THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A5. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Uses any method to find one unknown **[M1]**

Uses any method to find second unknown **[M1]**

$x = \frac{2}{3}$ **[A1]**

$y = -3$ **[A1]**

Question A2

0.7^2 and 0.3^2 seen **[M1]**

Adds their probabilities **[M1]**

0.58 or equivalent (Accept a fraction or a percentage) **[A1]**

Question A3

Please note: the Remainder Theorem must be used.

Substitutes $x = -2$ into first expression $(10 - 2k)$ **[M1*]**

Substitutes $x = -3$ into second expression $(51 - 3k)$ **[M1*]**

Doubles the second remainder or halves the first, sets expressions equal to each other and reaches a value for k. **[M1]**

$k = 23$ **[A1]**

Question A4

$4^4 + {}^4C_1 \times 4^3 \times (-6x) + {}^4C_2 \times 4^2 \times (-6x)^2 + {}^4C_3 \times 4 \times (-6x)^3 + (-6x)^4$

Any two unsimplified correct **(B1)**; all unsimplified correct **(B2)** [Allow ${}^xC_y$ for ${}^yC_x$]

$= 256 - 1536x + 3456x^2 - 3456x^3 + 1296x^4$

Any two correct **(B1)**; all correct **(B1)**

[B4]

Question A5

Takes logs correctly $[(4x-3)\log 7 = \log 33]$ **[M1]**

Rearranges correctly and reaches a value for x **[M1]**

$x = 1.1992 \ldots$ (can be implied) **[A1]**

$= 1.20$ to 3 significant figures (Allow follow through) **[A1ft]**

Question A6

$\sin\theta = \pm\frac{\sqrt{3}}{2}$ **[M1]**

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$. **(A1)** any two correct; **(A2)** all correct. **[A2]**

Extra solutions in the range loses 1 mark. Ignore solutions outside the range.

Question A7

$5\cos^4 x$ or $\pm\sin x$ seen **[M1]**

$\left(\frac{dy}{dx} = \right) 5\cos^4 x(-\sin x)$ **[A1]**

Substitutes $x = \frac{\pi}{6}$ into their $\frac{dy}{dx}$ **[M1]**

$= -\frac{45}{32}$ or equivalent, or anything rounding to -1.41 **[A1]**

Question A8

Attempts to integrate (sight of x^2 or x is sufficient for this mark) **[M1*]**

Substitutes limits into their integrated expression, subtracts the right way round, sets equal to 15 and forms a quadratic equation $[\,3a^2 - 4a - 15 = 0\,]$ **[M1]**

Factorises or uses formula $[(3a+5)(a-3) = 0$ or $a = \frac{4 \pm\sqrt{[(-4)^2 - 4 \times 3 \times -5]}}{2 \times 3}]$ **[M1]**

$a = -\frac{5}{3},\ 3.$ **[A1]**

Question A9

$f'(x) = 2x$ **[M1]**

Substitutes $x = 8$ into Newton-Raphson formula $\left(8 - \frac{8^2 - 68}{2 \times 8}\right)$ **[M1*]**

8.25 **[A1]**

Question A10

Finds $h^{-1}(x)$ by rearranging and exchanging x for y at any stage $\left(\frac{\cos^{-1}x}{2}\right)$ **[M1]**

Substitutes $x = \frac{1}{2}$ into their $h^{-1}(x)$ and finds a value [$30°$ but allow $\frac{\pi}{6}$] **[M1]**

[The first two marks can be gained by setting $h(x) = \frac{1}{2}$ **(M1)** and solving **(M1)**]

Applies composite function in the correct order **[M1]**

$\frac{1}{\sqrt{3}}$ or equivalent but must be exact. **[A1]**

Question A11

Finds the value of z which gives $\phi(z) = 0.67$ $\quad (z = 0.44)$ **[M1]**

$\frac{x - 120}{10} = 0.44$ **[M1]**

$x = 124.4$ (kg) (Allow 124) **[A1]**

Question A12

Separates variables $\left(\frac{1}{y}dy = \frac{1}{x + 5}\,dx\right)$ **[M1*]**

Attempts to integrate both sides [$\ln y$ or $\ln(x + 5)$ seen is sufficient for this mark] [$\ln y = \ln(x + 5) + c$]. There must be a constant on one side: if it is missing, this mark and all subsequent ones are lost. **[M1*]**

