NCUK

THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination 2017-18

Examination Session	**Time Allowed**
Semester Two	2 Hours 40 minutes (including 10 minutes reading time)

INSTRUCTIONS TO STUDENTS

SECTION A **Answer ALL questions. This section carries 45 marks.**

SECTION B **Answer 4 questions ONLY. This section carries 80 marks.**

The marks for each question are indicated in square brackets [ ].

- Answers must not be written during the first 10 minutes.
- A formula booklet and graph paper will be provided.
- An approved calculator may be used in the examination.
- Show **ALL** workings in your answer booklet.
- Examination materials must not be removed from the examination room.

DO NOT OPEN THIS QUESTION PAPER UNTIL INSTRUCTED BY THE INVIGILATOR

Section A
Answer ALL questions. This section carries 45 marks.

Question A1

Solve the equations $3x - 2y = 8$

$6x + 5y = -11$ **[4]**

Question A2

The probability that it rains on any given day is 0.7

Two consecutive days are chosen at random.

Find the probability that it either rains on both days, or rains on neither day. **[3]**

Question A3

When $3x^2 + kx - 2$ is divided by $(x + 2)$ the remainder is twice the remainder when $4x^2 - 5x - 3k$ is divided by $(x + 3)$.

Use the Remainder Theorem to find the value of k. **[4]**

Question A4

Expand $(4 - 6x)^4$. Give your answer in its simplest form. **[4]**

Question A5

Solve $7^{(4x-3)} = 33$. Give your answer to **3** significant figures.

In this question, 1 mark will be given for the correct use of significant figures. **[4]**

Question A6

Solve $\sin^2\theta = \frac{3}{4}$ $(0 \le \theta \le 2\pi)$.

Give your answers as exact multiples of π. **[3]**

Question A7

A curve has equation $y = \cos^5 x$.

Find $\frac{dy}{dx}$ and hence find the gradient of the tangent at $x = \frac{\pi}{6}$. **[4]**

Question A8

Find the values of a if

$$\int_0^a (6x - 4)\,dx = 15.$$ **[4]**

Question A9

The function $f(x)$ is defined as $f(x) = x^2 - 68$.

Starting with $x = 8$, apply the Newton-Raphson method once to obtain a better approximation to the equation $f(x) = 0$. **[3]**

Question A10

Function $g(x)$ is defined as $g(x) = \tan x \quad (0° \leq x < 90°)$.

Function $h(x)$ is defined as $h(x) = \cos 2x \quad (0° \leq x < 90°)$

Find the **exact** value of $g\left(h^{-1}\left(\frac{1}{2}\right)\right)$. **[4]**

Question A11

The masses of blocks of concrete are assumed to follow a Normal distribution with mean 120 kg and standard deviation 10 kg.

33% of blocks are above x kg.

Find the value of x. **[3]**

Question A12

Solve the differential equation

$$\frac{dy}{dx} = \frac{y}{x + 5}$$

subject to $y = 18$ when $x = 1$.

Give your answer in the form $y = f(x)$ which must contain no logarithms. **[5]**

Section B
Answer 4 questions ONLY. This section carries 80 marks.

Question B1

a) i. Solve the inequality $\frac{7x - 11}{5} > 2$ **[1]**

ii. Solve the inequality $2x^2 - 3x - 44 \leq 0$. **[4]**

iii. List the integers which satisfy both $\frac{7x - 11}{5} > 2$ and $2x^2 - 3x - 44 \leq 0$. **[1]**

b) i. Divide $3x^3 - 4x^2 - 59x + 20$ by $(3x - 1)$. **[3]**

ii. Hence factorise $3x^3 - 4x^2 - 59x + 20$ completely. **[1]**

c) An arithmetic series has common difference 11 and the 19th term is 36.

i. Find the first term. **[2]**

ii. Find the sum of the first 40 terms. **[3]**

d) A geometric series has common ratio 1.5 and the 11th term is 236196.

i. Find the first term. **[2]**

ii. Find the sum of the first 8 terms.

