THE NCUK INTERNATIONAL FOUNDATION YEAR

IFYME002 Mathematics Engineering Examination

2017-18

MARK SCHEME

Notice to Markers

This mark scheme should be used in conjunction with the NCUK Centre Marking and Recording results policy, available from the secure area of the NCUK website (http://www.ncuk.ac.uk). Contact your Principal/ Academic Manager if you do not have login details.

Significant Figures:

All correct answers should be rewarded regardless of the number of significant figures used, with the exception of question A6. For this question, 1 discretionary mark is available which will only be awarded to students who correctly give their answer to the number of significant figures explicitly requested.

Error Carried Forward:

Whenever a question asks the student to calculate - or otherwise produce - a piece of information that is to be used later in the question, the marker should consider the possibility of error carried forward (ECF). When a student has made an error in deriving a value or other information, provided that the student correctly applies the method in subsequent parts of the question, the student should be awarded the Method marks for the part question. The student should never be awarded the Accuracy marks, unless a follow through is clearly indicated in the mark scheme. (This is denoted by A1ft or B1ft.) When this happens, write ECF next to the ticks.

M=Method (In the event of a correct answer, M marks can be implied unless the M mark is followed by * in which case, the working must be seen.)
A=Answer
B = Correct answer independent of method
If a student has answered more than the required number of questions, credit should only be given for the first *n* answers, in the order that they are written in the student's answer booklet (*n* being the number of questions required for the examination). Markers should **not** select answers based on the combination that will give the student the highest mark. If a student has crossed out an answer, it should be disregarded.

Section A

Question A1

Finds gradient of AB $(= -\frac{1}{3})$ **[M1]**

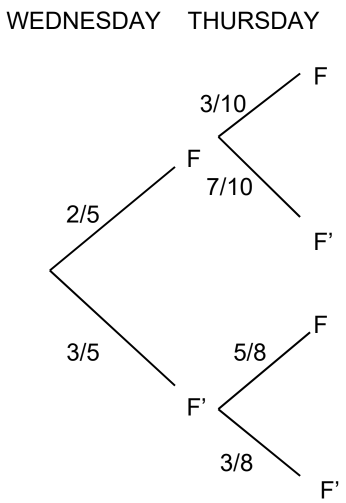
Inverts and changes sign (3) **[M1]**

Writes correct form of equation $[y - 2 =$ their gradient $(x - 7)]$ **[M1]**

$3x - y - 19 = 0$ or equivalent, but must be in this form. **[A1]**

Question A2

F denotes the event 'developing a fault'.



First set of branches correct **[B1]**

Second set of branches correct **[B1]**

Either $\frac{2}{5} \times \frac{7}{10}$ or $\frac{3}{5} \times \frac{5}{8}$ seen **[M1]**

Adds their probabilities **[M1]**

$\frac{131}{200}$ Accept equivalent fractions, decimals or percentages. **[A1]**

Question A3

Solves $3x^2 - 4x - 15 = 0$
$[(3x + 5)(x - 3) = 0 \text{ or } x = \frac{4 \;\pm\sqrt{[(-4)^2 - 4 \times 3 \times -15]}}{2 \times 3}]$ **[M1]**

Finds two critical values $(-\frac{5}{3}, 3)$ **[M1]**

$x < -\frac{5}{3}$ **(A1)** $x > 3$ **(A1)** **[A2]**
Please note: the two ranges can be separated by a space, a comma or the word 'or'. The final mark is lost if the word 'and' is seen.

Question A4

$(a = 8,\ r = 1.5)$ $\quad \dfrac{\text{their } a(\text{their } r^n - 1)}{\text{their } r - 1} = 15000$ **[M1]**

Rearranges and reaches $r^n = \cdots (938.5)$ **[M1]**

Takes logs correctly and obtains a value for n $(16.88 \ldots)$ **[M1]**

17 terms needed **[A1]**

Question A5

$\log_2\left[\dfrac{x(x+4)}{(x-4)(x+4)}\right] = \log_2 4$

Combines logs correctly on LHS **[M1*]**

Adapts RHS and removes logs at the right time **[M1*]**

Solves $\left(\dfrac{x}{x-4} = 4\right)$ **[M1]**

$x = \frac{16}{3}$ or equivalent, or anything rounding to 5.33 **[A1]**

[If the factorising and/or cancelling does not take place, the third M mark can be scored for forming a quadratic equation $(3x^2 - 4x - 64 = 0)$ and solving $(x = \frac{16}{3}, -4)$. If the -4 is not discarded (placing it in brackets is sufficient to indicate non-inclusion), the A mark is then lost].

