NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering
Time-Controlled Assessment

2019-20

INSTRUCTIONS TO STUDENTS

SECTION A	**Answer ALL questions. This section carries 40 marks.**
SECTION B	**Answer THREE questions ONLY. This section carries 60 marks.**
The marks for each question are indicated in square brackets [ ].	
Guide Time: 2 hours **The Guide Time is how long you are expected to spend completing this Time-Controlled Assessment. You are allowed 24 hours in total to complete and submit.**	

- You **MUST** show **ALL** of your working. This is very important. You will score no marks if there is not enough working shown – even if your answer is correct.
- An approved calculator may be used in the assessment.
- All work must be completed independently. The penalty for collusion is a mark of zero.
- Due to the nature of the questions, there should be no need to use external sources of information to answer them. If you do use external sources of information you must ensure you reference these. Plagiarism is a form of academic misconduct and will be penalised.
- Work must be submitted by the deadline provided. Your Study Centre can be contacted only for guidance on submission of work and cannot comment on the contents of the assessment.
- Your work can be word-processed or handwritten. Once complete, any handwritten work will need to be clearly photographed/scanned and inserted into a single word-processed file for submission
- Work must be submitted in a single word-processed file using the standard NCUK cover page.

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Section A
Answer ALL questions. This section carries 40 marks.

Question A1

Solve the equations $7c + 2d = 2$ **[4]**

$5c - 6d = 7$

Question A2

In a bag, there are x red beads and y blue beads. Two beads are drawn from the bag, one after the other, with no replacement.

Write down, in terms of x and y, the probability that both beads are red. (You do not have to simplify your answer). **[3]**

Question A3

(a) Write $x^2 - 10x + 5$ in the form $(x+a)^2 + b$ where a and b are integers. **[2]**

(b) Use your answer to part (a) to solve the equation $x^2 - 10x + 5 = 0$, giving your answers in surd form. **[2]**

Question A4

Use the binomial expansion to show that

$$(4 - \tfrac{1}{2}x)^4 = 256 - 128x + 24x^2 - 2x^3 + \tfrac{1}{16}x^4$$ **[3]**

Question A5

If $\log_2(8x^2) = c$, write $\log_2 x$ in terms of c. **[3]**

Question A6

Solve the equation $\cos 2\theta = -0.53$ $(0 < \theta < 180°)$ **[4]**

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Question A7

Evaluate **[4]**

$$\int_0^a (x-a)^2 \ dx$$

Where a is a constant, giving your answer in terms of a in its simplest form.

Question A8

Prove that **[3]**

$$\csc 2\theta - \cot 2\theta \ \equiv \tan\theta$$

Question A9

A curve has equation $y = \sin 3x \cos 2x$.

(a) Find $\frac{dy}{dx}$. **[2]**

(b) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$. Give your answer to **4** significant figures. **[3]**

In this question, 1 mark will be given for the correct use of significant figures.

Question A10

Function $f(x)$ is defined as $f(x) = x^2 - 6x \quad (x > 2)$

Function $g(x)$ is defined as $g(x) = 2x - 7 \quad (x > 2)$

Given that $f[g(a)] = 27$, find the value of a. **[3]**

(Remember to show every stage of your working)

Question A11

Use the substitution $u = x^2 - 2x + 5$ to find **[4]**

$$\int (x-1)(x^2-2x+5)^4 \ dx.$$

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Section B
Answer <u>THREE</u> questions ONLY. This section carries 60 marks.

Question B1

(a) Point A lies at $(-6, -2)$ and point B lies at $(9, 7)$. AB is perpendicular to the line with equation $10x + py - 7 = 0$.

Find the value of p. **[3]**

(b) **i.** Solve $2x - 10 \geq 3$. **[1]**

ii. Solve $x^2 \leq 64$. **[2]**

iii. State the range of values which satisfy both $2x - 10 \geq 3$ and $x^2 \leq 64$. **[1]**

iv. Is your answer to part iii an open interval or a closed interval? Give a reason for your answer. **[1]**

(c) When $x^3 - 2x^2 + kx + 2k$ is divided by $(x - 2)$, the remainder is the same as when $x^3 + 2x^2 - 2kx + k$ is divided by $(x + 3)$.