Give the *full* answer here, with no rounding off. **[2]**

iii. Explain why it is not possible to find the sum to infinity. **[1]**

Question B2

a) A small furnace is heated up and then allowed to cool. The temperature, θ℃, after t hours from when the furnace started to cool is given by the formula

$$\theta = 1500e^{-0.04t} + 20$$

i. State the temperature of the furnace when cooling started. **[1]**

ii. Find the temperature after 3 hours. **[2]**

iii. Find $\frac{d\theta}{dt}$. **[1]**

iv. After how many hours is the rate of cooling 47.2 ℃ per hour? **[3]**

b) Solve the equation

$$2\log_8 x - \log_8(x^2 - 3x) = \frac{2}{3} \quad (x > 3)$$

All working must be shown. **[4]**

c)

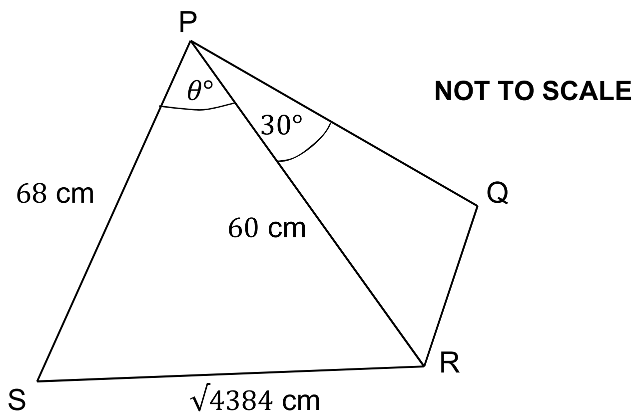

Figure 1

Figure 1 shows the quadrilateral PQRS which is made up of acute-angled triangles PQR and PRS.
PS = 68 cm, PR = 60 cm and SR = $\sqrt{4384}$ cm. Angle SPR = θ° and angle RPQ = 30°.

i. Find the value of $\cos\theta$. Give your answer in the form $\frac{m}{n}$ where m and n are integers. **[3]**

ii. Without finding the size of θ, show that $\sin\theta = \frac{15}{17}$. *Show all working*. **[2]**

The area of triangle PRS is 3 times larger than the area of triangle PQR.

iii. Find PQ. **[2]**

iv. Find angle PRS. **[2]**

Question B3

a)

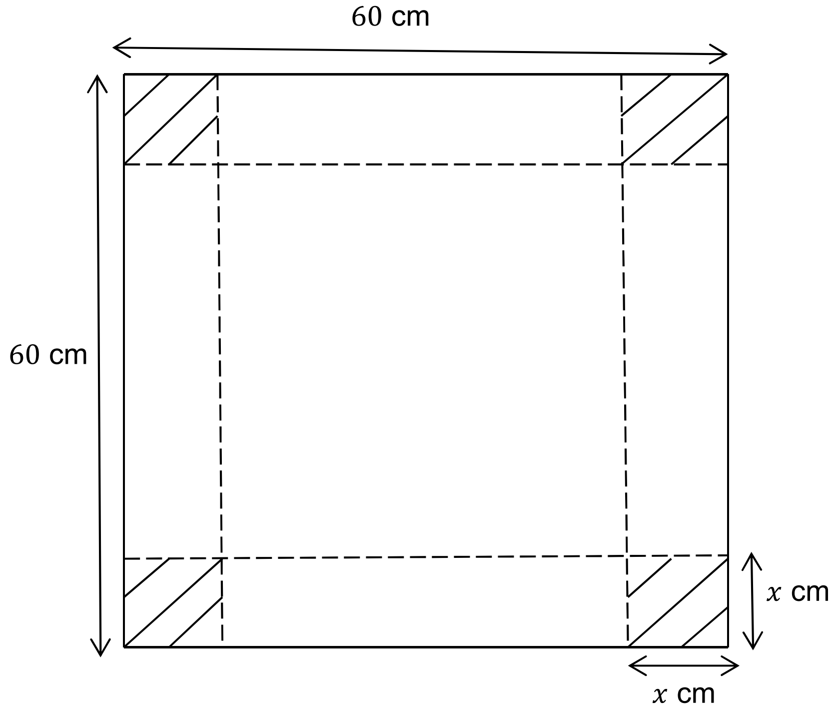

Figure 2

Figure 2 shows a square sheet of metal of side 60 cm. Four squares x cm by x cm are cut from each corner. The sides are then folded up to make a tray.

i. Show that the volume, V, of the tray is given by

$$V = 3600x - 240x^2 + 4x^3$$ **[2]**

ii. Use $\frac{dV}{dx}$ to find the stationary values of x.

