NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2016-17

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A Answer ALL questions. This section carries 45 marks.

SECTION B Answer 4 questions ONLY. This section carries 80 marks.

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Solve the simultaneous equations $8m + 9n = 6$

$6m - 3n = 11$ **[4]**

Question A2

During the day, a student has two breaks when she can choose tea or coffee.

In the first break, the probability she chooses tea is 0.6.

If she chooses tea in the first break, the probability she chooses tea in the second break is 0.2. If she chooses coffee in the first break, the probability she chooses coffee in the second break is 0.1.

Draw and label a tree diagram. Work out the probability that the student chooses different drinks in the two breaks. **[5]**

Question A3

The quadratic equation $2x^2 + kx + 8 = 0$ has no real roots.

Given that k is an integer, find the largest value k can take. **[3]**

Question A4

Find the sum of the first 65 terms of the arithmetic series $-100,\ -94,\ -88, \ldots$ **[3]**

Question A5

Write the expression

$$3\log_3 12 - \frac{1}{2}\log_3 49 + \log_3 21 - 3\log_3 4$$

in its simplest form which must contain no logarithms. *All working must be shown: just giving the answer on its own, even if it is correct, scores no marks.* **[4]**

Question A6

Solve the equation $\sin^2\theta = \frac{3}{4}$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Give your answers as exact multiples of π. **[3]**

Question A7

[4]

Find the equation of the normal to the curve $y = 2x^3 - 5x^2$ at the point $(2, -4)$.

Question A8

The function $f(x)$ is defined as $f(x) = \sqrt{(3e^x - 2)}$.

Find the **exact** value of $f^{-1}(7)$. [3]

Question A9

Point A lies at $(2,\ -1,\ 5)$ and point B lies at $(-3, -7,\ 8)$.

Find the length of $\overrightarrow{AB}$. Give your answer to 3 significant figures. [4]

In this question, one mark will be given for the correct use of significant figures.

Question A10

A curve has equation

$$y = \frac{x^3}{x^2 + 1}$$

Find the value of $\frac{dy}{dx}$ when $x = 1$. [4]

All working must be shown: just the answer on its own will score 0, even if it is correct.

Question A11

Use integration by parts to evaluate

$$\int_{\frac{1}{3}}^{1} 9x\, e^{3x}\ dx$$

Show all of your working. [5]

Question A12

Metal rods have a mean length of 350 cm and standard deviation 5 cm.

The lengths of the rods can be assumed to follow a Normal distribution.

33% of the rods are longer than x cm.

Find the value of x. [3]

Section B
Answer <u>4</u> questions ONLY. This section carries 80 marks.

Question B1

a) Solve the equation $x^2 - 4x - 3 = 0$ **by completing the square.**

Give your answers in the form $p + \sqrt{q}$ and $p - \sqrt{q}$. **[4]**

Each stage of your working must be clearly shown.

b) When $2x^2 + 5x - 30$ is divided by $(x - k)$ the remainder is the same as when $x^2 - 8x + 24$ is divided by $(x - 2k)$.

Given that k is an integer, use the Remainder Theorem to find the value of k. **[5]**

c) An arithmetic progression has common difference 6. The 11^{th} term is 5 times larger than the first term.

Find the first term. **[2]**

d) A geometric series has common ratio $\frac{3}{5}$ and the sum to infinity is 3125.

i. Find the first term. **[2]**

ii. Find the term in the series which is the first to fall below 2. **[4]**

e) In the expansion of $(3 + mx)^6$ the coefficient of the term in x^3 is -160.

Find the value of m. **[3]**

Question B2

a) Solve the equations. *All working must be shown.*

i. $3^{2x} - 10(3^x) + 9 = 0$ **[4]**

ii. $\frac{x^{11}}{(x^3)^4 \times \sqrt{(x^6)}} = \frac{1}{625}$ **[3]**

iii. $4^{8x-5} = 1024$ **[2]**

iv. $\ln(5x - 3) = 2$ **[2]**

b)

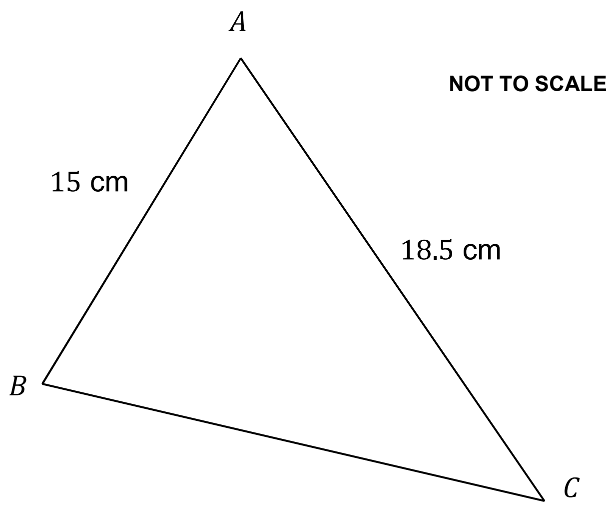

Figure 1

Figure 1 shows the acute-angled triangle ABC with $AB = 15$ cm and $AC = 18.5$ cm.