<u>Use the Remainder Theorem</u> to find the value of k. **[4]**

(d) The 3^{rd} term of a geometric series is 1152 and the 6^{th} term is 3888.

Find the common ratio and the first term. **[5]**

(e) An arithmetic series has common difference 7 and the sum of the first 20 terms is 1590.

Show that the 6^{th} term is 48. **[3]**

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Question B2

(a) The variables x and y are connected by the formula

$$y = e^{2x} + e^{x} - x.$$

i. Find the exact value of y when $x = \ln 3$, giving your answer in its simplest form. **[2]**

The graph of $y = e^{2x} + e^{x} - x$ has one stationary value.

ii. Find the exact value of the $x-$ coordinate where this stationary value occurs. **[4]**

(b) Given that

$$\log_x x^3 + \log_x 3 + \log_x (x-6) = 3\log_y y \quad (x > 0)$$

where y is a positive constant, find the value of x. **[4]**

(c) Find the value of u if

$$\left[\frac{40u^7 \div 5u}{3u^4 \times 9u^5}\right]^{2/3} = \frac{1}{16}$$ **[3]**

(d)

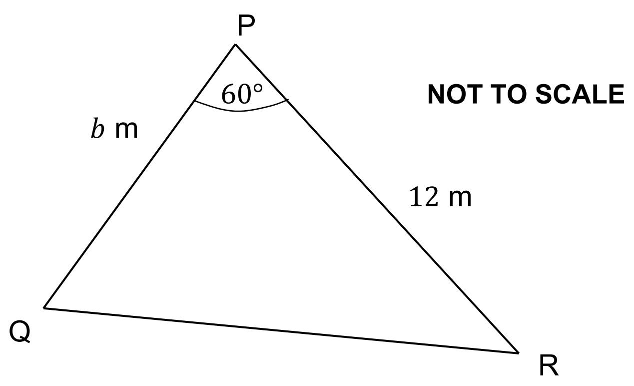

Figure 1

Figure 1 shows the acute-angled triangle PQR with PQ = b metres, PR = 12 metres and angle QPR = $60°$.

The area of triangle PQR is $33\sqrt{3}$ cm^2.

i. Find the value of b. **[2]**

ii. Find the length of QR, giving your answer in surd form. **[3]**

iii. Find angle PQR. **[2]**

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Question B3

(a)

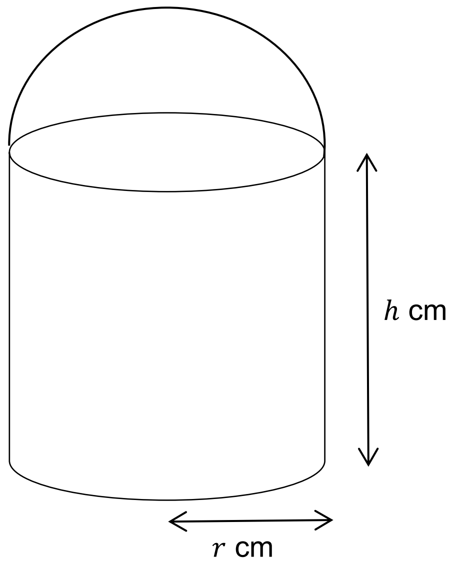

Figure 2

In this question, you are given the volume of a hemisphere (half of a sphere) with radius r is $\frac{2}{3}\pi r^3$ and its surface area is $2\pi r^2$.

Figure 2 shows a solid piece of metal which is made up of a cylinder of radius r cm and height h cm with a hemisphere attached to its top.

The total surface area of the solid is 180π cm^2.

i. Find h in terms of r. **[2]**

ii. Show that the volume of the solid, V, is given by

$$V = 90\pi r - \frac{5}{6}\pi r^3$$

[3]

iii. Use calculus to find the value of r which gives the maximum volume. **[4]**

iv. Confirm that your value of r give a maximum. **[3]**

Part (b) is on the next page.