Substitutes boundary conditions into equation to find the constant ($\ln 3$ if on RHS) **[M1]**

Combines logs correctly at any stage **[M1]**

$y = 3(x + 5)$ or equivalent but there must be no logs present. **[A1]**

Section B

Question B1

a) i. $x > 3$ **[B1]**

ii. Solves $2x^2 - 3x - 44 = 0$ by factorising or using formula **[M1]**

$[(2x-11)(x+4) = 0$ or $x = \frac{3 \pm \sqrt{[(-3)^2 - 4 \times 2 \times -44]}}{2 \times 2}]$

Finds two critical values $(-4, \frac{11}{2})$ **[M1]**

$x \geq -4$ **(A1)*** $x \leq \frac{11}{2}$ **(A1)*** or $-4 \leq x \leq \frac{11}{2}$ **(A1)** for each end **[A2]**

*Please note: if this version of the answers is quoted, the two ranges can be separated by a space, a comma or the word 'and'. The final mark is lost if the word 'or' is seen.

iii. $4, 5$ **[B1]**

b) i.

$$
\begin{array}{r|l}
 & x^2 - x - 20 \\
\hline
3x - 1 & 3x^3 - 4x^2 - 59x + 20 \\
 & \underline{3x^3 - x^2} \\
 & \quad -3x^2 - 59x \\
 & \quad \underline{-3x^2 + x} \\
 & \qquad -60x + 20 \\
 & \qquad \underline{-60x + 20} \\
 & \qquad \underline{\cdots\cdots\cdots}
\end{array}
$$

Correct first division **[M1]**

Any correct subsequent division **[M1]**

Correct quotient **[A1]**

ii. $(3x-1)(x-5)(x+4)$ **[A1]**

c) i. $36 = a + (19-1) \times 11$ **[M1]**

$a = -162$ **[A1]**

ii. $S_{40} = \frac{40}{2}[2 \times \text{their } a + (40-1) \times 11]$ **[M1]**

Calculates correctly in the right order **[M1]**

$= 2100$ **[A1]**

d) i. $a \times (1.5)^{10} = 236196$ **[M1]**

$a = 4096$ **[A1]**

ii. $S_8 = \frac{\text{their } a[(1.5)^8 - 1]}{1.5 - 1}$ **[M1]**

$= 201760$ (Only accept the full answer with no rounding off here) **[A1]**

iii. The series is not convergent (or similar explanation) [Accept $r > 1$] **[B1]**

Question B2

a) i. 1520 (degrees) **[B1]**

ii. Substitutes $t = 3$ into formula **[M1]**

Anything rounding to 1350 (degrees) **[A1]**

iii. $\left(\frac{d\theta}{dt} = \right) - 0.04 \times 1500 \times e^{-0.04t}$ (No need to simplify) **[B1]**

iv. $-47.2 = \text{their } \frac{d\theta}{dt}$ and reaches $e^{-0.04t} = \cdots \left(\frac{-47.2}{-0.04 \times 1500}\right)$ **[M1]**

Takes logs correctly and reaches $-0.04t = \cdots \left[\ln\left(\frac{-47.2}{-0.04 \times 1500}\right)\right.$ **[M1]**

Anything rounding to 6 (hours) **[A1]**

Special case: if the negative sign in front of the 47.2 is missed, but candidate proceeds correctly to second line, award 1 mark out of 3.