The area of triangle ABC is 45 cm^2.

i. Find $\sin A$ giving your answer in the form $\frac{m}{n}$ where m and n are integers. **[2]**

ii. Without working out the size of angle A, show that $\cos A = \frac{35}{37}$. **[2]**

iii. Find the length of BC. **[3]**

iv. Find angle B. **[2]**

Question B3

a)

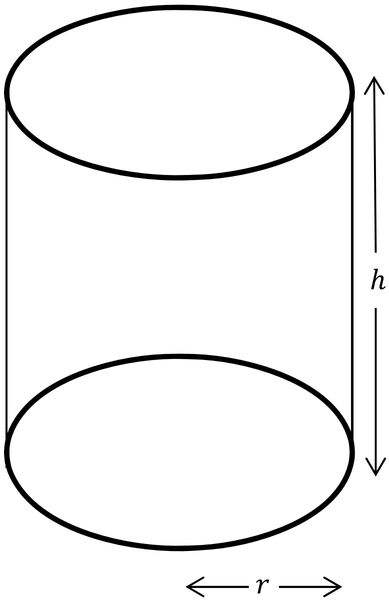

Figure 2

Figure 2 shows a solid cylinder with radius r cm and height h cm.

The volume of the cylinder is 6750π cm^3.

i. Find h in terms of r. **[1]**

ii. Show that the surface area, A, of the cylinder is given by

$$A = 2\pi r^2 + \frac{13500\pi}{r}$$

[3]

iii. Use $\frac{dA}{dr}$ to find the value of r which gives the minimum surface area. **[5]**

iv. Confirm that your value of r is a minimum. **[3]**

Part b) is on the next page.

Question B3 – (continued)

b)

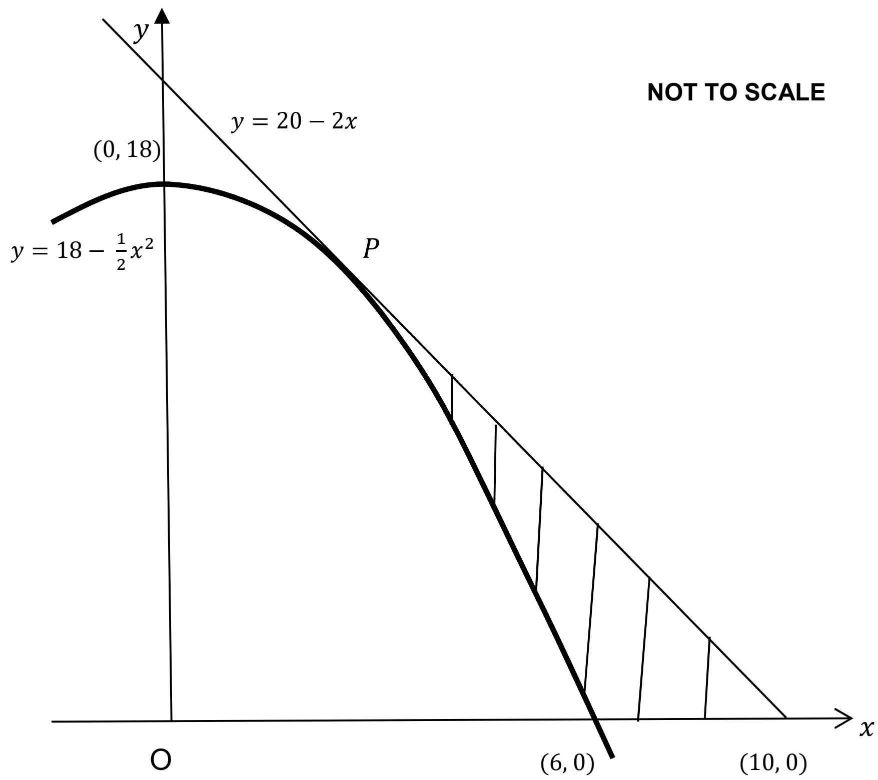

Figure 3

Figure 3 shows the curve $y = 18 - \frac{1}{2}x^2$. The line $y = 20 - 2x$ is also shown which is a tangent to the curve at point P.

The curve $y = 18 - \frac{1}{2}x^2$ crosses the $y -$ axis at $(0, 18)$ and the $x -$ axis at $(6, 0)$. The line $y = 20 - 2x$ crosses the $x -$ axis at $(10, 0)$.

i. Confirm that the coordinates of point P are $(2, 16)$. **[2]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = 18 - \frac{1}{2}x^2$, the $x -$ axis and the line $y = 20 - 2x$. **[6]**

Show all working.