Question B3 – (continued)

(b)

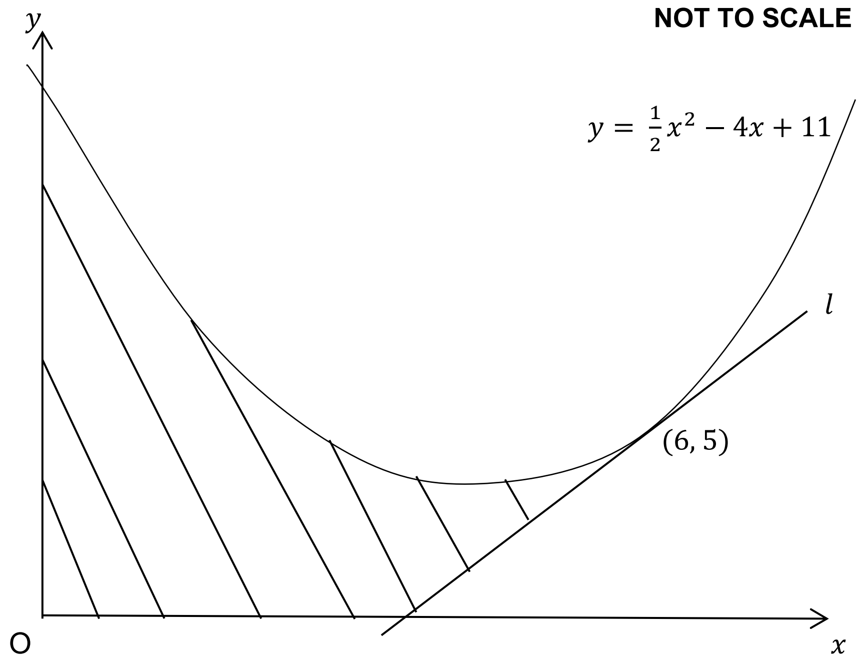

Figure 3

Figure 3 shows the curve $y = \frac{1}{2}x^2 - 4x + 11$ and line l which is a tangent to the curve at $(6, 5)$.

i. Find the equation of line l, giving your answer in the form $y = mx + c$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by the curve $y = \frac{1}{2}x^2 - 4x + 11$, line l and both axes. **[5]**

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Question B4

(a) Line l_1 has equation $\boldsymbol{r} = (3\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}) + \lambda(2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$

Line l_2 has equation $\boldsymbol{r} = (-3\boldsymbol{i} + 7\boldsymbol{j} + 17\boldsymbol{k}) + \mu(3\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k})$

where λ and μ are scalars.

i. Show that lines l_1 and l_2 intersect and find the coordinates of point A, which is their point of intersection. **[6]**

ii. Find the acute angle between lines l_1 and l_2 . **[3]**

Point B lies on l_2 and has coordinates $(a,\ b, -19)$.

iii. Find the values of a and b. **[1]**

iv. Find the shortest distance from point B to line l_1 . **[3]**

(b) i. Solve $3\cos 2\theta - \cos\theta + 1 = 0$ $\quad (0° \leq \theta \leq 360°)$ **[4]**

ii. If $\tan A = \frac{1}{p}$ and $\tan B = \frac{1}{q}$, show that $\tan(A + B) = \frac{p + q}{pq - 1}$. **[3]**

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Question B5

(a) <u>Use the Quotient Rule</u> to differentiate $y = \frac{x}{1 - x^2}$.

Simplify your answer. **[2]**

(b) Solve the differential equation $x^2 y \left(\frac{dy}{dx}\right) = x + 3$

subject to $y = 2$ when $x = 1$.

Give your answer in the form $y = f(x)$. **[4]**

(c) i. Show that $\int 6x^2 \ln x \; dx = 2x^3 \ln x - \frac{2}{3}x^3 + c$ **[3]**
where c is a constant

ii. Use the result in part i. to explain why **[2]**

$$\int 6x^2 \ln\left(\frac{1}{x}\right) dx = -2x^3 \ln x + \frac{2}{3}x^3 - c$$

(d) i. Show that $\frac{1}{(x+1)(x+m)}$ can be expressed in partial fractions in the form $\frac{1}{k}\left(\frac{1}{x+1} - \frac{1}{x+m}\right)$ where k and m are constants. Find k in terms of m. **[4]**

ii. Hence find

$$(m-1)\int_0^1 \frac{1}{(x+1)(x+m)} \; dx.$$

Give your answer as a single logarithm **[5]**

- This is the end of the Time-Controlled Assessment. -

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