Question A6

Uses cosine formula $(23^2 = 20^2 + 25^2 - 2 \times 20 \times 25 \times \cos Q$ or equivalent) **[M1]**

Calculates correctly and in right order **[M1]**

$= 1.05181 \ldots$ (can be implied) **[A1]**

$= 1.05$ to 3 significant figures. (Allow follow through) **[A1ft]**

Special case: if 60.3 (degrees) is quoted, allow 3 marks out of 4 (even if a more accurate answer is not seen earlier.)

Question A7

Attempts to differentiate [sight of the x term is sufficient for this mark $(6x - 10)$] **[M1*]**

Substitutes $x = 2$ into their $\frac{dy}{dx}$ **[M1]**

$y + 1 = 2(x - 2)$ or equivalent **[A1]**

Question A8

Finds $f(\ln 3) \quad (= 4)$ **[M1]**

Applies g to their $f(\ln 3)$ **[M1]**

$= 16$ **[A1]**

Question A9

$2 \sin A \cos A (\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A})$ **[M1*]**

$= 2\cos^2 A - 2\sin^2 A = 2(\cos^2 A - \sin^2 A)$ (At least one of these steps must be seen) **[M1*]**

$= 2\cos 2A$ (Both M marks scored and no errors seen) [Accept any other valid methods as long as the working is seen] **[A1]**

Question A10

Uses integration by parts in the right direction. **[M1*]**

$= 8x \times \frac{1}{4}\sin 4x$ **(A1)** $- \underline{\int (8 \times \frac{1}{4}\sin 4x) \; dx}$ [A1 for first part] **[A1]**

$= [\, 2x \sin 4x + \underline{\frac{1}{2}\cos 4x}]$ **(A1ft)** [This ft mark is for carrying out the second integration (relevant parts underlined)] **[A1ft]**

Substitutes limits into their integrated expression and subtracts the right way round. **[M1]**

$= \frac{\pi}{4} - \frac{1}{2}$ or anything rounding to 0.285 **[A1]**

Question A11

$24.9 \pm \frac{1.96 \times 2}{\sqrt{10}}$ **[M1]**

Anything rounding to 23.7 **(A1)** and 26.1 **(A1)** **[A2]**

Question A12

Please note: evidence of the trapezium rule must be seen in the answer.

Area $\approx \frac{0.25}{2}$ **(M1*)** $[(0.269 + 0.119) + 2(0.223 + 0.182 + 0.148)]$ **(M1*)** **[M2*]**

$=$ anything rounding to 0.187 **[A1]**

Section B

Question B1

a)

$x = 5 - 2y$ or $y = \frac{5\ -\ x}{2}$ **[M1*]**

Substitutes into second equation **[M1*]**

Reaches a quadratic equation $(2y^2 - 5y - 12 = 0$ or $x^2 - 5x - 24 = 0)$ **[M1*]**

Factorises or uses formula **[M1]**

$[(2y+3)(y-4) = 0$ or $y = \frac{5\ \pm\sqrt{[(-5)^2\ -\ 4\ \times\ 2\ \times -12]}}{2\ \times\ 2}]$

or $[(x-8)(x+3) = 0$ or $x = \frac{5\ \pm[(-5)^2\ -\ 4\times\ 1\ \times -24]}{2\ \times\ 1}]$

Substitutes their x values into original equation to find y or vice versa. **[M1]**

Solutions are $(-3,\ 4)$ and $(8, -\frac{3}{2})$ The solutions do not have to be written in coordinate form, but it must be clear that they have been matched up correctly. **[A1]**

b) i. Substitutes $x = -4$ into expression **[M1*]**

$6(-4)^3 + 17(-4)^2 - 31(-4) - 12 = 0$ [Some evidence is needed – it is not enough to just state '$= 0$'] **[A1]**

ii.

$$
\begin{array}{r|l}
 & 6x^2 - 7x - 3 \\
\hline
x + 4 & 6x^3 + 17x^2 - 31x - 12 \\
 & \underline{6x^3 + 24x^2} \\
 & \quad -7x^2 - 31x \\
 & \quad \underline{-7x^2 - 28x} \\
 & \qquad\quad -3x - 12 \\
 & \qquad\quad \underline{-3x - 12} \\
 & \qquad\quad \underline{\ldots\ldots\ldots}
\end{array}
$$

correct first division **[M1]**

any correct subsequent division **[M1]**

correct quotient **[A1]**

Parts c) and d) are on the next page.