Question B4

a) i. Give an example of an even function. **[1]**

Function $g(x)$ is defined as $g(x) = \sqrt[3]{(2x^3 + 80)}$ and function $h(x)$ is defined as $h(x) = \frac{7x + 3}{11}$.

ii. Find the value of $g\big(h(9)\big)$. **[3]**

b)

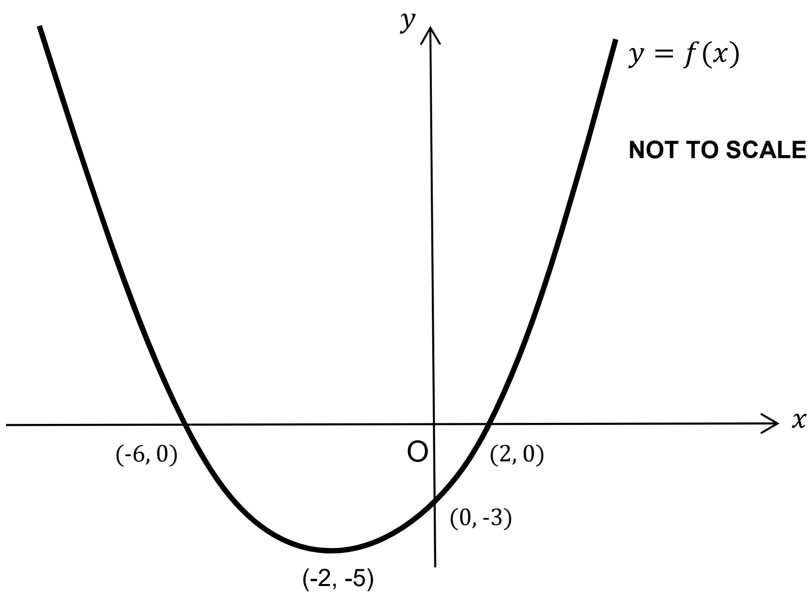

Figure 4

Figure 4 shows the function $y = f(x)$ which crosses the $x -$ axis at $(-6, 0)$ and $(2, 0)$; and crosses the $y -$ axis at $(0, -3)$. There is a stationary value at $(-2, -5)$.

On two separate sets of axes, draw sketches of the following. On each sketch, show clearly the coordinates of any stationary value and where the curve crosses the $x -$ axis and the $y -$ axis.

i. $y = f(x + 2)$ **[3]**

ii. $y = 2f(x)$ **[3]**

Parts c) and d) are on the next page.

Question B4 – (continued)

c) i. By using a suitable trigonometric formula, show that

$$1 + \cot^2\theta = \csc^2\theta$$ **[2]**

ii. Solve the equation

$$2\cot^2\theta - 3\csc\theta = 0 \qquad (0° \leq \theta \leq 360°)$$ **[4]**

Show all of your working.

d) By writing $\sin\theta\cos\theta$ as a **single trigonometric function**, evaluate

$$\int_0^{\frac{\pi}{6}} \sin\theta\cos\theta \; d\theta$$ **[4]**

All working must be shown: just the answer on its own will score 0, even if it is correct.

Question B5

a) Line l_1 has equation $\boldsymbol{r} = (3\boldsymbol{i} - \boldsymbol{j} + 8\boldsymbol{k}) + \lambda(-2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$ where λ is a scalar.

Line l_2 has equation $\boldsymbol{r} = (7\boldsymbol{i} + 8\boldsymbol{j} + 12\boldsymbol{k}) + \mu(5\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k})$ where μ is a scalar.

i. Show that lines l_1 and l_2 intersect and find the coordinates of A which is their point of intersection. **[6]**

ii. Find the **obtuse** angle between lines l_1 and l_2. **[3]**

iii. Show that point $B(-7,\ 4,\ 18)$ lies on line l_1. **[1]**

You are given that point $C(12,\ 11,\ 11)$ lies on line l_2.

iv. Find the area of triangle ABC. **[3]**

b) A curve has equation $-3x^2 + x^2y - 5y + 11 = 0$.

Find the gradient of the curve at the point $(1, 2)$. **[5]**

All working must be shown.

c) Differentiate $\sin^9 x$. **[2]**

Question B6

a) The table below shows the values of e^{x^2} (given to three decimal places where appropriate) for $x = 0,\ 0.25,\ 0.5,\ 0.75$ and 1.

x	0	0.25	0.5	0.75	1
e^{x^2}	1	1.064	c	1.755	2.718

i. Find the value of c. **[1]**

ii. Use the trapezium rule with four intervals to find an estimate of

$$\int_0^1 e^{x^2}\ dx$$

[3]

b) i. Write $\frac{x-5}{(x+1)(x-2)}$ in the form $\frac{A}{x+1} + \frac{B}{x-2}$ where A and B are constants to be determined. **[3]**

ii. Hence solve the differential equation

$$\frac{dy}{dx} = \frac{y(x-5)}{(x+1)(x-2)} \qquad (x > 2)$$

subject to $y = 16$ when $x = 3$.

Write your answer in the form $y = f(x)$ where $f(x)$ contains no logarithms. **[5]**

c) **Use the substitution** $u = x + 3$ to find

$$\int \frac{3x}{x+3}\ dx$$

[4]

Show all of your working.

d) The curve $y = \sqrt{(3x^2 + 4x + 2)}$ is rotated about the $x -$ axis between $x = 1$ and $x = 2$.

Find the volume formed. *Show all working.* **[4]**

This is the end of the examination.

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