b) $\log_8\left(\frac{x^2}{x(x - 3)}\right) = \log_8 4$

Combines logs correctly or uses correct power law **[M1*]**

Adapts RHS and removes logs at the right time **[M1*]**

Solves $\left(\frac{x}{x - 3} = 4\right)$ **[M1]**

$x = 4$ **[A1]**

[If the factorisation and cancelling are missed, the third M mark can be scored for forming a quadratic equation $(3x^2 - 12x = 0)$ and solving $(0, 4)$. If the 0 is not discarded (placing in brackets is good enough to show non-inclusion) the final mark is lost.]

c) i. Uses the cosine formula $(4384 = 68^2 + 60^2 - 2 \times 68 \times 60 \times \cos\theta)$ **[M1]**

Calculates correctly and in the right order **[M1]**

$\cos\theta = \frac{8}{17}$ or equivalent but must be in this form. **[A1]**

ii. Uses $\cos^2\theta + \sin^2\theta = 1$ or a right-angled triangle, or any valid method **[M1*]**

$\sin\theta = \frac{15}{17}$ (No errors seen and M mark scored) **[A1]**

iii. $\frac{1}{2} \times 68 \times 60 \times \frac{15}{17} = 3 \times \frac{1}{2} \times 60 \times PQ \times \sin 30$ **[M1]**

$PQ = 40$ (cm) **[A1]**

iv. Uses sine formula $\left(\frac{\sin PRS}{68} = \frac{15/17}{\sqrt{4384}}\right)$ **[M1]**

Anything rounding to 65 (degrees) or 1.13 radians **[A1]**

Question B3

a) i. $V = x(60 - 2x)(60 - 2x)$ **[M1*]**

At least one intermediate line of working

Reaches required result with no errors seen and the M mark scored **[A1]**

ii. Attempts to differentiate (sight of x or x^2 is sufficient for this mark) ($\frac{dV}{dx} = 3600 - 480x + 12x^2$) **[M1*]**

Sets equal to 0 (can be implied) **[M1]**

Factorises or uses formula **[M1]**

$[(x - 10)(x - 30) = 0 \text{ or } x = \frac{40 \ \pm\sqrt{[(-40)^2 - 4 \times 1 \times 300]}}{2 \times 1}]$

$x = 10,\ 30$ **[A1]**

iii. Attempts to differentiate a second time (sight of the constant term or x is sufficient for this mark) **[M1*]**

$\frac{d^2V}{dx^2} = -480 + 24x$ (Correct answer) **[A1]**

This is negative when $x = 10$ so there is a maximum. **[A1ft]**

This is positive when $x = 30$ so there is a minimum. (Allow follow through for their $\frac{d^2V}{dx^2}$) **[A1ft]**

or Takes a numerical value between 0 and 10 and shows $\frac{dV}{dx} > 0$ **(M1*)**

Takes a numerical value between 10 and 30 and shows $\frac{dV}{dx} < 0$ **(M1*)**

Takes a numerical value above 30 and shows $\frac{dV}{dx} > 0$ **(M1*)**

Thus there is a maximum when $x = 10$ and a minimum when $x = 30$. **(A1ft)** (Allow follow through for their $\frac{dV}{dx}$)

or an argument along the lines: when $x = 30, V = 0$ **(M1*)** thus it must be a minimum **(A1).** As a cubic function with two stationary values must have a maximum and a minimum **(M1*)**, $x = 10$ must be a maximum **(A1).**

Part iv and all of part b) are on the next page.

Question B3 – (continued)

iv. Substitutes their maximum x value into the formula for the volume **[M1]**

$= 16000$ (cm^3) (Allow follow through provided answer is positive) **[A1ft]**

b) i. Attempts to differentiate (sight of x term or $+\,8$ is sufficient for this mark) $(-2x+8)$ **[M1*]**

Substitutes $x = 3$ into their $\frac{dy}{dx}$ (2) inverts and changes sign $(-\frac{1}{2})$ **[M1]**

$y = -\frac{1}{2}x + \frac{19}{2}$ (must be in this form) **[A1]**

ii. Line l meets $y -$ axis at $(0, \frac{19}{2})$

Finds area below line l $\left(\frac{\text{their } \frac{19}{2} + 8}{2} \times 3\right)$ $(= \frac{105}{4})$ **[M1]**