Question B1 – (continued)

c) i. $44 = 8 + (n-1) \times 2$ **[M1]**

$n = 19$ or 19th day or 19 June, etc. **[A1]**

ii. $S_{30} = \frac{30}{2}[2 \times 8 + (30-1) \times 2]$ **[M1]**

Calculates correctly in the right order **[M1]**

$= 1110$ so not enough in stock [1110 must be seen, along with a conclusion]. **[A1]**

d) $^8C_2 \times 2^6 \times p^2$ **(M1*)** $^8C_3 \times 2^5 \times p^3$ **(M1*)** [Accept ^xC_y for ^yC_x and the presence of x] **[M2*]**

Multiplies second expression by 4 or divides first expression by 4 and sets equal to each other [there must now be no x present] and finds a value for p. **[M1]**

$p = \frac{1}{4}$ **[A1]**

Question B2

a) i. As a minimum, the candidate must set $t = 0$ and explain or clearly imply that $4096^0 = 1$. A conclusion is not needed. **[M1*]**

ii. Substitutes into formula and reaches $4096^{6k} = \cdots\left(\frac{80\ -\ 48}{0.5}\right)$ **[M1]**

Takes logs correctly and reaches $6k = \cdots\left(\frac{\log 64}{\log 4096}\right)$ **[M1]**

$k = \frac{1}{12}$ (must be in this form) **[A1]**

iii. Substitutes $t = 4$ into formula with their value of k **[M1]**

$= 56$ (Allow follow through for their k) **[A1ft]**

b) Recognises the 'hidden quadratic' equation **[M1]**

Factorises or uses formula **[M1]**
$[(\ln x + 5)(\ln x - 2) = 0$ or $\ln x = \frac{-3\ \pm\sqrt{[3^2\ -\ 4\ \times\ 1\ \times -10]}}{2\ \times\ 1}$

Finds two values of $\ln x\ (=\ -5,\ 2)$ (this can be implied) **[M1]**

$x =\ e^2,\ e^{-5}$ or anything rounding to 7.39 and 0.00674 **[A1]**

c) $2\left(x - \frac{\pi}{3}\right) = \frac{\pi}{6},\ \frac{7\pi}{6},\ \frac{13\pi}{6},\ \frac{19\pi}{6}$ (any one correct, which does not have to be in the range.) **[M1]**

Recognises that the search is in the range 0 to 4π **[M1]**

Divides by 2 at the right time. **[M1]**

$x = \frac{5\pi}{12},\ \frac{11\pi}{12},\ \frac{17\pi}{12},\ \frac{23\pi}{12}$. **(A1)** for any two correct; **(A2)** for all correct. Ignore any solutions outside the range. One mark is lost for any extra solutions in the range. **[A2]**

d) i. $\frac{1}{2} \times (5a - 2) \times 16\ \times \sin 60 = 72\sqrt{3}$ **[M1]**

Solves **[M1]**

$a = 4$ **[A1]**

ii. Uses the sine formula $\left(\frac{\sin B}{16} = \frac{\sin 53}{15}\right)$ **[M1]**

Angle B = anything rounding to 58.4 (degrees) or 1.02 radians **[A1]**

Question B3

a) i. Attempts to differentiate (sight of x^3 or x^2 is sufficient for this mark) **[M1*]**

$\frac{dy}{dx} = 12x^3 - 24x^2$ **[A1]**

ii. Substitutes $x = 0$ and $x = 2$ into their $\frac{dy}{dx}$ **[M1*]**

Shows $\frac{dy}{dx} = 0$ on both occasions (No follow through here) **[A1]**

or sets their $\frac{dy}{dx}$ equal to 0 and factorises $[12x^2(x-2) = 0]$ **(M1*)**

