addition or subtraction law **[M1*]**

$= \log_3 81$ **[A1]**

$= 4$ (If the correct answer appears with no working, this scores 0) **[A1]**

Question A6

$\sin\theta = \pm\frac{\sqrt{3}}{2}$ **[M1]**

$\theta = -\frac{\pi}{3}$ and $\frac{\pi}{3}$ **(A1)** for each correct answer **[A2]**

Question A7

$\frac{dy}{dx} = 6x^2 - 10x$ **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$ $(= 4)$ **[M1]**

Inverts and changes sign $(= -\frac{1}{4})$ **[M1]**

$y + 4 = -\frac{1}{4}(x - 2)$ or in any equivalent form. **[A1]**

Question A8

Rearranges and exchanges x and y $[f^{-1}(x) = \ln\left(\frac{y^2+2}{3}\right)]$ or uses any other valid method to find the inverse function **[M1*]**

Substitutes $x = 7$ into their $f^{-1}(x)$ **[M1]**

$= \ln 17$ **[A1]**

or sets $f(x) = 7$ and attempts to solve **(M1*)**

Reaches a value for e^x $(= 17)$ **(M1)**

$= \ln 17$ **(A1)**

Question A9

Length = $\sqrt{[(2--3)^2+(-1--7)^2+(5-8)^2]}$ or equivalent

Any one component correct **[M1]**

Correct expression (does not need to be simplified) **[M1]**

$= \sqrt{70}$ or $8.3666...$ **[A1+]**

$= 8.37$ to three significant figures **[A1ft]**

+This mark can be implied if this line is not seen but the 8.37 appears.
Allow follow through but a more accurate answer must be seen earlier.

Question A10

Quotient Rule used in its correct form **[M1*]**

$\frac{dy}{dx} = \frac{(x^2+1)(3x^2) - x^3(2x)}{(x^2+1)^2}$ Correct answer which does not need to be simplified **[A1]**

Substitutes $x = 1$ into their $\frac{dy}{dx}$ **[M1]**

$=1$ **[A1]**

Question A11

Uses integration by parts in right direction **[M1*]**

$= 9x \times \frac{1}{3}e^{3x}$ **(A1)** $-\int_{1/3}^{1} 9 \times \frac{1}{3}e^{3x}\ dx$ (Correct first part) **[A1]**

$= 3xe^{3x} - e^{3x}$ (Correct answer - ignore $+ c$ if this appears) **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$2e^3$ or anything rounding to 40.2 (Correct answer with no working scores 0) **[A1]**

Question A12

67% of rods are less then x cm gives $z = 0.44$ (Attempts to find value of z) **[M1]**

Their $z = \frac{x-350}{5}$ **[M1]**

$x = 352.2$ (cm) (allow 352) **[A1]**

Section B

Question B1

a) $(x-2)^2-4-3=0$ or $(x-2)^2-7=0$ **[M1*]**

Reaches $x = \pm\sqrt{}$... **[M1*]**

$2+\sqrt{7}$ or $2-\sqrt{7}$ **(A1)** for each or $2 \pm \sqrt{7}$ **(A2)** **[A2]**

b) Substitutes $x=k$ into $2x^2+5x-30$ $(2k^2+5k-30)$ **[M1*]**

Substitutes $x=2k$ into $x^2-8x+24$ $(4k^2-16k+24)$ **[M1*]**

Sets the expressions equal to each other and forms a quadratic equation $(2k^2-21k+54=0)$ **[M1*]**

Factorises or uses formula $[(2k-9)(k-6)=0$ or $k=\frac{21 \pm \sqrt{[(-21)^2-4\times 2\times 54]}}{2\times 2}]$ **[M1]**

[A1]

$k=6$ (ignore any reference to 4.5)

c) $5a = a+10\times 6$ or $a=\frac{1}{5}(a+10\times 6)$ **[M1]**

$a=15$ **[A1]**

d) i. $\frac{a}{1-\frac{3}{5}} = 3125$ **[M1]**

$a=1250$ **[A1]**

ii. Their $a\times(\frac{3}{5})^{n-1} = 2$ **[M1]**

Uses logs correctly **[M1]**

Reaches $n = \cdots$ (13.6) **[M1]**

14^{th} term **[A1]**

e) ${}^6C_3 \times 3^3 \times m^3(x^3)$ (Allow ${}^xC_y$ for ${}^yC_x$ and presence of x) **[M1]**

Sets coefficient equal to -160 and reaches $m=\cdots$ (There must now be no x) **[M1]**

[A1]