Attempts to integrate $\int_1^3 (-x^2 + 8x - 7)\,dx$ (Sight of any $x -$ term with the index raised by 1 is sufficient for this mark) $(-\frac{x^3}{3} + 4x^2 - 7x)$ **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. $6 - (-\frac{10}{3})$ **[M1]**

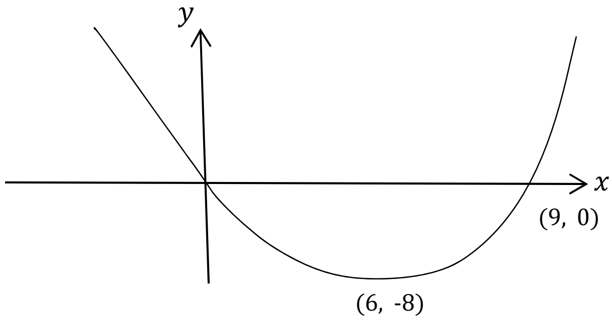
Subtracts areas $(\text{their } \frac{105}{4} - \text{their } \frac{28}{3})$ **[M1]**

$= \frac{203}{12}$ or equivalent or anything rounding to 16.9 **[A1]**

[If the $y -$ coordinate of the intersection of line l with the $y -$ axis, is wrong, it is still possible to score the first four marks]

Question B4

a)



correct shape **[B1]**

(6, -8) shown **[B1]**

(0, 0)* and (9, 0) shown. *(0, 0) does not need to be marked if the curve clearly goes through the origin **[B1]**

b) i. $g(x) > 1$ (Accept $y > 1$ but not $x > 1$) **[B1]**

ii. Finds $g^{-1}(x)$ [A clear showing of correct rearranging and exchanging x and y (if used) at some stage] $[\frac{1}{3}\ln(2x-1)]$ **or** sets $g(x) = \frac{9}{2}$ **[M1]**

Substitutes $x = \frac{9}{2}$ into their $g^{-1}(x)$ **or** solves $g(x) = \frac{9}{2}$ **[M1]**

$= \ln 2$ (Must be in this form) **[A1]**

iii. Finds $h^{-1}(x)$ $[\frac{12x + 7}{5}]$ [Same criteria as for finding $g^{-1}(x)$] **[M1]**

sets equal to $h(x)$ and solves **[M1]**

$x = -1$ **[A1]**

c) i. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ **[M1*]**

$= \frac{1}{\sin\theta\cos\theta}$ **[M1*]**

$= \frac{1}{½\sin 2\theta}$ **[M1*]**

$= \frac{2}{\sin 2\theta}$ (All 3 M marks scored and no errors seen) **[A1]**

ii. Uses previous result, sets $\frac{2}{\sin 2\theta} = -4$, and reaches $\sin 2\theta = \cdots (-\frac{1}{2})$ **[M1*]**

Realises search is from 0 to 360 degrees **[M1]**

Divides by 2 at the right time **[M1]**

$\theta = 105,\ 165$ (degrees) [One mark is lost for any extra solutions in the range. Ignore any solutions outside the range.] **[A1]**

d) i. $-\frac{4}{5}$ **[B1]**

ii. $\frac{25}{16}$ (Allow follow through.) **[B1ft]**

Question B5

a) i. $3 + 2\lambda = -2 + 3\mu$; $-2 = 4 - 2\mu$; $2 - 3\lambda = -1$ **[M1]**

From second and third equations finds values for λ $(=1)$ **(M1)** and μ $(= 3)$ **(M1)** **[M2]**

Confirms values of λ and μ do not satisfy first equation, so lines do not meet (conclusion needed) **[A1]**

ii. $3 + 2\lambda = -1$; $(-2 = -2)$; $2 - 3\lambda = 8$ and shows $\lambda = -2$ for the first and third equations. [Allow if no mention is made of the consistent $y -$ coordinate] **[M1*]**

iii. $\overrightarrow{CA} = (13\boldsymbol{i} - 14\boldsymbol{j} + 9\boldsymbol{k})$; $\overrightarrow{CB} = (21\boldsymbol{i} - 14\boldsymbol{j})$