$x = 0,\ 2.$ **(A1)**

iii. Attempts to find $\frac{d^2y}{dx^2}$ (sight of x^2 or x is sufficient for this mark) **[M1*]**

$\frac{d^2y}{dx^2} = 36x^2 - 48x$ **[A1]**

This is 0 and changes sign when $x = 0$ so there is a point of inflexion. (the phrase about changing sign must be seen but it does not need to be shown) (Allow follow through provided there is a point of inflexion) **[A1ft]**

This is positive when $x = 2$ so there is a minimum
(Allow follow through for their $\frac{d^2y}{dx^2}$) **[A1ft]**

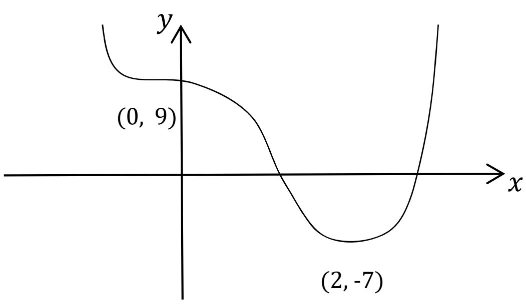
or Takes a numerical value less than 0 and shows $\frac{dy}{dx} < 0$ **(M1*)**

Takes a numerical value between 0 and 2, and shows $\frac{dy}{dx} < 0$ **(M1*)**

Takes a numerical value more than 2 and shows $\frac{dy}{dx} > 0$ **(M1*)**

Thus there is a point of inflexion at $x = 0$ and a minimum at $x = 2$.
(Allow follow through on their $\frac{dy}{dx}$ provided there is a POI at $x = 0$.)
(A1ft)

iv.



Correct shape (must have POI and min.) **[B1]**

$(0,\ 9)$ shown at point of inflexion **[B1]**

$(2,\ \text{-}7)$ shown at minimum **[B1]**

Part b) is on the next page.

Question B3 – (continued)

b) i. Multiplies out $[9-\frac{12}{x}+\frac{4}{x^2}]$ **[M1*]**

Attempts to integrate (sight of $x, \ln x$ or reciprocal x is sufficient for this mark) **[M1*]**

$9x-12\ln x-\frac{4}{x}+c$ or equivalent **[A2]**

(A1) for any two terms correct; **(A2)** for all correct and $+c$

ii. Area = $\int_0^2 (x^3+4)\,dx,\quad \int_2^4 (16-x^2)\,dx$ (Limits must be correct) **[M1]**

Attempts to integrate both parts (index of x raised by 1 or an x term in each part is sufficient evidence) $[\frac{1}{4}x^4+4x,\quad 16x-\frac{1}{3}x^3]$ **[M1*]**

Substitutes limits into both integrated expressions and subtracts the right way round $(12-0$ and $42\frac{2}{3}-29\frac{1}{3})$ **[M1]**

Adds areas **[M1]**

$=25\frac{1}{3}$ or equivalent or anything rounding to 25.3 **[A1]**

Or

Area $=\int_0^4 (16-x^2)\,dx,\quad \int_0^2 [(16-x^2-(x^3+4)]\,dx$ (Limits must be correct) **(M1)**

Attempts to integrate both parts (index of x raised by 1 or an x term in each part is sufficient evidence) $[\,16x-\frac{1}{3}x^3,\ 12x-\frac{1}{3}x^3-\frac{1}{4}x^4\,]$ **(M1*)**

Substitutes limits into both integrated expressions and subtracts the right way round $(42\frac{2}{3}-0$ and $17\frac{1}{3}-0\,)$ **(M1)**

Subtracts areas **(M1)**

$=25\frac{1}{3}$ or equivalent or anything rounding to 25.3 **(A1)**

Question B4

a) i. $f(x) > 1$ (Allow $y > 1$ but not $x > 1$) **[B1]**

ii. Makes x the subject of $e^{2x} + 1$ and exchanges x and y at some stage. **[M1]**