$m=-\frac{2}{3}$ or equivalent, or anything rounding to - 0.667

Question B2

a) i. *Please note: Working has been requested throughout part a. If just the correct answer appears without working, this scores 1 mark only.*

Recognises the 'hidden' quadratic equation **[M1*]**

Factorises or uses formula **[M1]**

[$(3^x - 1)(3^x - 9) = 0$ or $3^x = \frac{10 \pm \sqrt{[(-10)^2 - 4 \times 1 \times 9]}}{2 \times 1}$]

$3^x = 1$ or 9 **[M1]**

$x = 0$ or 2 **[A1]**

ii. $\frac{x^{11}}{x^{12} \times x^3}$ Correct handling of indices when multiplying or dividing **[M1*]**

$x^{-4} = \frac{1}{625}$ Correct handling of indices when using a square root **[M1*]**

$x = 5$ **[A1]**

iii. Writes $4^{8x - 5} = 4^5$ or uses logs correctly **[M1*]**

$x = \frac{5}{4}$ or equivalent **[A1]**

iv. Removes log on LHS and writes RHS as an exponential ($5x - 3 = e^2$) **[M1*]**

[A1]

$x = \frac{e^2 + 3}{5}$ or anything rounding to 2.08

b) i. $\frac{1}{2} \times 15 \times 18.5 \times \sin A = 45$ **[M1]**

$\sin A = \frac{12}{37}$ **[A1]**

Please note: this is a 'show that' question so all working must be seen.

ii. Uses a right-angled triangle, or a formula, or any valid method **[M1*]**

$\cos A = \frac{35}{37}$ (M mark scored and no errors seen) **[A1]**

iii. Uses cosine formula correctly in any form **[M1]**
($BC^2 = 15^2 + 18.5^2 - 2 \times 15 \times 18.5 \times \frac{35}{37}$)

Calculates in the correct order **[M1]**

$BC = 6.5$ (cm) **[A1]**

iv. Uses correct version of the sine formula $\left(\frac{\sin B}{18.5} = \frac{\sin \mathrm{A}}{\text{their BC}}\right)$ **[M1]**
Angle $B =$ anything rounding to 67.4 (degrees) or 1.18 (radians). **[A1]**

Question B3

a) i. $(\pi r^2 h = 6750\pi) \qquad h = \frac{6750}{r^2}$ **[B1]**

ii. *Please note: this is a 'show that' question so all working must be seen.*

$A = 2\pi r^2 + 2\pi rh$ **[M1*]**

Substitutes their h into their formula **[M1*]**

Reaches $A = 2\pi r^2 + \frac{13500\pi}{r}$ with no errors seen **[A1]**

iii. $\frac{dA}{dr} = 4\pi r - \frac{13500\pi}{r^2}$ Attempts to differentiate (sight of r in the first term or reciprocal r^2 in the second is sufficient for this mark) **[M1*]**

Correct answer **[A1]**

Sets equal to 0 and multiplies through by r^2 **[M1]**

Reaches $r^3 = \cdots (3375)$ **[M1]**

$r = 15$ (cm) **[A1]**

iv. $\frac{d^2A}{dr^2} = 4\pi + \frac{27000\pi}{r^3}$ Attempts to differentiate a second time (sight of the constant term or reciprocal r^3 is sufficient evidence for this mark). **[M1*]**

Correct answer **[A1]**

This is positive (when $r = 15$) so there is a minimum (or similar reason and conclusion) **[A1ft]**
Allow follow through if their $\frac{d^2A}{dr^2}$ is positive for their value of r.

or takes a numerical value below 15 and shows $\frac{dA}{dr} < 0$ **(M1*)**

takes a numerical value above 15 and shows $\frac{dA}{dr} > 0$ **(M1*)**

Thus there is a minimum (conclusion) (**A1ft)**

Allow follow through for their value of r and their value of $\frac{dA}{dr}$ provided a minimum occurs.

Part b) is on the next page.

Question B3 – (continued)

b) i. **[M1*]**

Substitutes $x = 2$ into one equation and shows $y = 16$.

[A1]

Substitutes $x = 2$ into second equation and shows $y = 16$.

or sets two equations equal to each other and reaches $x = 2$ **(M1*)**

Shows $y = 16$ using one equation **(A1)**

ii. Area of triangular part $= 64$ **[B1]**

Attempts to find $\int_2^6 (18 - \frac{1}{2}x^2)\ dx$ (sight of the x or x^3 term is sufficient for this mark) **[M1*]**

$18x - \frac{x^3}{6}$ Correct answer **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

Subtracts their areas **[M1]**

$= \frac{80}{3}$ or equivalent or anything rounding to 26.7 **[A1]**

(If the correct answer appears with no evidence of integration, give 4 marks out of 6.)