All working must be shown. **[4]**

iii. For each stationary value, determine if it is a maximum or a minimum. **[4]**

All working must be shown. Your answers must be backed up by sufficient evidence.

iv. Find the maximum volume. **[2]**

Part b) is on the next page.

Question B3 – (continued)

b)

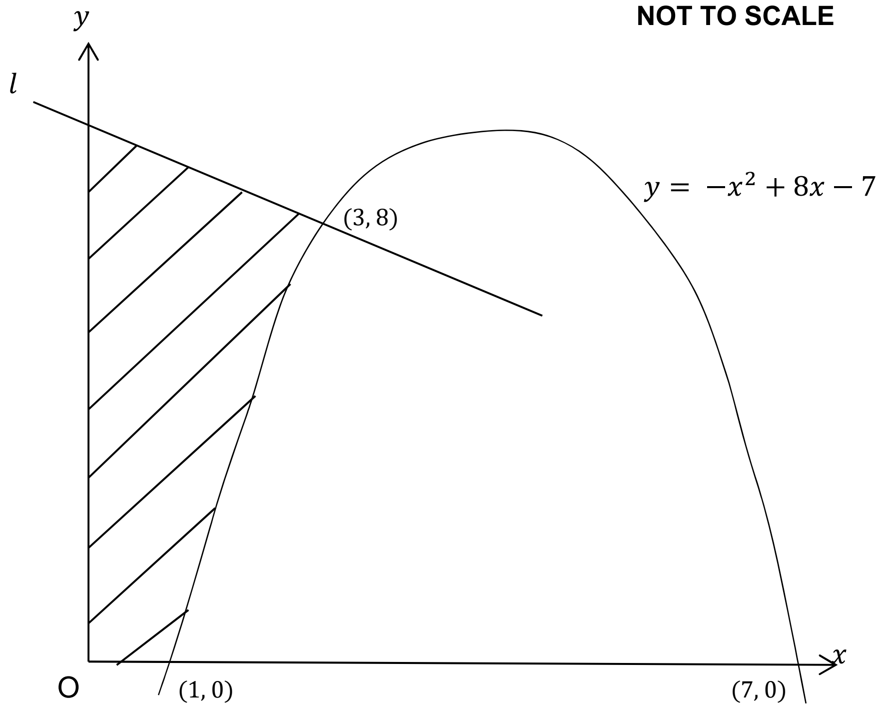

Figure 3

Figure 3 shows the curve $y = -x^2 + 8x - 7$ and line l which is a normal to the curve at the point $(3, 8)$. The curve $y = -x^2 + 8x - 7$ meets the $x -$ axis at $(1, 0)$ and $(7, 0)$.

i. Find the equation of line l. Give your answer in the form $y = mx + c$. **[3]**

ii. Find the area, which is shaded on the diagram, that is bounded by the $x -$ and $y -$ axes, line l and the curve $y = -x^2 + 8x - 7$. **[5]**

Question B4

a)