$[f^{-1}(x) =]\ \frac{1}{2}\ln(x-1)$ or equivalent **[A1]**

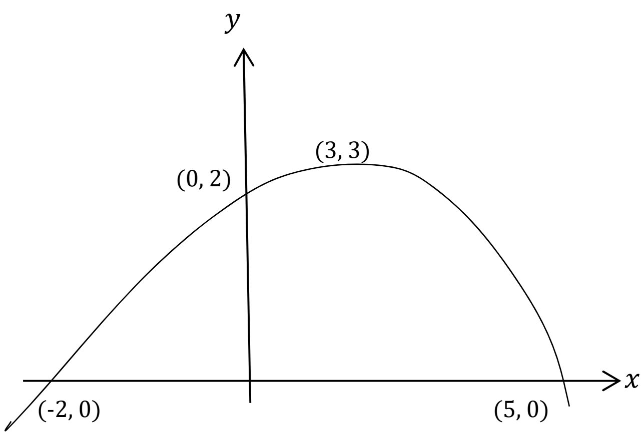
iii. $x > 1$ (Allow follow through from their answer to part i) **[B1ft]**

iv. $g^{-1}(x) = \frac{3x+1}{5}$ **[M1]**

Sets $g(x) = \text{their } g^{-1}(x)$ and solves **[M1]**

$x = \frac{1}{2}$ or equivalent **[A1]**

b) i.

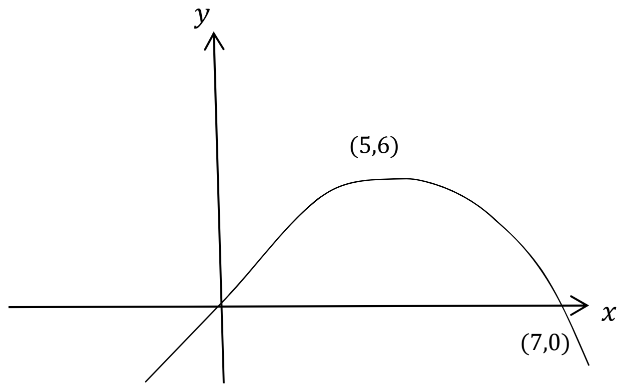


Correct shape **[B1]**

(-2,0), (5,0) **[B1]**

(0,2), (3,3) **[B1]**

ii.



Correct shape **[B1]**

(7,0), (0,0)* [*(0,0) does not need to be shown if the curve clearly goes through the origin] **[B1]**

(5,6) **[B1]**

Part c) is on the next page.

Question B4 – (continued)

c) i. $\tan(\theta + \theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$ **[M1*]**

$= \frac{2\tan\theta}{1 - \tan^2\theta}$ (M mark scored and no errors seen) **[A1]**

ii. Uses previous result and writes an equation in $\tan\theta$ **[M1*]**

$(\frac{2\tan\theta}{1 - \tan^2\theta} + \tan\theta = 0)$

Multiplies through by $(1 - \tan^2\theta)$ and reaches $\tan\theta(3 - \tan^2\theta) = 0$ **[M1]**

or takes out $\tan\theta$ and reaches $\tan\theta(\frac{3 - \tan^2\theta}{1 - \tan^2\theta}) = 0$.

$\tan\theta = 0$ or $(\pm)\sqrt{3}$ (This mark is not lost if the $\pm$ is missed.) **[M1]**

$\theta = 0,\ 60,\ 120,\ 180$ (degrees)

(A1) for any two correct; **(A2)** for all correct. [One mark is lost for any extra solutions in the range; ignore any solutions outside the range.] **[A2]**

Question B5

a) i. Sets $5 - 3\mu = 2$; $-2 + 4\mu = 2$; $4 - \mu = 3$ **[M1*]**

Confirms $\mu = 1$ for all three. **[A1]**

ii. Finds $\overrightarrow{CA}.\overrightarrow{CB}$ $[(-8\boldsymbol{i} + 18\boldsymbol{j} - 8\boldsymbol{k}).(4\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}) = 36]$ **[M1]**

Finds magnitudes of $\overrightarrow{CA}$ and $\overrightarrow{CB}$ $(\sqrt{452},\ 6)$ and applies **[M1]**

$\cos\theta = \frac{\text{their scalar product}}{\text{product of their magnitudes}}$

$\theta =$ anything rounding to 73.6 (degrees) or 1.28 radians **[A1]**

iii. $\frac{1}{2}$ × their $\overrightarrow{CA}$ × their $\overrightarrow{CB}$ × their $\sin\theta$ **[M1]**