Question B4

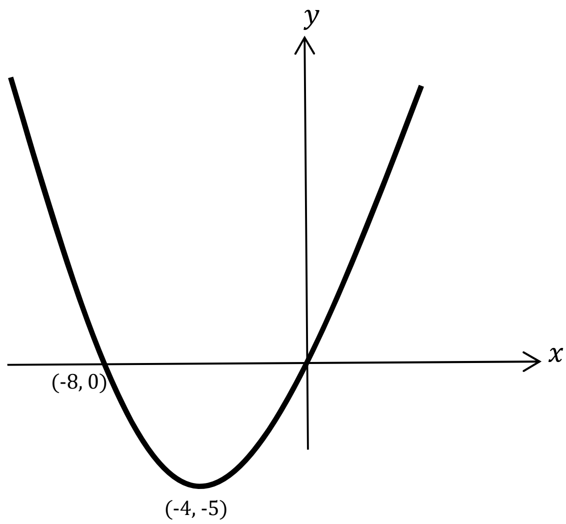
a) i. Any correct example (*e.g.* $y = \cos x$, $y = x^n$ (n even), $y = |x|$, etc. **[B1]**

ii. Substitutes $x = 9$ into their expression at some stage **[M1]**

Indicates functions being carried out in the correct order. (Sight of evaluation of $g(6)$ is sufficient evidence for this mark.) **[M1]**

$= 8$ **[A1]**

b) i.

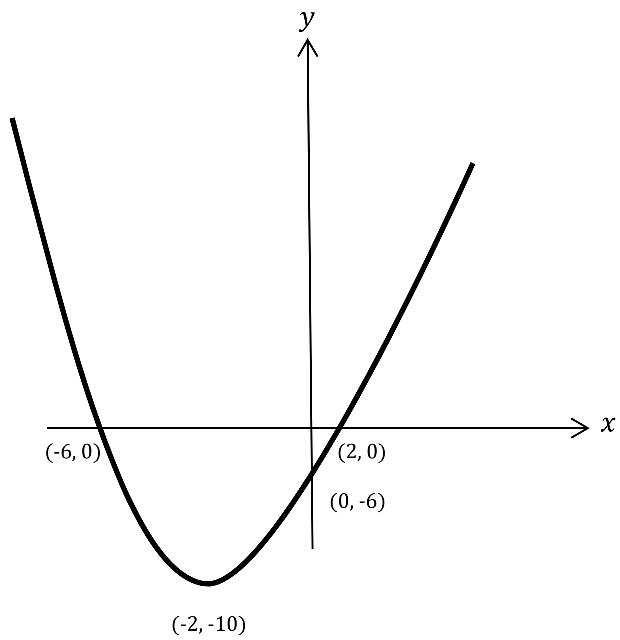


Correct shape **[B1]**

(-4, -5) **[B1]**

(-8, 0) and (0, 0)* **[B1]**
*The (0, 0) does not need to be marked if the curve clearly goes through the origin.

ii.



Correct shape **[B1]**

(-6, 0) and (2, 0) **[B1]**

(-2, -10) and (0, -6) **[B1]**

Parts c) and d) are on the next page.

Question B4 – (continued)

c) ii. *Please note: this is a 'show that' question so all working must be seen.*

Uses $\cos^2\theta + \sin^2\theta = 1$ and divides by $\sin^2\theta$ **[M1*]**

giving $1 + \cot^2\theta = \csc^2\theta$ M mark scored and no errors seen. **[A1]**

iii. Uses above result and forms a quadratic equation in $\csc\theta$. **[M1*]**
$(2\csc^2\theta - 3\csc\theta - 2 = 0)$

Factorises or uses formula **[M1]**
$[\ (2\csc\theta + 1)(\csc\theta - 2) = 0 \text{ or } \csc\ \theta = \frac{3 \pm \sqrt{[(-3)^2 - 4 \times 2 \times -2]}}{2 \times 2}]$

$\csc\theta = -\frac{1}{2} \text{ or } 2; \quad \text{or} \quad \sin\theta = -2 \text{ or } \frac{1}{2}$ **[A1]**

$\theta = 30 \text{ and } 150$ (degrees) **[A1]**

d) Writes $\sin\theta\cos\theta$ as $\frac{1}{2}\sin 2\theta$ and attempts to integrate (sight of $\cos 2\theta$ is sufficient for this mark) **[M1*]**

$= -\frac{1}{4}\cos 2\theta$ **[A1]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{1}{8}$ **[A1]**

<u>Please note</u>: a candidate scores no marks if no working is shown, or if a different method is used.