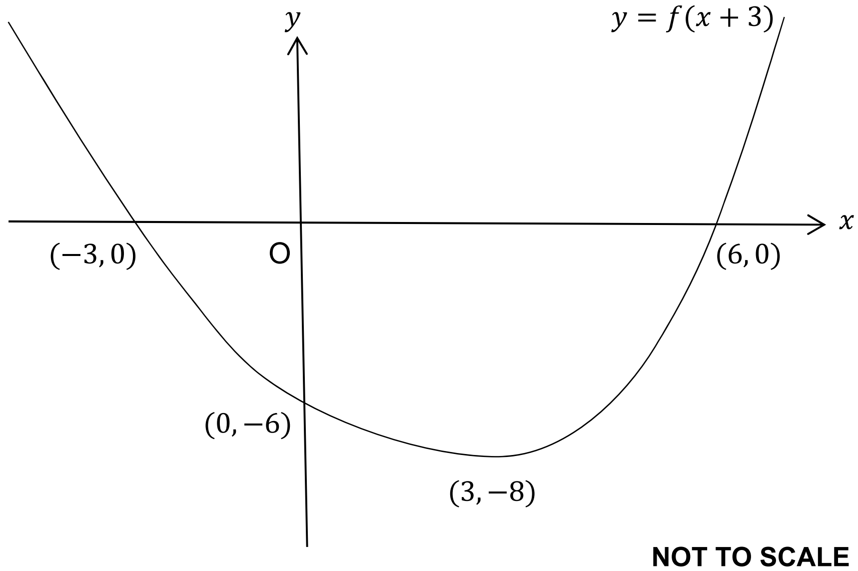

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Figure 4

Figure 4 shows the curve $y = f(x+3)$. The curve crosses the $x-$ axis at $(-3, 0)$ and $(6, 0)$; and the $y-$ axis at $(0, -6)$. There is a stationary value at $(3, -8)$.

Draw a sketch of the graph $y = f(x)$. On your sketch, show clearly the coordinates where the curve crosses the $x-$ axis and the $y-$ axis; and also the coordinates of any stationary values. **[3]**

b) Function $g(x)$ is defined as $g(x) = \frac{e^{3x} + 1}{2} \quad (x > 0)$

i. State the range of $g(x)$. **[1]**

ii. Find $g^{-1}\left(\frac{9}{2}\right)$. Give your answer in the form $\ln k$ where k is an integer. **[3]**

iii. Function $h(x)$ is defined as $h(x) = \frac{5x - 7}{12} \quad (-\infty < x < \infty)$

Solve the equation $h(x) = h^{-1}(x)$. **[3]**

Parts c) and d) are on the next page.

Question B4 – (continued)

c) i. Prove that $\tan\theta + \cot\theta = \frac{2}{\sin 2\theta}$.

<u>Each</u> stage of your working must be clearly shown. **[4]**

ii. Hence solve the equation $\tan\theta + \cot\theta = -4 \quad (0° \leq \theta \leq 180°)$ **[4]**

d) Angle A is obtuse and $\sin A = \frac{3}{5}$.

<u>Without</u> finding the size of angle A, write down each of the following, giving your answers in the form $\frac{p}{q}$ where p and q are integers.

i. $\cos A$ **[1]**

ii. $\sec^2 A$ **[1]**

Question B5

a) Line l_1 has equation $\boldsymbol{r} = (3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}) + \lambda(2\boldsymbol{i} - 3\boldsymbol{k})$ where λ is a scalar.

Line l_2 has equation $\boldsymbol{r} = (-2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}) + \mu(3\boldsymbol{i} - 2\boldsymbol{j})$ where μ is a scalar.

i. Show that lines l_1 and l_2 do not intersect. **[4]**

ii. Show that point $A(-1, -2, 8)$ lies on line l_1. **[1]**

You are given both point $B(7, -2, -1)$ and point $C(-14, 12, -1)$ lie on line l_2.

iii. Find the acute angle between lines CA and CB. **[3]**

iv. Find the area of triangle ACB. **[2]**

The vector $\overrightarrow{CA}$ is denoted by vector $\boldsymbol{u}$.

v. Find the unit vector $\boldsymbol{u}$. **[1]**

b) A curve has equation $3x^2 - xy + y^2 = 80$.

i. Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

Points P and Q lie on the curve and the gradient of the tangents to the curve at both points P and Q is $-\frac{4}{3}$.

ii. Show that at points P and Q, $y = kx$ where k is an integer. **[3]**

iii. Hence find the coordinates of points P and Q. **[2]**

Question B6

a) A curve has equation

$$y = \frac{x^2 - 8}{x + 1}$$

Use the Quotient Rule to find $\frac{dy}{dx}$ and hence show that the curve has no real stationary values. **[5]**

b) i. Write $\frac{3x^2 + 9x + 7}{(x + 1)(x + 2)^2}$ in the form $\frac{A}{x + 1} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$

where A, B and C are constants to be determined. **[4]**

ii. Hence evaluate

$$\int_0^2 \frac{3x^2 + 9x + 7}{(x + 1)(x + 2)^2} \, dx.$$

Give your answer in the form $\ln p + \frac{q}{r}$ where p, q and r are integers. **[5]**

c) i. Use integration by parts to find

$$\int 4x \, e^{2x} \, dx.$$

[3]

ii. The curve $y = 2x^{½} \, e^x$ is rotated about the $x -$ axis between $x = 0$ and $x = \ln 2$.

Find the volume formed. **[3]**

This is the end of the examination.

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