Anything rounding to 61.2 (Allow follow through) **[A1ft]**

iv. $(4 \times -3 + 2 \times 4 + -4 \times -1) = 0$ (No conclusion needed.) **[M1*]**

v. Finds magnitude of AB $(= \sqrt{416})$ **or** of directional vector $(\sqrt{26})$ **[M1]**

$\frac{1}{\sqrt{416}}(12\boldsymbol{i} - 16\boldsymbol{j} + 4\boldsymbol{k})$ (or equivalent) **or** $\frac{1}{\sqrt{26}}(-3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k})$ **[A1]**

vi. *Please note: just showing the result is not enough here*.
Triangle ACB is right-angled (at angle B) **[M1*]**

By Pythagoras, $AB^2 + BC^2 = AC^2$ [Reference to Pythagoras must be seen]. **[A1]**

b) i. $8x - 2y - 2x\frac{dy}{dx} + 3\frac{dy}{dx} = 0$

Use of Product Rule (sight of $\pm 2y \pm 2x\frac{dy}{dx}$ is sufficient for this mark) **[M1*]**

Correct implicit differentiation (sight of $x\frac{dy}{dx}$ or $3\frac{dy}{dx}$ is sufficient) **[M1*]**

Takes $\frac{dy}{dx}$ terms on to one side and factorises (this mark is available only if there are at least two $\frac{dy}{dx}$ terms) **[M1]**

$\frac{dy}{dx} = \frac{2y - 8x}{3 - 2x}$ **[A1]**

ii. Substitutes $x = 2,\ y = 7$ into their $\frac{dy}{dx}$ $(= 2)$ **[M1]**

Inverts and changes sign $(= -\frac{1}{2})$ **[M1]**

$y - 7 = -\frac{1}{2}(x - 2)$ or equivalent [Allow follow through] **[A1ft]**

iii. $k = 4$ **[B1]**

Question B6

a) Uses Quotient Rule $[(\frac{dy}{dx}=)\ \frac{2x(x+1)-(x^2+1)}{(x+1)^2}]$ **[M1*]**

Sets equal to $\frac{7}{8}$ and forms a quadratic equation $(x^2+2x-15=0)$ **[M1]**

Factorises or uses formula $[(x+5)(x-3)=0 \text{ or } x=\frac{-2\ \pm\sqrt{[2^2-\ 4\times 1\times -15]}}{2\times 1}]$ **[M1]**

Finds at least one value of x and substitutes into original equation to find a value for y. **[M1]**

$\left(-5,-\frac{13}{2}\right);\ \ (3,\ \frac{5}{2})$ **[A1]**

b) i. $1=A(x+3)+B(x+2)$ **[M1]**

$A=1$ **(A1)** $B=-1$ **(A1)** **[A2]**

ii. Separates the variables $[\frac{1}{y}\ dy=\frac{dx}{(x+2)(x+3)}]$ **[M1*]**

Integrates both sides and adds a constant to one side (if the constant is missed, this mark and all subsequent marks are lost) **[M1*]**
$[\ln y=\ln(x+2)-\ln(x+3)+c\ ]$

Substitutes boundary conditions into their expression and finds a value for c ($=\ln 2$ if on RHS) **[M1]**

Correctly combines logs at any stage **[M1]**

$y=\frac{2(x+2)}{x+3}$ (Must be in this form with no logs) **[A1]**

c) i. $du=2x+4 \text{ or } 2(x+2)\ dx$ **[M1*]**

Writes integral in terms of u $(\int\frac{1}{2}\ u^4\ \ du\ )$ **[M1*]**

Attempts to integrate (sight of u^5 is sufficient) $[\frac{1}{10}u^5\ (+c)]$ and writes answer in terms of x **[M1]**

[A1]

$\frac{1}{10}(x^2+4x-3)^5+c$

ii. Volume $=\ \pi\int_0^1(x+2)(x^2+4x-3)^4\ \ dx$ **[M1*]**

Uses previous result (or any other valid method), substitutes in limits and subtracts the right way round (this mark is not lost if the π is dropped) **[M1*]**

[A1]

$=\frac{55}{2}\pi$ or anything rounding to 86.4

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