Question B5

a) i. $3 - 2\lambda = 7 + 5\mu \qquad -1 + \lambda = 8 + 3\mu \qquad 8 + 2\lambda = 12 - \mu$ **[M1]**

Takes any two equations and uses a correct method to find one unknown **[M1]**

Uses a correct method to find second unknown **[M1]**

$\lambda = 3$ **[A1]**

$\mu = -2$ **[A1]**

Confirms third equation is satisfied and states the coordinates of point $A\,(-3,\ 2,\ 14)$. **[A1]**

ii. $(-2i + \boldsymbol{j} + 2\boldsymbol{k}).(5\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}) = -10 + 3 - 2 = -9$ **[M1]**
Takes scalar product

Their scalar product = $\sqrt{[(-2)^2 + 1^2 + 2^2]} \times \sqrt{[5^2 + 3^2 + (-1)^2]} \cos\theta$ **[M1]**

$\theta = 120.5$ (degrees) or 2.10 (radians) (anything rounding to these figures) **[A1]**

iii. Solves one of $3 - 2\lambda = -7$; $-1 + \lambda = 4$; $8 + 2\lambda = 18$.
and confirms value of $\lambda (= 5)$ satisfies other two equations; or solves other two equations. **[M1*]**

iv. Finds the magnitudes of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. $(6 \text{ and } \sqrt{315})$ **[M1]**

$\frac{1}{2} \times \text{their } |\overrightarrow{AB}| \times \text{ their } |\overrightarrow{AC}| \times \sin(\text{their } \theta)$ **[M1]**

= anything rounding to 45.9 (square units). (Allow ft for their θ.) **[A1ft]**

b) $-6x + 2xy + x^2\frac{dy}{dx} - 5\frac{dy}{dx} = 0$ Correct use of Product Rule **[M1*]**
Correct implicit differentiation (sight of $x^2\frac{dy}{dx}$ or $5\frac{dy}{dx}$ is sufficient) **[M1*]**

Factorises and obtains an expression for $\frac{dy}{dx}$. (This mark can only be given if there are at least two $\frac{dy}{dx}$ terms. $\left(= \frac{6x - 2xy}{x^2 - 5}\right)$ **[M1]**

Substitutes $x = 1$ and $y = 2$ into their $\frac{dy}{dx}$. **[M1]**
$= -\frac{1}{2}$ **[A1]**

c) Correct use of Chain Rule (sight of $9\sin^8 x$ or $\cos x$ is sufficient for this mark) **[M1]**

$= 9\sin^8 x \cos x$ **[A1]**

Question B6

a) i. Any answer rounding to 1.284 **[B1]**

ii. Area $\approx \frac{0.25}{2}[1 + 2.718 + 2(1.064 + \text{their } c + 1.755)]$ sight of $\frac{0.25}{2}$ **[M1*]**

Contents of square brackets **[M1*]**

= anything rounding to 1.49 **[A1]**

b) i. $x - 5 = A(x - 2) + B(x + 1)$ **[M1]**

$A = 2$, $B = -1$. **(A1)** for each **[A2]**

ii. Separates the variables $\left(\frac{dy}{y} = \frac{(x-5)\ dx}{(x+1)(x-2)}\right)$ **[M1*]**

Attempts to integrate both sides (Sight of $\ln$ ... on each side is sufficient for this mark) **[M1*]**

$\ln y = 2\ln(x+1) - \ln(x-2) + c$ (If the constant is not present, this mark is lost and no further marks can be scored.) **[A1]**

Substitutes $x = 3$ and $y = 16$ into their equation and finds a value for c $(= 0)$. **[M1*]**

$y = \frac{(x+1)^2}{x-2}$ **[A1]**

c) $\frac{du}{dx} = 1$ or $du = dx$ **[M1*]**

Writes integral in terms of u $\left(\int \frac{3(u-3)}{u}\ du\right)$ **[M1]**

Splits integrand and attempts to integrate (a term in u or $\ln u$ is sufficient) **[M1]**

$3(x+3) - 9\ln(x+3) + c$ (No marks are given for any other method.) **[A1]**

d) Uses correct formula $[\pi \int_1^2 (3x^2 + 4x + 2)\ dx]$ **[M1*]**

Attempts to integrate (sight of x^3, x^2 or x is sufficient for this mark). This mark, and the next one, are not lost if the π has been dropped. $[\pi(x^3 + 2x^2 + 2x)]$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= 15\pi$ or anything rounding to 47.1 (If the correct answer appears with no working, award 1 mark out of 4.) **[A1]**

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