Finds scalar product (469) **[M1]**

Finds magnitudes of $\overrightarrow{CA}$ and $\overrightarrow{CB}$ $(\sqrt{446},\ \sqrt{637})$ and applies $\cos\theta = \frac{\text{their scalar product}}{\text{product of their magnitudes}}$ **[M1]**

Anything rounding to 28.4 (degrees) or 0.5 radians **[A1]**

[Instead of finding $\overrightarrow{CB}$, the directional vector of l_2 can be used. If this happens, the scalar product is 67 and the magnitude is $\sqrt{13}$]

iv. $\frac{1}{2} \times |\text{their } CA| \times |\text{their } CB| \times \sin(\text{their } \theta)$ **[M1]**

Anything rounding to 127 or 128 (units^2) **[A1]**

v. $\frac{1}{\sqrt{446}}(13\boldsymbol{i} - 14\boldsymbol{j} + 9\boldsymbol{k})$ (Allow follow through) **[B1ft]**

b) i. $6x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ Correct Use of Product Rule ($\pm y \pm x\frac{dy}{dx}$ seen) **[M1*]**

Uses implicit differentiation ($x\frac{dy}{dx}$ or $2y\frac{dy}{dx}$ seen) **[M1*]**

Takes $\frac{dy}{dx}$ terms on to one side and factorises (this mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{y - 6x}{2y - x}$ **[A1]**

ii. Sets their $\frac{dy}{dx}$ equal to $-\frac{4}{3}$. **[M1]**

Solves **[M1]**

$y = 2x$ **[A1]**

iii. Substitutes their $y = 2x$ into the original expression and finds values for x (± 4) **[M1]**

Coordinates are $(4,\ 8)$; $(-4, -8)$ **[A1]**

Question B6

a) Uses Quotient Rule **[M1*]**

$\left(\frac{dy}{dx} =\right) \frac{2x(x+1) - (x^2 - 8)}{(x+1)^2}$ (correct answer) **[A1]**

Sets expression (or just top line) equal to 0 and forms a quadratic equation $[x^2 + 2x + 8 = 0]$ **[M1]**

Finds the discriminant $(2^2 - 4 \times 1 \times 8)$ and confirms this is negative. **[M1]**

Thus there are no real stationary values (conclusion needed) (Allow follow through provided their discriminant is negative) **[A1ft]**

b) i. $3x^2 + 9x + 7 = A(x+2)^2 + B(x+1)(x+2) + C(x+1)$ **[M1]**

$A = 1$ **(A1)** $B = 2$ **(A1)** $C = -1$ **(A1)** **[A3]**

ii. Uses previous result $\int_0^2 \frac{1}{x+1} + \frac{2}{x+2} - \frac{1}{(x+2)^2}\ dx$ **[M1*]**

Attempts to integrate (sight of a log term or reciprocal $(x+2)$ is sufficient for this mark. $[\ln(x+1) + 2\ln(x+2) + \frac{1}{x+2}]$ **[M1*]**

Combines logs or uses power law at any stage **[M1*]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$\ln 12 - \frac{1}{4}$ (must be in this form) **[A1]**

c) i. Uses integration by parts in the right direction **[M1*]**

$4x \times \frac{1}{2}e^{2x}$ **(A1)** $-\int 4 \times \frac{1}{2}e^{2x}\ dx$ (A1) for first part only **[A1]**

$2xe^{2x} - e^{2x} + c$ **[A1]**

ii. Volume $= \pi \int_0^{\ln 2} 4x\, e^{2x}\ dx$ **[M1*]**

Uses previous result (or goes through process of integration by parts again), substitutes limits into expression and subtracts the right way round (if the π is dropped, this mark is not lost). **[M1]**

[A1]

$= (8\ln 2 - 3)\pi$ or anything rounding to 8